Complementary Goods: Creating, Capturing, and Competing for Value*

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Abstract

This paper studies the strategic interaction between firms producing strictly complementary products. With strict complements, a consumer derives positive utility only when both products are used together. We show that value-capture and value-creation problems arise when such products are developed and sold by separate firms ("non-integrated" producers). Although the firms tend to price higher for given quality levels, their provision of quality is so low that in equilibrium prices are set well below what an integrated monopolist would choose. When one firm can mandate a royalty fee from the complementor producer (as often occurs in arrangements between hardware and software makers), we find that the value-capture problem is mitigated to some extent and consumer surplus rises. However, because royalty fees greatly reduce the incentives of the firm paying them to invest in quality, the arrangement exacerbates the value-creation problem and leads to even lower total quality. Surprisingly, this result can reverse with competition. Specifically, when the firm charging the royalty fee faces a vertically differentiated competitor, the value-creation problem is greatly reduced— opening the door for the possibility of a Pareto improving outcome in which all firms and consumers are better off. Notably, this outcome cannot be achieved by giving firms the option of introducing a line of product variants; competition serves as a necessary "commitment" ingredient.

Keywords: *Complementary Goods, Product Quality, Royalty Fees, Competition, Game-Theory*

1 Introduction

In a number of prominent markets, consumers have to purchase and use multiple products simultaneously to derive positive utility. The goods involved in consumption are, therefore, highly complementary and value is derived from their joint consumption. The complex technology and know-how involved in developing each of the products can require specialized organizational skills. Thus, separate firms often produce each type of good.

There are several noteworthy examples of such an interaction. In the emerging category of smart phones, one firm typically designs and produces the device and operating platform while other firms create applications for it (as is the case with the iPhone). The video game industry is another example that embodies characteristics of strictly complementary goods: a game title such as Guitar Hero has no use without a gaming platform and a game console and has no use without titles.[1] In the case of computers, one firm produces the central processor while another produces the operating system, and with electric instruments, one firm typically focuses on the instrument itself (say the synthesizer) while another firm focuses on the amplifying equipment.

Because of the joint consumption characteristic, there is, in many instances, a quality interdependence among the goods produced: the utility consumers derive from one product depends not only on that product's quality but also on the quality of the complementary good. For example, a more advanced operating system delivers much better performance when the microprocessor is capable of handling the operating system's increased code complexity. But improving quality requires costly upfront research and development (R&D) investments that can complicate matters, as each producer relies on its counterpart's efforts. Indeed, given the need for the complementary products to work together, they are typically designed sequentially, with the second product developed according to specifications set by the first one. For instance, decisions regarding hardware architecture are usually made before

[1] We acknowledge that in hardware-software settings other factors can be relevant, for example, network effects and the ability to use one of the products without the other. We discuss the former issue in Section 2 (Related Literature) and the latter in Section 6 in connection with an extension we studied.

software code is written.

The fact that consumers need to purchase both goods has strategic implications for the firms involved. Specifically, if we view the revenue pie as consisting of the total amount consumers spend on the two complementary products, the question arises as to how this pie is split. With video games, for example, total industry revenue in the United States reached $24.3 billion in 2011 (NPD Group, 2012) from the sale of consoles, games, and accessories. The desire to capture a greater share of this soaring revenue stream generates a pricing tension between console makers and game publishers. Each would like to price higher but an increase by both could make the total price the consumer pays too high. Furthermore, there may be competition in one of the markets, such as when more than one hardware platform can run the software application or game title,[2] resulting in pressure to lower prices to appeal to consumers.

Given these challenges to capturing as much value as possible, utilizing royalty fees is a prominent feature in many complementary product markets. Game console producers charge a royalty fee for the right to publish games for their consoles, and a similar arrangement exists for applications sold separately for smart phones. As one might expect, the firm that levies the royalty fee attempts to appropriate a portion of the value otherwise captured by the complementary producer. But whether the firm charging such a fee is vastly better off depends on how the royalty arrangement impacts firms' actions, particularly the incentives to create value through quality level choices.

In this paper, we study the strategic interaction among firms producing highly complementary products and focus on the following research questions:

- How do firms make pricing and quality investment decisions in light of the joint consumption of their products?

- How does a royalty structure affect the interaction between firms in this context? Does

[2] For example both Sony's PlayStation 3 and Microsoft's Xbox 360, two competing video game consoles, run Guitar Hero.

it lead to a larger or smaller incentive to invest in quality?

- What impact does competition have on each of the complementary firms? Should we necessarily expect the payoff of a firm facing a direct competitor to decrease?

To answer these questions, we construct a stylized model in which two firms develop complementary products and one firm must design its product (product A, the "hardware") before the complementary product (product B, the "software") can be designed. The analysis reveals that, for given quality levels, the joint consumption but separate production and sale of complementary goods yields an incentive for firms to price higher as compared to the benchmark case of an integrated firm that produces and sells both goods. This is due to the fact that the prices of complementary products sold by separate firms form strategic substitutes, i.e., if one firm tries to cut its price to stimulate demand, the other firm has an incentive to increase its price. Yet higher prices by both producers leads to limited demand—hence the firms are not capturing as much value as they possibly can.

We further find that each producer wants to shirk on quality and let the other producer carry most of the quality-provision burden. But because the qualities of complementary products form strategic complements, if one firm decreases quality to save on development costs, the other firm's best response is to decrease the quality of its product. Consequently, there is a value-creation problem. In fact, the resulting product qualities are so low that the firms end up charging prices that are well below those charged by an integrated firm.

We then show that the ability of one firm to mandate a royalty fee from the complementor firm has advantages and disadvantages. It enables the firm receiving the royalty payments to capture a larger slice of the revenue pie, but shrinks the size of that pie. Specifically, a royalty arrangement exacerbates the value-creation problem because it causes the second mover to invest even less in product development, thus leading to even lower quality of the composite complement pair.

Interestingly, we find that the presence of a direct competitor can mitigate the value-creation problem and result in a Pareto improvement for all firms and consumers. Specifically, when the first mover A firm faces competition from a vertically differentiated product, prices in that market tend to plummet. But because consumers only care about the total price they pay, this allows the B firm to charge relatively high prices and make considerable profits. This, in turn, provides the B firm a strong incentive to select a higher quality level. The result is a greater overall level of quality for the complement pair and an increase in the size of the revenue pie. For the first mover that faces competition, the benefit of getting a piece of a larger pie (through the royalty arrangement) outweighs the fact that competition causes it to have to reduce its price. Consequently, this leads to a win-win-win-win outcome in which all the firms in the market and consumers are better off. Notably, the role of competition in yielding this outcome cannot be overstated. In particular, if the A firm were given the option to introduce a lower-quality product variant, it could not credibly commit to the same pricing levels as under competition, and the B firm would not invest sufficiently in quality. The A firm is thus strictly better off when a competitor introduces the lower-quality product.

The strategic interaction between firms producing substitute products has been well researched (Desai, 2001; Schmidt-Mohr and Villas-Boas, 2008). The incentives of such firms are naturally conflicting. One could intuitively suggest that the incentives of firms producing strictly complementary products should be more closely aligned. Our work reveals that this is not necessarily the case, because with product complements there exists a double moral hazard problem involving costly investment in quality by both firms. Thus, producers of strict complements have conflicting incentives in their efforts to create and capture value, and prescriptions for dealing with these conflicts may be counterintuitive. In particular, we show that mitigating the value-creation problem for complementary products can sometimes be achieved by encouraging competitors to enter the market.

The rest of the paper is organized as follows. In the next section we relate our work to the extant literature. Section 3 presents the model setup. Section 4 first solves the benchmark case of an integrated firm (a single firm that produces both complementary products) and then analyzes the strategic interaction between two non-integrated producers with and without a royalty fee structure. Section 5 introduces competition in the first mover's market. Section 6 discusses several model extensions and Section 7 concludes and provides managerial implications. All proofs have been relegated to the Appendix.

2 Related Literature

One of the first analyses of the interaction between producers of complementary products was by Cournot (1838). He modeled two firms that produce complementary goods (zinc and copper) that are combined to make a composite product (brass). He showed that both firms share profits equally regardless of differences in marginal costs. The division of profits between complement producers has received new interest recently with a stream of literature looking at "one-way" complements, whereby one of the products (A) has value for consumers by itself while the other (B) is useless without the first one. That makes one of the products "essential" and its value can be enhanced by the "non-essential" product. Cheng and Nahm (2007), for example, examined how prices are influenced by the value of the essential good (A) relative to the value of the bundle (AB). Chen and Nalebuff (2006) explored how the firm producing the essential product can appropriate some of the value from the non-essential B product by imposing royalty fees or by introducing its own B product. However, in all of these papers, product qualities are exogenous. By contrast, we consider the case in which quality levels are endogenous and introduce the possibility of competition in the A product market.

Economides (1999) examined the quality decisions of two-way-complement firms. In his model, the composite product's quality is equal to the minimum quality of the two products. As he notes, this approach is appropriate for long-distance telecom services, which require the use of a long-distance line as well as local lines at the two terminating points. In this case, the final sound quality will be the minimum of the qualities of the different services used. In our model, product qualities are supermodular: the impact on the composite product's quality from an incremental increase in the quality of one component depends on the absolute quality level of the other component. This approach better suits the hardware-software complementary product pairs that we have in mind.

Farrell and Katz (2000) and Casadesus-Masanell et al. (2007) considered competition with strict complementarities. In Farrell and Katz (2000) one of the complements (A) is monopolized and the other (B) is supplied by a competitive sector. They found that the monopolist may want to supply its own version of B and destroy the incentives to innovate in the competitive B sector. However, the quality of the monopolized complementary product is fixed. We, on the other hand, look at the incentives to innovate in both markets. Casadesus-Masanell et al. (2007) showed that when there are two firms in the A market and a single B firm, the lower-quality A firm cannot have positive sales. We show that allowing the B firm to discriminate in prices and enabling royalty mechanisms can result in all three firms having positive demand at positive prices. Importantly, we characterize conditions under which the firms make greater profits in the presence of competition than in its absence—effectively leading to a Pareto improvement.

Our model has an indirect network externalities flavor in the spirit of Chou and Shy (1990). Economides and Viard (2010) establish an equivalence between a model that has a base (essential) and a complementary (non-essential) good and a model that has a base good and a reduced form of network externalities. In this framework, they consider quality improvement decisions. Our analysis differs in that we investigate the interplay between royalty arrangements and competition and in that our model has two-way complementarity. We acknowledge that in some of our motivating examples (for example, video games) there are aspects of direct network effects in addition to strict complementarity issues; the latter of course being the focus of our paper.

Lastly, our paper is loosely related to the literature on product bundling (McAfee et al., 1989; Venkatesh and Kamakura, 2003), since in the integrated case (which we solve as a benchmark) a single firm can sell the complementary products as a bundle. Our work is different because we examine perfect complements and hence the value (reservation price) is zero for each product by itself. By contrast, the bundling literature typically deals with goods that have value when consumed separately. Furthermore, we focus on the strategic case in which the products are sold by separate profit-maximizing firms and on potential monetary arrangements between them (royalty fees).

3 Model Setup

Products Consider a market with two strictly complementary products, A and B. Consumers derive positive utility only if they use both products together as a composite good that we denote as AB. The quality of the composite good is modeled as a multiplication of the quality levels of each component; thus capturing the notion of strict complementarity. More formally, the quality of the composite product, q, is given by $q = \alpha\beta$ where α is the quality of product A and β is the quality of product B.[3]

With this specification we have, $\frac{\partial q}{\partial \alpha} = \beta$ and $\frac{\partial q}{\partial \beta} = \alpha$, which implies that the impact on the composite quality of increasing one's own product quality level is greater when the complementing product is of higher quality. For example, a computer with a 64-bit processor is more powerful when paired with an x64 edition of the operating system because that version can better use the capabilities of the advanced processor.

Consumers We assume that the market is composed of a unit mass of consumers who have the same preference ordering of (potential) composite products that are offered at the same price. All consumers prefer higher quality over lower quality, but they are heterogeneous in their willingness to pay for quality. The marginal valuation of quality, θ, is distributed uniformly on $[0, 1]$. The utility a consumer derives from buying product A and product B

[3] "Quality" can be thought of simply as an attribute (or collection of attributes) that consumers always prefer more of for the same price.

with quality levels α and β at prices p_A and p_B is equal to $U = \theta\alpha\beta - p_A - p_B$. A consumer buys a product pair if her valuation is greater than the sum of the prices, p_A and p_B (i.e., $U_i \geqslant 0$). Thus, the indifferent consumer has the taste parameter $\widehat{\theta} = \frac{p_A + p_B}{\alpha\beta}$. All consumers of types $\theta \in \left[\widehat{\theta}, 1\right]$ will purchase both products. Demand, which is equivalent for both products in a given complementary pair, is thus equal to $1 - \widehat{\theta}$. Note that in this set-up, as long as prices are positive, it will always be the case that some consumers with valuations close to zero will not be served. Said differently, in all the equilibria we characterize throughout the paper the market will be only partially covered.

Cost Structure Firms incur costs to develop products. As one would expect, it is increasingly more costly to deliver greater quality levels.[4] To capture this in the model, we assume that the cost function is increasing and convex in the quality level selected. Specifically, the cost functions for developing products with quality levels α and β are $c(\alpha) = \frac{1}{n}k_A\alpha^n$ and $c(\beta) = \frac{1}{n}k_B\beta^n$, respectively, where k_A and k_B are development cost parameters. For mathematical tractability, we solve for the case of $n = 3$.[5] Since our objective is to understand the strategic interaction between firms' quality choices, we wish to avoid outcomes that are generated merely by asymmetries in development costs. Hence, we assume that $k_A = k_B = k$ for the analysis presented in the paper.[6] Variable production costs are assumed to be constant and are normalized to zero.[7]

4 Model Analysis

We start with the analysis of a case in which both complementary goods (A and B) are produced by a single profit-maximizing firm. This benchmark case will prove useful in

[4]Given our motivating examples, this is a plausible assumption. For instance, in the video game industry, upfront R&D costs are large. The latest generation of consoles required an investment on the order of billions of dollars and the budget for a typical video game ranged from \$10 to \$35 million (Ofek, 2008).

[5]Our analysis applies whenever the cost function is sufficiently convex (the results hold for any $n>2$).

[6]All results hold for the case of asymmetric costs. For details see the Technical Appendix.

[7]This assumption captures well the situation in industries such as software and electronics, where the main cost burden comes from upfront R&D. In Section 6 we discuss adding variable costs to the model.

understanding the strategic forces that govern firm behavior in the non-integrated case, in which separate firms produce and sell each of the complementary products.

4.1 Benchmark Case: An Integrated Producer

The integrated firm is a monopolist in the market for both products and chooses the quality levels of products A and B (α and β) and their prices (p_A and p_B). Since consumers need both products to derive value, the integrated firm effectively charges a single price, $p_I = p_A + p_B$, for the composite pair. Consumers with taste parameters that exceed $\widehat{\theta}_I = \frac{p_I}{\alpha\beta}$ buy the products and the firm solves the following problem:

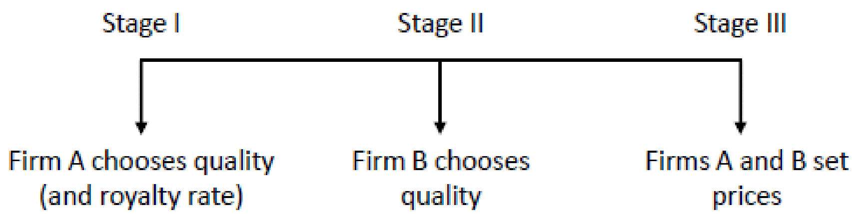
$$\max_{p_I,\alpha,\beta} \pi^I = \left(1 - \widehat{\theta}_I\right) p_I - \frac{1}{3}k\alpha^3 - \frac{1}{3}k\beta^3. \tag{1}$$

The optimal solution of the integrated firm is given in Table 1 (under the heading "Integrated"). As can be seen, the profit-maximizing price is $p_I^* = \frac{\alpha\beta}{2}$. At this price, the indifferent consumer has a taste for quality $\widehat{\theta}_I = 0.5$, so half of the market is covered. The integrated firm chooses quality levels that trade off the increased revenue generated by higher-quality products against the greater cost of development. Note from Table 1 that the profit-maximizing quality level of the two products is the same for the integrated firm.

4.2 Non-integrated Producers

A single firm may not have the technology or know-how required to develop both products, and strictly complementary products developed and sold by separate firms is common in the marketplace. For instance, many firms that produce hardware (processors, game consoles, and smart phones) depend on other firms to develop complementary software (operating systems, game titles, and applications). Given the need for both products to work together, strictly complementary products are typically developed sequentially, with development decisions about the subsequent product based on the specifications of the leading product.

Figure 1: Timeline of the Game



For example, Microsoft needs to know the planned architecture of a processor before it develops a compatible operating system. To capture this, we analyze a multistage game in which one firm chooses a level of quality for product A (the "hardware") before the maker of complementary product B (the "software") chooses its quality level.[8]

A common practice in many industries with complements is the use of royalty fees. In the video game industry, console makers charge game publishers a royalty fee per game copy in return for permission to develop games compatible with the console (for example, Activision has to pay Microsoft a royalty of about 10% for each game sold). Smart phones and applications have a similar royalty structure (with about 30% in royalties). To incorporate this aspect of complement markets, we allow firm A, the first mover that develops and sells the A product, to impose a royalty fee on firm B, which develops and sells the B product. Consistent with industry practice, the royalty fee, r, is defined as a percentage of product B's retail price and firm B knows this rate when finalizing its quality decision. The general timeline of the game is shown in Figure 1.

We next analyze the case of non-integrated complement firms. We divide the analysis into two parts: first, we consider the case of non-integrated complements that lack a royalty fee structure, and then we look at how the addition of royalty fees affects the results.

[8]The assumption of sequential development is not crucial for the paper's results. In fact, a more pronounced problem of quality under-provision arises with simultaneous development. See Section 6 for details.

4.2.1 The Case of No Royalty Fees

When there are no royalty fees, the sequence of moves is as follows. In the first stage firm A chooses the quality level for its product, in the second stage firm B chooses its quality level, and in the final stage firms set prices for their products simultaneously. The marginal consumer's valuation is $\widehat{\theta} = \frac{p_A + p_B}{\alpha\beta}$, and the firms' optimization problems are given by

$$\max_{\alpha, p_A} \pi_A = \left(1 - \widehat{\theta}\right) p_A - \frac{1}{3} k\alpha^3, \tag{2}$$

$$\max_{\beta, p_B} \pi_B = \left(1 - \widehat{\theta}\right) p_B - \frac{1}{3} k\beta^3. \tag{3}$$

We solve the model by working backward from the final pricing stage. From the solution to the Non-integrated (no-royalty) case in Table 1, we see that firms' equilibrium prices are $p_A^* = p_B^* = \frac{\alpha\beta}{3}$. For given quality levels, these prices are clearly higher than what the integrated firm would charge as $p_A^* + p_B^* = \frac{2\alpha\beta}{3} > p_I^* = \frac{\alpha\beta}{2}$. This results in lower demand for the products at given quality levels and reflects the *value-capture* problem of non-integrated firms that produce complementary products. This problem occurs because when one firm cuts its price, this has a positive externality on its rival's pricing. More specifically, if one firm decreases its price, the other firm benefits from the resulting increase in demand as well, and its best response is to increase price. Formally, prices of complementary products form strategic substitutes for given quality levels. Thus, even if both firms would be better off with lower prices, neither wants to deviate unilaterally from the high price they charge.

Taking the last stage pricing equilibrium into account, we can see from the non-integrated solution in Table 1 that firms' quality selections are lower than what the integrated firm chooses. In fact, the qualities are so much lower that the following outcome holds.

Proposition 1 *In equilibrium, non-integrated firms select lower levels of product quality and lower prices than those chosen by an integrated firm; even though the price per unit of quality is higher.*

Proposition 1 reflects the severe *value-creation* problem between firms that produce complementary products. Each firm has an incentive to free-ride on the investments of its counterpart. Specifically, if firm A increases its product's quality, firm B's incentive to increase quality is not as high because it can simply increase its price to capture some of the value from firm A's quality improvement. As with a price decrease, a quality improvement by one firm thus produces a positive externality on the other firm.[9] Such free-riding is a serious drawback because it results in firms under-supplying overall quality in equilibrium. And given that the prices firms charge are a function of the composite quality of the offerings ($\frac{\alpha^*\beta^*}{3}$), such low-levels of quality ultimately lead to lower prices. In other words, the value-creation problem dominates and $p_A^* + p_B^* < p_I^*$. This result contrasts with Economides (1999), whose model yields that the total price asked by non-integrated firms is higher.[10]

We note that although both firms shirk on quality relative to the integrated case (per Proposition 1), they do not end up with products of the same quality in equilibrium (despite having the same development cost parameter). The second mover has an advantage because it chooses quality after the first mover has already committed to its quality level. The ability of the second mover to choose a lower quality level translates into greater profit: the firms charge the same price and face the same demand but firm B saves on R&D investment. We also point out that the severity of the value-capture problem results in lower market coverage.[11]

In the next section, we let firm A impose a royalty fee on firm B and study how that affects the value-creation and value-capture problems between complementor firms.

4.2.2 Incorporating Royalty Fees

In the first stage of the game, let firm A set a royalty fee, $r \in [0, 1]$, that is defined as a percentage of firm B's price p_B^R (See Figure 1 for the timeline). The marginal consumer's

[9] We thank the Area Editor for this insight and terminology.

[10] Economides (1999) uses $q = \min(\alpha, \beta)$ for composite quality, and thus assumes away free-riding and coordination problems in qualities between complementors.

[11] A numerical example is provided in Table 2 for easier comparison.

valuation is denoted $\widehat{\theta}_R$ and the firms' profit functions are as follows:

$$\pi_A^R = \left(1 - \widehat{\theta}_R\right)\left(p_A^R + rp_B^R\right) - \frac{1}{3}k\alpha_R^3, \tag{4}$$

$$\pi_B^R = \left(1 - \widehat{\theta}_R\right)(1 - r)\, p_B^R - \frac{1}{3}k\beta_R^3. \tag{5}$$

The equilibrium solution is given in Table 1 (under the heading "Non-integrated, with Royalty"). The following proposition highlights the effect of royalty fees on firm behavior.

Proposition 2 *With royalty fees, the gap in quality selections between the firms increases while the quality of the composite good decreases. The total price for the two products decreases and market coverage increases.*

In order to understand the intuition behind Proposition 2 it is critical to examine how firms' incentives change under the royalty structure. From Table 1 we can see that compared to the non-integrated case without royalty fees, firm A increases its quality investment but firm B shirks even more— hence the quality gap between the products is exacerbated. This happens because firm A's stake in the industry's overall revenue is greater than in the case of no royalty fees, as it can now capture value both directly from the sale of its product and indirectly from the sale of product B through the royalty payments. This, in turn, increases firm A's willingness to invest in quality to attract more consumers to buy. On the other hand, firm B's incentive to invest in quality declines with the inclusion of royalty fees. The optimal quality level for firm B is $\beta_R^*(\alpha_R) = \frac{\sqrt{(1-r)\alpha_R}}{\sqrt{k}(3-r)}$, which increases with the quality of firm A's product (due to strategic complementarity) but decreases sharply with the royalty rate (as firm B cannot appropriate all of the returns on its quality investment because it must hand over a portion, r, of its revenue to firm A).

Note further that when imposing a royalty fee, firm A offers a higher-quality product but actually charges consumers a lower price (See Table 1). This is because firm A seeks to stimulate demand by setting a lower price and can compensate for the decrease in the revenue per unit through the royalty transfer. Firm B, on the other hand, reacts to the price decrease of firm A by raising its price (strategic substitutes) and ends up selling a lower-quality product at a higher price. Because firm A lowers its price more than firm B increases its price, the total price paid by a consumer for both goods is lower. The impact on total price is so dramatic that, although overall product quality is lower, consumers are better off with royalty fees as the expenses they incur drop precipitously. From firm A's standpoint, despite the lower quality of the composite product and the lower price it charges, the royalty arrangement is beneficial because it allows earning larger profits relative to the case where it did not mandate these fees. Firm B suffers from the implications of these fees and earns lower profits.

To summarize, using a royalty fee structure relieves the value-capture problem for firm A, but does so at the expense of exacerbating the value-creation problem by causing a decline in overall quality provision. In the next section we show that, surprisingly, a direct competitor to firm A can alleviate the value-creation problem in a way that leaves all parties better off— including the competing A firms.

5 Competition

We now seek to understand how competition in the A market impacts our findings. One might think that such competition could only benefit the B firm because it reduces firm A's market power. But, as we will show, competition in the A market results in more intricate effects that can alleviate the decline in overall quality and leave all firms better off.

Consider two vertically differentiated firms that produce an A-type product and are denoted A_H and A_L. Let α_H and α_L be the quality levels and p_{AH} and p_{AL} be the prices of the high- and low-quality products in the A market, respectively, and where $\alpha_L \leqslant \alpha_H$. There is a single B firm that produces the complementary product in two versions, one compatible with the A_H product and the other with the A_L product. The B product's level of quality is again denoted by β. As a result, there are two product pairs available to consumers, $A_H B$ and $A_L B$. Firm B selects a price, p_{BH} and p_{BL}, for each version of its product.

Our focus is on understanding how the existence of competition in the A market affects the actions of the firm that sells the higher-quality A product and of the B firm (these firms can be thought of as the two players in the analysis of the previous sections). Thus, we treat α_L as exogenous. This would capture, for example, the situation in which a console maker faces competition from personal computers (PCs) that also serve as hardware gaming platforms and it has to take their existence into account, along with the game developer's ability to sell PC-compatible titles. To simplify our analysis, we further assume that firm B pays royalties only to firm A_H.[12] In Section 6 we discuss several extensions and robustness checks of the competitive model setup.

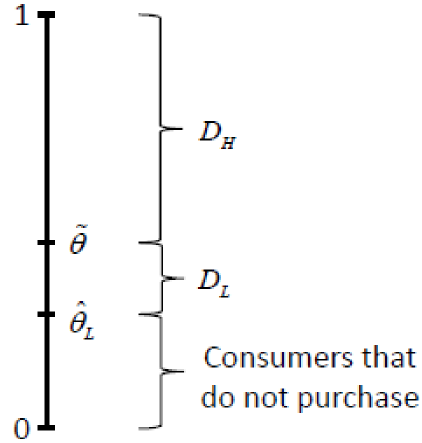
The sequence of moves is similar to that depicted in Figure 1, except that firm A_L also prices its product in the final stage. We define $\widehat{\theta}_i$ as the lowest type consumer that gets non-negative utility from purchasing $A_i B$ for $i = \{H, L\}$ and $\widetilde{\theta}$ as the consumer who is indifferent between the high-quality pair and the low-quality pair. We have $\widehat{\theta}_H = \frac{p_{AH}+p_{BH}}{\alpha_H \beta}$, $\widehat{\theta}_L = \frac{p_{AL}+p_{BL}}{\alpha_L \beta}$, and $\widetilde{\theta} = \frac{p_{AH}+p_{BH}-(p_{AL}+p_{BL})}{(\alpha_H-\alpha_L)\beta}$. D_i denotes the demand for $A_i B$. Figure 2 depicts the consumer space and demand structure when $\widetilde{\theta} > \widehat{\theta}_H > \widehat{\theta}_L$ is satisfied, which is the condition for both product pairs to have positive sales. We note that in the equilibria we characterize, the market will never be fully covered.

The profit functions in the competitive case for the high-quality A firm, the low-quality A firm, and the B firm are given below. We use superscript CR to denote relevant quantities for the model with competition and royalty payments and continue to use superscript R for the baseline non-integrated model that lacks competition but includes royalty payments (per the solutions in Section 4.2.2).

$$\pi_{AH}^{CR} = \left(1 - \widetilde{\theta}\right)\left(p_{AH} + r^{CR} p_{BH}\right) - \frac{1}{3} k \alpha_H^3, \tag{6}$$

[12] This is true, for instance, for video games where the game publisher pays a royalty fee only to the console maker and not to PC manufacturers for each game title sold.

Figure 2: Demand Structure When There is Competition in the A Market



$$\pi_{AL}^{CR} = \left(\widetilde{\theta} - \widehat{\theta}_L\right) p_{AL} - \frac{1}{3} k \alpha_L^3, \tag{7}$$

$$\pi_B^{CR} = \left(1 - \widetilde{\theta}\right) \left(1 - r^{CR}\right) p_{BH} + \left(\widetilde{\theta} - \widehat{\theta}_L\right) p_{BL} - \frac{1}{3} k \beta^3. \tag{8}$$

Since there are two implications of adding a rival in the A market, namely, there is now competition in this market and there is another product pair available to consumers, the reader may wonder about the role of each. Thus, it is useful to first consider the case where we let the high-quality A firm produce its own low-quality variant at level α_L, i.e., allow increased product variety but not competition. The profit functions in this case are:

$$\pi_A = \left(1 - \widetilde{\theta}\right) \left(p_{AH} + r p_{BH}\right) + \left(\widetilde{\theta} - \widehat{\theta}_L\right) p_{AL} - \frac{1}{3} k \alpha_H^3 - \frac{1}{3} k \alpha_L^3, \tag{9}$$

$$\pi_B = \left(1 - \widetilde{\theta}\right) \left(1 - r\right) p_{BH} + \left(\widetilde{\theta} - \widehat{\theta}_L\right) p_{BL} - \frac{1}{3} k \beta^3. \tag{10}$$

Solving for the optimal prices in equations (9) and (10) it is straightforward to show (see the Appendix, start of the proof of Proposition 3) that the single A firm always prefers to set prices such that its low-quality variant has no sales. In other words, a monopolist on the A side of the market will never sell its full product line. The reason for this is that the monopolist prefers to push the higher-quality version to as many customers as it can because of the higher margins it can earn. This extends the results of previous literature on a firm's behavior when having the option to introduce a product line (e.g., Stockey 1979, Salant 1989, Anderson and Dana 2009), to the case where there is a complement product to pair up with.[13] Thus, the mere ability to introduce a product variant, in and of itself, does not alter the findings reported earlier: the firms would behave in such a way that yields the exact same outcome of section 4.2.2.

We now return to solve the model with competition between the two A firms (equations (6)-(8)) and characterize the properties of the unique equilibrium under competition.

Proposition 3 *There exists an $\overline{\alpha}_L$ such that for $\alpha_L \in (0, \overline{\alpha}_L]$, adding a competitor to the A market results in a unique equilibrium in which the profits of all firms, consumer surplus, and social welfare are greater relative to the non-competitive case.*

This result is surprising because it shows that there are conditions under which competition is beneficial for all parties involved, including the direct competitors. Specifically, if the quality of A_L is not too high, there is a *Pareto improving* situation in which all three firms and consumers are better off— a win-win-win-win outcome.

To understand how such an outcome can arise, when it does not occur if the single A firm has the option to introduce a second product of its own, we need to examine the impact competition has on firms' desire to invest in quality and on their pricing considerations. As we might expect, competition on the A side of the market drives down the prices of the A products. This in turn gives firm B the opportunity to increase its prices (per the strategic substitutability of prices between complementary products). Moreover, the existence of two product-pair offerings in the market, $A_H B$ and $A_L B$, provides more choices to consumers and, importantly, coupled with lower total prices paid this expands the market and renders consumers better off. In the numerical example we provide in Table 3, market coverage increases by 17% (from 42% to 59%; with 41% of the market remaining not covered).

[13] We thank the Area Editor for pointing this out to us.

But why does the high-quality A firm benefit from competition? In addition to the familiar negative effect of competition, which hurts the A firms through intensified price rivalry and the division of potential sales between them, competition in a context involving complementary products also has an indirect effect on quality decisions. If competition induces higher levels of quality, all firms may benefit from the greater value created.

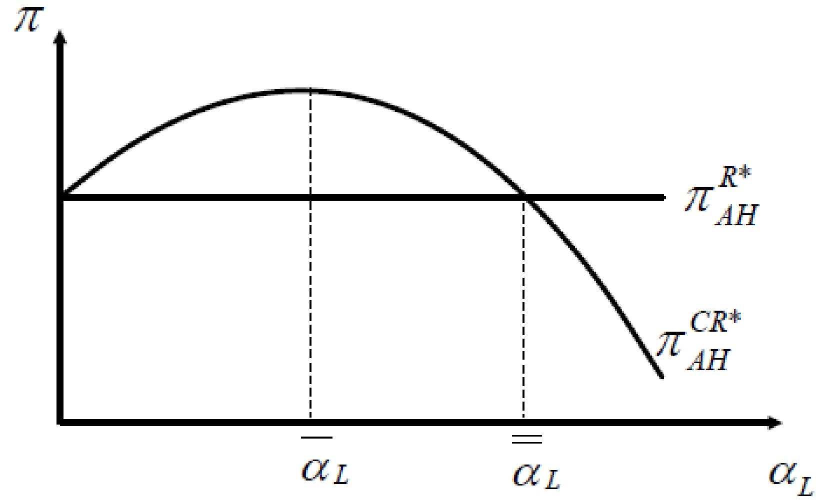
Proposition 4 *For $\alpha_L \in (0, \overline{\alpha}_L]$, compared to the case without competition:*

- *Product B's quality is higher: $\beta^{CR*} > \beta^{R*}$.*
- *Product A_H's quality is higher: $\alpha_H^{CR*} > \alpha^{R*}$.*
- *The quality of the composite good $A_H B$ is higher: $q_H^{CR*} > q^{R*}$.*
- *The high-quality A firm chooses a higher royalty rate: $r^{CR*} > r^{R*}$.*

Proposition 4 reveals that competition in the A market can induce firms to offer higher-quality products, thus alleviating the value-creation problem. Firm A_H captures some of this added value through increased consumer demand, and through the royalty fees it receives from firm B. As long as α_L is not too high, the negative effect of price competition between the A firms that results in firm A_H getting a smaller share of the pie is overshadowed by the positive effect of higher quality levels that increase the size of the pie. A numerical example, that compares the findings with and without competition, is given in Table 3 (which also provides illustrative values for the conditions needed for the Pareto improving result to hold).

Figure 3 depicts how the A_H firm's equilibrium profit changes as a result of these two forces. At $\alpha_L = 0$, the model is equivalent to the royalty model without competition that was analyzed in Section 4, and $\pi_{AH}^{CR*} = \pi_{AH}^{R*}$. As α_L increases, both the direct negative effect of price competition and the indirect positive effect of rising qualities get stronger— but they do not intensify at the same rate. When α_L is low, the quality improvement effect dominates because firm A_H can achieve relatively high differentiation in the A market by choosing a greater level of quality, thus confining its loss from price competition. In that case, firm A_H benefits greatly from the increase in β and the resulting increase in the composite quality

Figure 3: Firm A_H's Profits without Competition (π_{AH}^{R*}) and with Competition (π_{AH}^{CR*})



level delivered by the $A_H B$ product pair. Initially then, $\frac{\partial \pi_{AH}^{CR*}}{\partial \alpha_L} > 0$. However, as α_L increases, firm A_H cannot profitably maintain as much differentiation by selecting much higher quality (which comes at an increasing cost), hence price competition between the A firms intensifies and the negative effect of direct competition starts to grow more rapidly than the positive effect of greater total quality provision. At some point, denoted by $\overline{\alpha}_L$ in Figure 3, any further increase in α_L will cause π_{AH}^{CR*} to decrease. Eventually, when α_L increases beyond $\overline{\overline{\alpha}}_L$, π_{AH}^{CR*} will be lower than π_{AH}^{R*}. This results in an inverse-U pattern for the profit of A_H as the quality of its rival's product, α_L, increases.

We would like to highlight the interplay between competition and royalty fees, which is central for the results to hold. For firm B, which now sells two product versions, the market expands and it prices the version that complements A_H higher. This creates an incentive to increase quality. For firm A_H, direct competition from firm A_L generates a desire to increase differentiation, which is accomplished by improving α_H. But to support the necessary R&D investment, while having to drop its price in the face of competition, firm A_H seeks to raise the royalty rate— and this is something that firm B is willing to tolerate because the market has expanded and it has raised its price. Thus, both firms have an incentive to increase quality. Lastly, we note that the high quality of firm B's product, and sufficient differentiation by firm A_H, ensure that firm A_L can make positive profits in equilibrium, so it benefits from being an active player in this market.[14]

In sum, the presence of competition in the A market coupled with a royalty fee structure can have a "coordinating" effect on firms' behavior and ameliorate the value-creation problem to the benefit of all parties involved.

6 Model Extensions and Robustness Checks

We analyzed a number of extensions and performed a series of robustness checks by relaxing key model assumptions. We discuss our findings below and present formal details in the Technical Appendix.

Simultaneous Quality Decisions If quality decisions are made simultaneously instead of sequentially, we find that all the propositions presented in the paper hold and, in fact, the magnitude of the effects described become even more pronounced. To understand why, note that when qualities are decided simultaneously both firms are always worse off compared to the sequential development case: each firm would like its counterpart to select a higher quality level but with simultaneous development neither can credibly commit to it. By contrast, when quality decisions are sequential, the first mover can commit to a higher quality level, and because quality levels are strategic complements this prompts a somewhat higher quality level by the second mover (relative to the simultaneous case).

The Cost of Quality Has a Per-product (Variable) Component We examined two model extensions. In the first, we assumed that in addition to the convex development cost incurred there is also a marginal cost per unit sold that increases linearly with the product's quality (to reflect the fact that higher-quality products are also more costly to manufacture). Specifically, firm A incurs a manufacturing cost of $m\alpha$ for each unit of its product and firm

[14] A_L's profits remain positive even if we account for the cost of developing a product with quality α_L in the range $\alpha_L \in [0, \overline{\overline{\alpha}}_L]$.

B incurs $m\beta$. We find that all the results hold if m is not too high. In the competitive scenario, since the low-quality firm's price is low to begin with, an increase in m hurts this producer more than the other players; hence after some point the low-quality firm cannot break even and the three firm equilibrium collapses.

In the second extension, we considered the case in which there is only a variable cost, i.e., there is no upfront investment in quality. For the second order conditions to hold, this marginal cost should be sufficiently convex and we use $\frac{1}{3}m\alpha^3$ and $\frac{1}{3}m\beta^3$. In such a specification, the non-integrated firms choose the same quality levels that the integrated firm chooses, with or without a royalty fee structure. Consequently, there is no value-creation problem. This happens because the margin functions across models become multiples of each other with constant coefficients once optimal prices are substituted into the profit functions. And this is also true for the demand functions. Hence the first order conditions are maximized at the same quality levels in each case. When facing competition, the high-quality A firm has an incentive to set its price and royalty rate such that the low-quality firm has no sales and the Pareto improving result no longer holds.

Non-strict Complementarity: A Flexible Composite Quality Function We examined a specification in which $q = z_1\alpha + z_2\beta + z_3\alpha\beta$. Note that the model analyzed in the paper is a special case of this general model with $z_1 = z_2 = 0$. This flexible specification also nests the non-essential complements case with $q = z_1\alpha + z_3\alpha\beta$. The analysis using the general function shows that all our results continue to hold as long as there is some degree of complementarity, i.e., z_3 is non-negligible compared to z_1 and z_2. For example, all the findings reported in the paper hold for z_1=$z_2 = 1$ and $z_3 = 3$.

Enriching the Competitive Setup Our goal in analyzing the competitive case was to understand how a low-cost rival in the A market, by virtue of the price pressure it exerts on the high-quality firm, affects firms' incentives to invest in quality and to price products. To focus on these issues, we simplified the setup in a number of ways by assuming that: the B firm has one quality level; the quality level of the A_L firm is exogenous; the A_L firm cannot impose a royalty fee. We examined relaxing these assumptions one by one. First, let firm B offer two different quality levels, β_H and β_L, the former compatible with the A_H product and the latter with the A_L product. Define the quality ratio $\gamma \equiv \frac{\beta_L}{\beta_H}$ and note that $\gamma \in (0, 1]$. With this specification, the two product pairs available to consumers are $A_H B_H$ and $A_L B_L$. We show that all the results reported in the paper continue to hold in this expanded setting. To see why, note that one can define $\widetilde{\alpha}_L = \gamma\alpha_L$ and replace α_L with $\widetilde{\alpha}_L$ throughout the analysis to reach the equilibrium quality levels and prices.

As for the quality level of the A_L firm, we note that the Pareto improvement result that we find requires sufficient quality separation in the A market. This can occur either because there exists some product that can minimally serve the purpose of the complement in the A market, as is the case with average PCs that can run video games but possess much lower quality than high-powered consoles, or because endogenously two firms would choose sufficient differentiation on quality to avoid fierce price competition[15]. For example, Amazon's decision to introduce a much lower-quality tablet relative to Apple's iPad (with significant price differentials as well). Our model structure presented in the paper is consistent with the former case and we provide some numerical analysis relevant for the latter case in the Technical Appendix. We leave for future research the full analytic characterization of the endogenous A_L quality case.

Lastly, several numerical examples allowing the A_L firm to impose a royalty fee show that we can find ranges of the parameter space where the pareto improvement findings continue to hold. We provide these analyses in the Technical Appendix.

Horizontal Differentiation in the A Market We explored whether horizontal competition would have similar effects in mitigating the value-creation problem. We found that horizontal competition drives down prices in the A market. Because of strategic substi-

[15]The desire to sufficiently differentiate and not co-locate in vertical models of consumer preferences is consistent with classic papers in this literature, such as Shaked and Sutton (1982) and Moorthy (1988).

tutability, the B firm increases its price and captures more of the value. This increases firm B's incentive to invest in quality, thus alleviating the value-creation problem as in our vertical differentiation model. If the cost parameter (k) is low enough, then the B firm's quality improvement may more than compensate the original A firm's losses due to price competition. Interestingly, the consumer horizontal taste parameter, t, needs to be in a mid-range for the win-win-win-win result to hold. It needs to be low enough to induce sufficient price competition between the A firms to incentivize firm B, but it also needs to be high enough so that the loss from price competition is confined.

Other Contractual Agreements The contractual instrument we studied in this paper was that of a royalty rate. We analyzed other contracts such as: two-part-tariffs, non-linear royalty fees, revenue sharing, and fixed royalty fees (instead of a percentage rate). Some of these pricing arrangements can improve performance, but do not succeed in fully coordinating the firms' decisions, and may even exacerbate the quality-gap problem. Consider, for example, a two-part tariff contract. In this case, firm A charges firm B a fixed fee plus royalty per each unit sold. This arrangement clearly improves firm A's profits, and since it nests the model we have analyzed in the paper, it can also improve total profits. However, in this arrangement firm A extracts all of firm B's profits, and since firm A still maintains control of its retail price (to consumers) firm B is not a residual claimant and we find that this contract does not provide the necessary incentives to fully coordinate the firms' decisions (for details see the Technical Appendix). Similar issues arise with the other contractual arrangements.

The failure of these contracts to fully coordinate the system stems from two sources. First, because the firms need to coordinate both their pricing and quality levels, an arrangement that incentivizes firm B to reduce price toward the optimal level reduces its incentive to increase quality toward the optimal level. Second, the firms set their prices simultaneously. Thus, in contrast to the typical channel coordination setting, firm A doesn't fully commit to its final consumer price at the time of selecting quality, regardless of the pricing arrangement between the firms, and this puts further restrictions on the ability to fully coordinate all the decisions.

We can also consider the possibility that the A firm conditions the arrangement on firm B's quality. In the most extreme case, the A firm can require the B firm to produce the optimal quality and refuse to deal with it otherwise. These types of arrangements fail to achieve full coordination because they are not renegotiation-proof. If B's quality deviates from the requirements, the A firm still has an incentive to deal with the B firm. Second, even if firm B's quality is optimal, firm A would not produce the optimal quality given the structure of the game, and full coordination is not achieved.

7 Conclusion

In this paper, we analyzed the strategic interaction between complementor firms that need to make decisions about price and quality and examined the impact of royalty fees and competition on their interactions. Our study has yielded several important insights.

We have shown that an integrated firm is much more effective than non-integrated firms in producing complementary goods because the former can internalize all of the gains from its actions. Non-integrated firms, on the other hand, price selfishly (leading to value-capture problems) and have an incentive to free-ride on each other's quality investment (leading to value-creation problems). We find that the quality levels can be so low that, in contrast to the extant literature, separate firms end up pricing below the integrated firm's price (despite the tendency to price higher for given quality levels). Moreover, the second mover free-rides extensively, resulting in a gap in the qualities of the two complementary products.

Royalty fees allow the first mover to extract surplus from the second mover. As a result, the first mover's profit increases, compensating for the disadvantage of having to set quality first. On the positive side, this induces the first firm (e.g., the hardware producer) to select a higher level of quality for its product. On the negative side, royalties prompt the second firm to shirk even more on quality. Consequently, the quality-gap problem is exacerbated, resulting in even lower quality for the composite product pair.

We find that one way to benefit from royalty fees, while mitigating the value-creation problem, is to have a vertically differentiated competitor. The presence of competition in the A market results in an equilibrium in which all firms and consumers can be better off relative to the case in which only one A firm and one B firm are active. The existence of a lower-quality A firm increases the incentives of firm B and also of the higher-quality A firm to invest in quality, thereby alleviating the value-creation problem. The result is that all three firms share a bigger pie and, in equilibrium, are all better off. In addition, consumers benefit from the greater value provided and the lower prices the A firms charge. Notably, this outcome cannot be obtained by simply giving the A firm the option to introduce a low-quality variant— the inability to commit to pricing in the last stage renders such an approach ineffective.

These findings present a number of implications for managers. First, for a firm that develops the platform-side of a complement pair and that typically announces its specifications first, our results suggest that mandating a royalty fee is a good way to capture additional value. At the same time, managers of such firms should realize that royalty fees will prompt the complement producer (the "software" side of the pair) to select a lower level of quality. Hence, royalty fees only work if the firm instituting them increases quality sufficiently and lowers price drastically so that the composite good remains attractive to consumers. Second, we show that royalty fees can be even more conducive for the platform manufacturer in the presence of a competitor. Because the competitor creates an incentive for the complement producer to increase quality, all the firms can benefit. But to obtain this benefit, the platform-side firm must (a) increase quality further (to maintain differentiation from the low-quality rival), (b) lower price further (to keep the high-quality pair attractive to consumers in the face of competition), and (c) charge a higher royalty rate (to capture back some of the value from its pro-consumer activities). Thus, while common wisdom might have suggested that a direct rival should make a firm worse off, our findings suggest to managers of complementary good firms that they may gain from inviting competition. Furthermore, although an initial reaction to the presence of competition might be to lower the royalty rate to lure firm B to cater more to the high-end A product, the more profitable equilibrium reaction is to raise the royalty rate while concomitantly investing in greater quality and lowering price. For its part, the B firm should "play along": invest in higher quality and introduce a version of its product that works with the lower-quality A product.

In closing, we acknowledge that our analysis is based on a stylized model that involves a number of simplifying assumptions. The focus has been to understand how various mechanisms and market structures (royalties and competition) could mitigate the value-creation and value-capture problems inherent to firms that produce strictly complementary products, particularly for the first mover in the A market. In Section 6 we described a number of extensions and robustness checks that we examined in detail. However, there remain issues that we leave for future research. The most obvious one is competition in the B market. Intuitively, one would expect such competition to benefit the A firm but leave the original B firm worse off. It is important to note that it is the interplay between facing a direct competitor and being able to charge royalty fees that yields the main result reported in this paper— namely that the firm producing the high-quality product should not oppose but rather welcome entry by a low-quality competitor. In practice, the B firm in a complementary market (e.g., a software firm) moves second on product design so it is typically not in a position to mandate royalties. Hence, the win-win-win-win result is unlikely to emerge by only introducing competition in the B market. We note that if the B market contains multiple products that are sufficiently differentiated horizontally (for example, Grand Theft Auto is a rather unique type of game that does not face direct competition from games in other genres) all our results continue to hold.

Table 1: Analytical Equilibrium Results

	Integrated	Non-integrated	
		No Royalty	**With Royalty**
Quality	$\alpha_I^* = \frac{1}{2^2 k}$ $\beta_I^* = \frac{1}{2^2 k}$ $q_I^* = \frac{1}{2^4 k^2}$	$\alpha^* = \frac{1}{2^{2/3} 3^{4/3} k}$ $\beta^* = \frac{1}{2^{1/3} 3^{5/3} k}$ $q^* = \frac{1}{2 \times 3^3 k^2}$	$\alpha_R^* = \frac{5^{5/3}}{2^{13/3} 3^{4/3} k}$ $\beta_R^* = \frac{5^{4/3}}{2^{11/3} 3^{5/3} k}$ $q_R^* = \frac{5^3}{2^8 3^3 k^2}$
Price	$p_I^* = \frac{1}{2^5 k^2}$	$p_A^* = p_B^* = \frac{1}{2 \times 3^4 k^2}$	$p_A^{R*} = \frac{5^3}{2^9 3^4 k^2}$ $p_B^{R*} = \frac{5^4}{2^{10} 3^4 k^2}$
Royalty Rate	-	-	$\frac{3}{5}$
Demand	$1 - \widehat{\theta}_I = \frac{1}{2}$	$1 - \widehat{\theta} = \frac{1}{3}$	$1 - \widehat{\theta}_R = \frac{5}{12}$
Profit	$\pi_I^* = \frac{1}{2^6 3 k^2}$	$\pi_A^* = \frac{1}{2^2 3^5 k^2}$ $\pi_B^* = \frac{1}{3^6 k^2}$ $\pi_A^* + \pi_B^* = \frac{7}{2^2 3^6 k^2}$	$\pi_A^{R*} = \frac{5^5}{2^{13} 3^5 k^2}$ $\pi_B^{R*} = \frac{5^4}{2^{10} 3^6 k^2}$ $\pi_A^{R*} + \pi_B^{R*} = \frac{23 \times 5^4}{2^{13} 3^6 k^2}$
Consumer Surplus	$\frac{1}{2^7 k^2}$	$\frac{1}{2^2 3^5 k^2}$	$\frac{5^5}{2^{13} 3^5 k^2}$
Social Welfare	$\frac{5}{2^7 3 k^2}$	$\frac{5}{2 \times 3^6 k^2}$	$\frac{19 \times 5^4}{2^{12} 3^6 k^2}$

Table 2: Numerical Equilibrium Results

	Integrated	Non-integrated	
		No Royalty	**With Royalty**
Quality	$\alpha_I^* = 2.50$ $\beta_I^* = 2.50$ $q_I^* = 6.25$	$\alpha^* = 1.46$ $\beta^* = 1.27$ $q^* = 1.85$	$\alpha_R^* = 1.68$ $\beta_R^* = 1.08$ $q_R^* = 1.81$
Price	$p_I^* = 3.13$	$p_A^* = p_B^* = 0.62$	$p_A^{R*} = 0.30$ $p_B^{R*} = 0.75$
Royalty Rate	-	-	60%
Demand	$1 - \widehat{\theta}_I = 50\%$	$1 - \widehat{\theta} = 33\%$	$1 - \widehat{\theta}_R = 42\%$
Profit	$\pi_I^* = 0.520$	$\pi_A^* = 0.103$ $\pi_B^* = 0.137$ $\pi_A^* + \pi_B^* = 0.240$	$\pi_A^{R*} = 0.157$ $\pi_B^{R*} = 0.084$ $\pi_A^{R*} + \pi_B^{R*} = 0.241$
Consumer Surplus	0.780	0.102	0.157
Social Welfare	1.300	0.343	0.398

The numerical values have been calculated assuming $k = 0.1$

Table 3: Comparison of Royalty Cases With and Without Competition

	No Competition	**Competition**
Quality	$\alpha_R^* = 1.68$ $\beta_R^* = 1.08$ $q_R^* = 1.81$	$\alpha_{CR}^* = 1.69$ $\beta_{CR}^* = 1.14$ $q_{CR}^* = 1.93$
Price	$p_A^{R*} = 0.30$ $p_B^{R*} = 0.75$	$p_{AH}^{CR*} = 0.051$ $p_{AL}^{CR*} = 0.036$ $p_{BH}^{CR*} = 0.990$ $p_{BL}^{CR*} = 0.082$
Royalty Rate	60%	69%
Demand	$1 - \widehat{\theta} = 42\%$	$1 - \widetilde{\theta} = 44\%$ $\widetilde{\theta} - \theta_L = 15\%$
Profit	$\pi_A^{R*} = 0.157$	$\pi_{AH}^{CR*} = 0.160$
	$\pi_B^{R*} = 0.084$	$\pi_B^{CR*} = 0.099$
Consumer Surplus	0.157	0.206
Social Welfare	0.398	0.465

The numerical values have been calculated assuming $k = 0.1$ and $\alpha_L = 0.25$

At $k = 0.1$, $\overline{\alpha}_L$=0.2 and $\overline{\overline{\alpha}}_L$=0.4

Appendix

Proof of Proposition 1. Integrated firm: In order to find the sub-game perfect equilibrium we solve the game backward and start from the firm's pricing decisions followed by the firm's quality decisions. The solution for the first order condition of (1) with respect to price yields $p_I^* = \frac{\alpha\beta}{2}$. Substituting it into the (integrated) firm's profit function ($\pi_I = \frac{\alpha\beta}{4} - \frac{1}{3}k\alpha^3 - \frac{1}{3}k\beta^3$) and solving the first order conditions with respect to α and β simultaneously yield $\alpha_I^* = \beta_I^* = \frac{1}{2^2 k}$. It is easy to see that, the two second order conditions are negative.

Non-integrated firms: At the pricing stage, Firm A's profit function is $\pi_A = (1 - \frac{p_A + p_B}{\alpha\beta})p_A$. The first order condition of the profit function with respect to p_A yields $p_A^* = \frac{\alpha\beta - p_B^*}{2}$. Similarly, $p_B^* = \frac{\alpha\beta - p_A^*}{2}$. Solving these two functions simultaneously we get $p_A^* = p_B^* = \frac{\alpha\beta}{3}$.

Substituting these prices into firm B's profit function ($\pi_B = \frac{\alpha\beta}{9} - \frac{1}{3}k\beta^3$) and solving the first order condition with respect to β yields : $\beta^*(\alpha) = \frac{\sqrt{\alpha}}{3\sqrt{k}}$. Consequently firm A's profit function in the first stage is updated and the optimal quality level of firm A is $\alpha^* = \frac{1}{2^{2/3}3^{4/3}k}$ and thus, $\beta^* = \frac{1}{2^{1/3}3^{5/3}k}$. The second order conditions in all stages are satisfied.

It is easy to see that $\alpha^* = \frac{1}{2^{2/3}3^{4/3}k} < \alpha_I^* = \frac{1}{2^2k}$ and $\beta^* = \frac{1}{2^{1/3}3^{5/3}k} < \beta_I^* = \frac{1}{2^2k}$ and that $p_A^* + p_B^* = \frac{1}{3^4k^2} < p_I^* = \frac{1}{2^5k^2}$. ■

Proof of Proposition 2. In order to find the sub-game perfect equilibrium we solve the game backward and start with the analysis of pricing decisions. The solutions of the first order conditions of the profit functions in (4) and (5) with respect to prices (p_A^R and p_B^R) yield: $p_A^{R*} = \frac{(1-r)\alpha_R\beta_R}{3-r}$ and $p_B^{R*} = \frac{\alpha_R\beta_R}{3-r}$. Next we solve firm B's quality decision. The solution of firm B's profit $\pi_B^R = \frac{(1-r)\alpha_R\beta_R}{(3-r)^2} - \frac{1}{3}k\beta_R^3$ with respect to its quality β_R yields; $\beta_R^*(\alpha_R^*) = \frac{\sqrt{(1-r)\alpha_R}}{(3-r)\sqrt{k}}$. Next, we analyze firm's A royalty rate decision and the solution of the first order condition of firm A's profit $\pi_A^R = \frac{\sqrt{(1-r)\alpha_R^3}}{(3-r)^3\sqrt{k}} - \frac{1}{3}k\alpha_R^3$ with respect to the royalty rate r yields: $r^* = \frac{3}{5}$. Finally we solve firm A's quality decision and the solution of this optimization problem is $\alpha_R^* = \frac{5^{5/3}}{2^{13/3}3^{4/3}k}$ and thus $\beta_R^* = \frac{5^{4/3}}{2^{11/3}3^{5/3}k}$, $p_A^{R*} = \frac{5^3}{2^9 3^4 k^2}$, $p_B^{R*} = \frac{5^4}{2^{10}3^4k^2}$ and $\widehat{\theta}_R = \frac{5}{12}$. The second order conditions in all stages are satisfied.

It is easy to show that relative to the non-integrated case without royalty fees, firm A provides a higher-quality product as $\alpha_R^* - \alpha^* = \frac{5^{5/3}}{2^{13/3}3^{4/3}k} - \frac{1}{2^{2/3}3^{4/3}k} = \frac{5^{5/3}-2^{11/3}}{2^{13/3}3^{4/3}k} > 0$ and firm B provides a lower-quality product as $\beta_R^* - \beta^* = \frac{5^{4/3}}{2^{11/3}3^{5/3}k} - \frac{1}{2^{1/3}3^{5/3}k} = \frac{5^{4/3}-2^{10/3}}{2^{11/3}3^{5/3}k} < 0$. The quality of the composite product is also lower as $\alpha_R^*\beta_R^* - \alpha^*\beta^* = \frac{5^3}{2^8 3^3 k^2} - \frac{1}{2\times 3^3k^2} = \frac{5^3-2^7}{2^8 3^3 k^2} < 0$.

It is also easy to show that compared to the non-integrated case without royalty fees, firm A charges a lower price as $p_A^{R*} - p_A^* = \frac{5^3}{2^9 3^4 k^2} - \frac{1}{2\times 3^4k^2} = \frac{5^3-2^8}{2^9 3^4 k^2} < 0$ and firm B charges a higher price as $p_B^{R*} - p_B^* = \frac{5^4}{2^{10}3^4k^2} - \frac{1}{2\times 3^4k^2} = \frac{5^4-2^9}{2^{10}3^4k^2} > 0$. Finally, sales under a royalty structure ($\frac{7}{12}$) are higher than sales of the non-integrated firms ($\frac{1}{3}$). ■

Proof of Proposition 3. We start by showing that competition is essential for obtaining the Pareto improvement and then proceed to proving the claims in the Proposition 3. ■

Proof that competition is essential for pareto improvement. The monopolist in the A market and the B firm set prices in the last stage. The first order conditions of the profit functions given in (9) and (10) with respect to prices p_{AH}, p_{AL}, p_{BH}, and p_{BL} yield $p^*_{AH} = \frac{\alpha_H \beta(1-r)}{(3-r)}$, $p^*_{AL} = \frac{\alpha_L \beta}{(3-r)}$, $p^*_{BH} = \frac{\alpha_H \beta}{(3-r)}$, and $p^*_{BL} = \frac{\alpha_L \beta(1-r)}{(3-r)}$. The second order conditions are met, so these are the optimal prices. Substituting these for the marginal consumers' valuations, we see that $\widetilde{\theta} = \widehat{\theta}_H = \widehat{\theta}_L = \frac{(2-r)}{(3-r)}$. The low-quality variant has no sales and therefore actually producing a low-quality variant and incurring the development cost is a net loss for the monopolist for any α_L. ■

Proof that a unique equilibrium exists. Note that our focus is on an equilibrium with three firms and therefore, we assume that $\alpha_L < \frac{2}{25k}$. As we show later, for higher levels of α_L, firm B deviates from the three firm equilibrium. In order to find the subgame equilibrium we solve the game backward starting from the pricing stage. We derive the first order conditions of the firms' profits (as given in 6, 7, and 8) with respect to the prices p^{CR}_{AH}, p^{CR}_{AL}, p^{CR}_{BH}, and p^{CR}_{BL}. This system yields the following prices and demands: $p^{CR*}_{AH} = \frac{\alpha_H \beta\left[3(1-r)^2 \alpha_H - (3-2r)\alpha_L\right]}{(1-r)[3(3-r)\alpha_H - \alpha_L]}$, $p^{CR*}_{AL} = \frac{\alpha_L \beta(\alpha_H - \alpha_L)}{3(3-r)\alpha_H - \alpha_L}$, $p^{CR*}_{BH} = \frac{\alpha_H \beta[3(1-r)\alpha_H + \alpha_L]}{(1-r)[3(3-r)\alpha_H - \alpha_L]}$, $p^{CR*}_{BL} = \frac{\alpha_H \alpha_L \beta(4-3r)}{3(3-r)\alpha_H - \alpha_L}$, $D_H = \frac{3\alpha_H}{[3(3-r)\alpha_H - \alpha_L]}$, and $D_L = \frac{\alpha_H}{[3(3-r)\alpha_H - \alpha_L]}$. Note that all the demand expressions are positive and that $p^{CR*}_{AL} > 0$. Consequently, given that consumer valuations for quality are distributed in the range $[0, 1]$, the market will only be partially covered as consumers with valuations close to 0 will not purchase the good.

If a deviation makes one of the product pairs dominant, then one of the A firms will be driven out of the market and the demand structure given in Figure 2 will not be valid anymore. In such a case the firm that produce good A that remains in the market and firm B will interact as two non-integrated complementors and the deviation may be profitable. There are four possible deviations of this kind and we analyze them below. The superscript d denotes the corresponding variables arising from the analyzed deviation.

i) Firm A_H can decrease price such that $\widehat{\theta}^d_H \leqslant \widehat{\theta}_L$ holds. (Firm A_L loses all demand.)

In order to do this A_H needs to choose $p^d_{AH} \leqslant \frac{\alpha_H \beta[(2-3r)(1-r)\alpha_H - (2-r)\alpha_L]}{(1-r)[3(3-r)\alpha_H - \alpha_L]}$. Since firm A_H's best response to p^{CR*}_{BH} is higher than this threshold value and A_H's profit is concave in p_{AH} $\left(\frac{\partial^2\pi_{AH}}{\partial p_{AH}^2}<0\right)$; the most profitable deviation is $p_{AH}^d = \frac{\alpha_H\beta[(2-3r)(1-r)\alpha_H-(2-r)\alpha_L]}{(1-r)[3(3-r)\alpha_H-\alpha_L]}$. The resulting profit from deviation $\pi_{AH}^d = \frac{8\alpha_H^2\beta(\alpha_H-\alpha_L)}{[3(3-r)\alpha_H-\alpha_L]^2} - C(\alpha_H)$ is less than $\pi_{AH}^{CR*} = \frac{9\alpha_H^2\beta(\alpha_H-\alpha_L)}{[3(3-r)\alpha_H-\alpha_L]^2} - C(\alpha_H)$. This deviation is not profitable.

ii) Firm A_L can decrease price such that $U_H\left(p_{AH}^{CR*}, p_{BH}^{CR*}, \theta\right) \leqslant U_L\left(p_{AL}^d, p_{BL}^{CR*}, \theta\right)$ holds for all θ. (Firm A_H loses all demand.)

Since $U_H\left(p_{AH}^{CR*}, p_{BH}^{CR*}, \theta=1\right) > U_L\left(p_{AL}=0, p_{BL}^{CR*}, \theta=1\right)$, firm A_L can never make the low-quality pair the dominant choice for all consumers. In other words even if A_L decreases its price to zero, there will be some customers that strictly prefer the high-quality pair. This deviation is not feasible.

iii) Firm B can increase p_{BH} to infinity and destroy A_HB sales. (Firm A_H loses all demand.)

In this case deviation price is $p_{BH}^d = \frac{\alpha_H\beta[6(1-r)\alpha_H+(2-r)\alpha_L]}{2(1-r)[3(3-r)\alpha_H-\alpha_L]}$, subsequent profit from deviating is $\pi_B^d = \frac{\alpha_H\beta[6(1-r)\alpha_H+(2-r)\alpha_L]^2}{4(1-r)[3(3-r)\alpha_H-\alpha_L]^2} - C(\beta)$ and subgame equilibrium profit from not deviating is given by $\pi_B^{CR*} = \frac{\alpha_H\beta[6(1-r)\alpha_H+(2-r)\alpha_L]^2}{[3(3-r)\alpha_H-\alpha_L]^2} - C(\beta)$. In order to compare these two, we need the equilibrium royalty rate, r. Assuming the CR equilibrium holds, the equilibrium royalty rate is $r^* = \frac{27\alpha_H^2+36\alpha_H\alpha_L+\alpha_L^2}{45\alpha_H^2+15\alpha_H\alpha_L}$(See below for derivation). When we substitute r^* into the profit functions we see that $\pi_B^d < \pi_B^{CR*}$ for $\frac{\alpha_L}{\alpha_H} < 0.465$.

iv) Firm B can increase p_{BL} to infinity and destroy A_LB sales. (Firm A_L loses all demand.)

In this case deviation price is $p_{BL}^d = \frac{\alpha_H\alpha_L\beta(8-3r)}{2[3(3-r)\alpha_H-\alpha_L]}$ and subsequent profit from deviating is $\pi_B^d = \frac{\alpha_H^2\alpha_L\beta(8-3r)^2}{4[3(3-r)\alpha_H-\alpha_L]^2} - C(\beta)$ is less than subgame equilibrium profit $\pi_B^{CR*} = \frac{\alpha_H^2\beta[9(1-r)\alpha_H+(7-3r)\alpha_L]}{[3(3-r)\alpha_H-\alpha_L]^2} - C(\beta)$. When we substitute r^{CR*} into the profit functions we see that $\pi_B^d < \pi_B^{CR*}$ again holds for $\frac{\alpha_L}{\alpha_H} < 0.465$.

From *iii* and *iv*, we see that if the quality differentiation is low, the equilibrium with three firms making positive sales may break down. As we will illustrate, the initial assumption ensures that $\frac{\alpha_L}{\alpha_H} < 0.465$.

Using the prices from the subgame equilibrium, we revise firm B's profit function in the second stage to $\pi_B^{CR} = \frac{\alpha_H^2 \beta[9(1-r)\alpha_H+(7-3r)\alpha_L]}{[3(3-r)\alpha_H-\alpha_L]^2} - \frac{1}{3}k\beta^3$. Firm B selects the profit-maximizing quality level $\beta^{CR*} = \frac{\alpha_H \sqrt{9(1-r)\alpha_H+(7-3r)\alpha_L}}{\sqrt{k}[3(3-r)\alpha_H-\alpha_L]^2}$. Substituting β^{CR*} we get the high-quality firm's profit function $\pi_{AH}^{CR} = \frac{9\alpha_H^3(\alpha_H-\alpha_L)\sqrt{9(1-r)\alpha_H+(7-3r)\alpha_L}}{\sqrt{k}[3(3-r)\alpha_H-\alpha_L]}$.

Next, in the first stage, the high-quality firm chooses royalty rate. We differentiate π_B^{CR} with respect to r, set the first order condition to zero and find the optimal $r^{CR*} = \frac{27\alpha_H^2+36\alpha_H\alpha_L+\alpha_L^2}{45\alpha_H^2+15\alpha_H\alpha_L}$. All of the second order conditions are met. Note that this value can only be higher than $\frac{3}{5}$, the optimal r^* value with no competition. Substituting r^{CR*}, we update the profit function to $\pi_{AH}^{CR} = \frac{25\sqrt{5}\alpha_H^{5/2}(\alpha_H-\alpha_L)(3\alpha_H+\alpha_L)^3}{24\sqrt{k}\left(18\alpha_H^2-\alpha_H\alpha_L-\alpha_L^2\right)^{5/2}} - \frac{1}{3}k\alpha_H^3$.

It turns out that characterizing a closed form solution for α_H is difficult. Instead we will first show that there is a unique pure-strategy equilibrium and then derive some results regarding the characteristics of the equilibrium.

In order to ensure that equilibrium exists, we will first characterize the parameter values such that π_{AH}^{CR} is positive and then show that π_{AH}^{CR} is quasiconcave with a single peak. We define $t = \frac{\alpha_H}{\alpha_L}$ as the quality ratio between the product pairs. Since $\alpha_H > \alpha_L$, it follows that $t > 1$. $\pi_{AH}^{CR} > 0$ if and only if $\frac{25\sqrt{5}(\alpha_H-\alpha_L)(3\alpha_H+\alpha_L)^3}{8\sqrt{\alpha_H}\left(18\alpha_H^2-\alpha_H\alpha_L-\alpha_L^2\right)^{5/2}} > k^2$ holds. We divide the terms in the numerator and the denominator with appropriate powers of α_L and get $\frac{25\sqrt{5}(t-1)(3t+1)^3}{8\sqrt{t}(18t^2-t-1)^{5/2}} > k^{3/2}\alpha_L^{3/2}$. Call the function on the left-hand side of the inequality as $G_1(t)$ and the combination of parameters on the right as K. This means that $\pi_{AH}^{CR} > 0$ if $G_1(t) > K$ is satisfied. If firm A_H's choice of α_H results in a quality ratio t such that $G_1(t) > K$, firm A_H will have positive profits; hence we may have an equilibrium. We will analyze $G_1(t)$ over the region $t \in (1, \infty)$. We have $G_1(t=1) = 0$ and $\lim_{t\to\infty} G_1(t) = 0$. We also have $\frac{\partial G_1(t)}{\partial t} > 0$ for $t \in (1, 1.43)$ and $\frac{\partial G_1(t)}{\partial t} < 0$ for $t \in (1.43, \infty)$; hence $G_1(t)$ reaches maximum value $G_1(t = 1.43) = 0.054$. If $K < 0.054$, for some α_H, $\pi_{AH}^{CR} > 0$ will hold and we may have an equilibrium.

Now we will study the shape of the profit function and demonstrate that it is quasiconcave. We know that: $\frac{\partial \pi_{AH}^{CR}}{\partial \alpha_H} = \frac{25\sqrt{5}\alpha_H^{3/2}(3\alpha_H+\alpha_L)^2\left(162\alpha_H^4-132\alpha_H^3\alpha_L+67\alpha_H^2\alpha_L^2+26\alpha_H\alpha_L^3+5\alpha_L^4\right)}{48\sqrt{k}\left(18\alpha_H^2-\alpha_H\alpha_L-\alpha_L^2\right)^{7/2}} - k\alpha_H^2$. When we set this first order condition equal to zero and rearrange the terms we find that

$\frac{\partial \pi_A^{CR}}{\partial \alpha_H} = 0$ iff. $\frac{25\sqrt{5}(3\alpha_H+\alpha_L)^2\left(162\alpha_H^4-132\alpha_H^3\alpha_L+67\alpha_H^2\alpha_L^2+26\alpha_H\alpha_L^3+5\alpha_L^4\right)}{48\sqrt{\alpha_H}\left(18\alpha_H^2-\alpha_H\alpha_L-\alpha_L^2\right)^{7/2}} = k^{3/2}$ holds. We, again, divide the terms in the numerator and the denominator with appropriate powers of α_L and get $\frac{25\sqrt{5}(3t+1)^2\left(162t^4-132t^3+67t^2+26t+5\right)}{48(18t^2-t-1)^{7/2}} > k^{3/2}\alpha_L^{3/2}$. Call the function on the left-hand side of the inequality as $G_2(t)$ and note that the combination of parameters on the right is K. This means that $\frac{\partial \pi_{AH}^{CR}}{\partial \alpha_H} = 0$ if $G_2(t) = K$ is satisfied and the t value that satisfies this equality is a candidate for equilibrium. Of course, we also need to check the sign of $\frac{\partial \pi_{AH}^{CR}}{\partial \alpha_H}$ around this value. Specifically, if we have a t^* such that $\frac{\partial \pi_{AH}^{CR}}{\partial \alpha_H} > 0$, or equivalently $G_2(t^*) > K$, for $t < t^*$ and $\frac{\partial \pi_{AH}^{CR}}{\partial \alpha_H} < 0$, or equivalently $G_2(t^*) < K$, for $t > t^*$; then t^* maximizes π_{AH}^{CR} and it is the unique equilibrium. This is indeed the case for $K < 0.14$ as we have $G_2(t=1) = 0.14$, $\lim_{t\to\infty} G_2(t) = 0$, and $\frac{\partial G_2(t)}{\partial t} < 0$ for $t \in (1, \infty)$; hence $G_2(t)$ is a strictly and monotonically decreasing function. This means that for $K < 0.14$, there will be a single t^* that satisfies $G_2(t^*) = K$ and hence maximizes π_{AH}^{CR}. We know that $\pi_{AH}^{CR} > 0$ if $K < 0.054$. That is, if $K < 0.054$ then we have a unique equilibrium at $t^* = G_2^{-1}(K)$. In short, using the definition of t a unique equilibrium $\alpha_H^* = \alpha_L G_2^{-1}\left(k^{3/2}\alpha_L^{3/2}\right)$ exists for $K = k^{3/2}\alpha_L^{3/2} < 0.054$. Since our focus is on equilibrium with three firms we need $t^* > \frac{1}{0.465}$. This condition ensures that firm B does not want to deviate from the pricing subgame equilibrium (See the deviation analysis at the start of the proof.) This is satisfied with the initial assumption $\alpha_L < \frac{2}{25k}$. ■

Proof that $\pi_{AH}^{CR*} > \pi_{AH}^{R*}$. $\pi_{AH}^{CR*} = \frac{25\sqrt{5}\alpha_H^{5/2}(\alpha_H-\alpha_L)(3\alpha_H+\alpha_L)^3}{24\sqrt{k}\left(18\alpha_H^2-\alpha_H\alpha_L-\alpha_L^2\right)^{5/2}} - \frac{1}{3}k\alpha_H^3$ and $\pi_{AH}^{R*} = \frac{25\sqrt{5}\alpha_H^{3/2}}{864\sqrt{2}\sqrt{k}} - \frac{1}{3}k\alpha_H^3$. It is easy to see that $\pi_{AH}^{CR*}(\alpha_L = 0) = \pi_{AH}^{R*}(\alpha_L = 0)$. Thus we need to show that $\frac{d\pi_{AH}^{CR*}}{d\alpha_L} > 0$ to prove that $\pi_{AH}^{CR*} > \pi_{AH}^{R*}$.

The total derivative is $\frac{d\pi_{AH}^{CR*}}{d\alpha_L} = \frac{\partial \pi_{AH}^{CR*}}{\partial \alpha_L} + \frac{\partial \pi_{AH}^{CR*}}{\partial \alpha_H^*}\frac{d\alpha_H^*}{d\alpha_L}$, and since $\frac{\partial \pi_{AH}^{CR*}}{\partial \alpha_H^*} = 0$ the second part of the equation equals zero at the equilibrium. An inspection of the following derivative $\frac{\partial \pi_{AH}^{CR*}}{\partial \alpha_L} = \frac{25\sqrt{5}\alpha_H^{5/2}(3\alpha_H+\alpha_L)^2\left(15\alpha_H^3-124\alpha_H^2\alpha_L-17\alpha_H\alpha_L^2-2\alpha_L^3\right)}{48\sqrt{k}\left(18\alpha_H^2-\alpha_H\alpha_L-\alpha_L^2\right)^{7/2}}$ shows that all of the terms are positive with the exception of the following parenthesis: $(15\alpha_H^3 - 124\alpha_H^2\alpha_L - 17\alpha_H\alpha_L^2 - 2\alpha_L^3)$. We can divide all the terms by α_L^3 and get $(15t^{*3} - 124t^{*2}\alpha_L - 17t^* - 2)$. This expression is positive for $t^* > 8.4$, which is satisfied for $\alpha_L < \frac{1}{50k}$, and negative otherwise. Therefore, $\frac{\partial \pi_{AH}^{CR*}}{\partial \alpha_L} > 0$ holds for $\alpha_L \in [0, \overline{\alpha}_L]$ where $\overline{\alpha}_L = \frac{1}{50k}$. It follows that there can only be one other point that satisfies $\pi_{AH}^{CR*} = \pi_{AH}^{R*}$ which is at $\alpha_L = \overline{\overline{\alpha}}_L \simeq \frac{1}{25k}$. ∎

Proof that $\pi_B^{CR*} > \pi_B^{R*}$. $\pi_B^{CR*} = \frac{5\sqrt{5}\alpha_H^{3/2}(\alpha_H-\alpha_L)(3\alpha_H+\alpha_L)^3}{324\sqrt{k}\left(18\alpha_H^2-\alpha_H\alpha_L-\alpha_L^2\right)^{3/2}} - \frac{1}{3}k\beta_H^3$ and $\pi_B^R = \frac{5\sqrt{5}\alpha_H^{3/2}}{648\sqrt{2}\sqrt{k}} - \frac{1}{3}k\beta_H^3$. It is, again, easy to see that $\pi_B^{CR*}(\alpha_L = 0) = \pi_B^{R*}(\alpha_L = 0)$. Next we need to show that $\frac{d\pi_B^{CR*}}{d\alpha_L} = \frac{\partial\pi_B^{CR*}}{\partial\alpha_L} + \frac{\partial\pi_B^{CR*}}{\partial\alpha_H^*}\frac{d\alpha_H^*}{d\alpha_L} > 0$. As $\frac{\partial\pi_B^{CR*}}{\partial\alpha_L} = \frac{5\sqrt{5}\alpha_H^{5/2}(3\alpha_H+\alpha_L)^3(39\alpha_H+5a_L)}{216\sqrt{k}\left(18\alpha_H^2-\alpha_H\alpha_L-\alpha_L^2\right)^{3/2}} > 0$ and $\frac{\partial\pi_B^{CR*}}{\partial\alpha_H} = \frac{5\sqrt{5}\alpha_H^{1/2}(3\alpha_H+\alpha_L)^2\left[54\alpha_H^3-24\alpha_H^2\alpha_L-9\alpha_H\alpha_L^2-\alpha_L^3\right]}{216\sqrt{k}\left(18\alpha_H^2-\alpha_H\alpha_L-\alpha_L^2\right)^{3/2}} > 0$, we only need to show that $\frac{d\alpha_H^*}{d\alpha_L} > 0$. Using the envelope theorem this is equivalent to showing that $\frac{d\pi_{AH}^{CR*}}{d\alpha_L}$, which was proven above. Hence $\frac{d\pi_B^{CR*}}{d\alpha_L} > 0$ and $\pi_B^{CR*} > \pi_B^{R*}$. ∎

Proofs of consumer surplus and social welfare results. First we show that the demand for product A_H under competition is higher than the demand for A without competition. The demand for A_H under competition is $D_{AH}^{CR} = \frac{3\alpha_H}{3(3-r^{CR})\alpha_H-\alpha_L}$, and the demand for A without competition is $D_A^R = \frac{1}{3-r}$. We have $D_{AH}^{CR}(\alpha_H, r^*, \alpha_L = 0) = D_A^{R*}$. Since $r^{CR*} > r^*$ (See proof of Proposition 4), $D_{AH}^{CR}\left(\alpha_H, r^{CR*}, \alpha_L = 0\right) > D_A^{R*}$.

Now we will consider the case when $\alpha_L > 0$: Using the full differential we can write $D_{AH}^{CR}\left(\alpha_H, r^{CR*}, \alpha_L\right) = D_{AH}^{CR}\left(\alpha_H, r^{CR*}, \alpha_L = 0\right) + \frac{\partial D_{AH}^{CR}}{\partial\alpha_L}\alpha_L + \frac{\partial D_{AH}^{CR}}{\partial\alpha_H}\frac{\partial\alpha_H}{\partial\alpha_L} = D_{AH}^{CR}\left(\alpha_H, r^{CR*}, \alpha_L = 0\right) + 3\alpha_H + \alpha_L - 3\alpha_L\frac{\partial\alpha_H}{\partial\alpha_L}$. From the optimality and incentive conditions it must be that $\frac{\partial\alpha_H}{\partial\alpha_L} < \alpha_L$, and since $\alpha_H > \alpha_L$, the full differential above is positive. Therefore, $D_{AH}^{CR}\left(\alpha_H, r^{CR*}, \alpha_L\right) > D_{AH}^{CR}\left(\alpha_H, r^{CR*}, \alpha_L = 0\right) > D_A^{R*}$.

It follows directly that consumers' surplus increase under competition and as consumers' surplus and profits increase so does the social welfare. ∎

Proof of Proposition 4. The individual proofs are below. ∎

Proof that $\alpha^{CR*} > \alpha^{R*}$. When $\alpha_L = 0$ the competitive case converges to the case of royalty without competition. Thus, $\alpha^{CR*}(\alpha_L = 0) = \alpha^{R*}$. From the proof of Proposition 3 we know that $\frac{\partial\alpha_H}{\partial\alpha_L} > 0$ under the specified conditions, therefore $\alpha^{CR*} > \alpha^{R*}$. ∎

Proof that $\beta^{CR*} > \beta^{R*}$. We know that $\beta^{CR*} = \frac{\sqrt{5}\sqrt{\alpha_H}(3\alpha_H+\alpha_L)}{6\sqrt{k}\sqrt{18\alpha_H^2-\alpha_H\alpha_L-\alpha_L^2}}$ and $\beta^{R*} = \frac{\sqrt{5}\sqrt{\alpha_H}}{6\sqrt{2k}}$. It is easy to see that $\beta^{CR*}(\alpha_L = 0) = \beta^{R*}$. Hence, $\frac{d\beta^{CR*}}{d\alpha_L} > 0$ if $\frac{\partial\alpha_H}{\partial\alpha_L} > 0$. Therefore $\beta^{CR*} > \beta^{R*}$ as long as $\frac{\partial\alpha_H}{\partial\alpha_L} > 0$. ∎

Proof that $q^{CR*} > q^{R*}$. Follows immediately from above. ∎

Proof that $r^{CR*} > r^*$. We know that $r^{CR*} = \frac{27\alpha_H^2 + 36\alpha_H\alpha_L + \alpha_L^2}{45\alpha_H^2 + 15\alpha_H\alpha_L}$. At $\alpha_L = 0$ we see that $r^{CR*} = r^{R*}$, and since $\frac{\partial r^{CR*}}{\partial \alpha_L} > 0$ it follows $r^{CR*} > r^*$. ■

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