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ON THE MANY KINDS OF GROWTH: A NOTE*

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In this note we synthesize exogenous and endogenous sources of economic growth in a stochastic dynamic log linear general equilibrium model. Endogenous growth could be the result of internal constant returns to scale, external increasing returns to scale in the production of human capital or in the production of goods. We get a closed form log linear representation for the dynamic laws of motion for the human and physical capital stocks. Using the solution we distinguish between different sources of growth that combine exogenous technical progress with endogenous sources of growth that jointly can generate many possible patterns of economic growth.

1. INTRODUCTION

In this note we specify a two-sector stochastic dynamic general equilibrium model that synthesizes both exogenous and endogenous sources of economic growth. The model shares several of the main characteristics of the existing endogenous growth models. We take the production of consumption goods to be separate from that of investment goods (Rebelo 1991) and we draw upon explicit specifications of preferences and technologies. In particular, we assume that preferences and the production technology are characterized by translog functional forms and that the accumulation of human and physical capital are characterized by Cobb-Douglas functional forms. Exogenous technological parameters are allowed to evolve at geometric growth rates that follow Markovian laws of motion.

We provide closed-form characterization for the competitive equilibrium of a model that encompasses specifications that are directly linked to the neoclassical growth model as well as the recent literature on endogenous growth. The laws of motion for all endogenous variables have a log-linear representation, which makes it relatively easy to study. This equilibrium exhibits a plethora of potential growth patterns that can be classified by exogenous and endogenous sources. Moreover, we allow for nonbalance growth equilibrium to characterize the economy.

Of special interest are the possibilities of steady-state growth due to exogenous sources with transitional dynamics dominated by endogenous growth sources and vice versa. This result implies that inference on the sources of economic growth based on reduced-form specifications cannot distinguish between the long run and transitional sources of economic growth (see also Caballé and Santos 1993). As such, countries may have the same long-run growth rate, but their transitional dynamics (convergence) to this steady state may be completely different due to differences in

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endogenous growth aspects of technology or preferences. The equilibrium laws of motions can exhibit monotone convergence to steady-state growth path, as in Kydland and Prescott (1982) as well as cyclical convergence, as in Benhabib and Nishimura (1985). A necessary, but not sufficient condition for cyclical transitional dynamics is the presence of external increasing returns.

Following Mankiw, Romer, and Weil (1992) we can test the Solow model and the convergence hypothesis as nesting hypothesis to an alternative economic specification using single country data.[2] Moreover, the model allows for the joint analysis of long-run growth aspects and the business cycle aspects of the aggregate economic data.[3]

2. THE ENVIRONMENT

We consider a closed economy with two final products—consumption goods (C) and investment goods (I), and two factors of production—capital (K) and labor (L). Labor quality changes with the stock of human capital (H). Goods are produced by many identical firms. The representative firm's production is characterized by the translog transformation function,

$$\text{(1)} \qquad \ln C_t \leq F\left(\theta_t^c, \overline{H}_t, \overline{K}_t, I_t, K_t, L_t\right) \equiv \varphi' X_t^c + \tfrac{1}{2} X_t^{c\prime} \phi X_t^c,$$

where C_t is the amount of the consumption good produced in period t, $X_t^c = (\ln \theta_t^c, \ln \overline{H}_t, \ln \overline{K}_t, \ln I_t, \ln K_t, \ln L_t)$, θ_t^c is a technological shock in period t, $\overline{H}_t$ is the average quality of the labor input in the economy in period t and the average stock of human capital in the economy at the beginning of the same period, $\overline{K}_t$ is the average stock of physical capital in the economy at the beginning of period t, I_t is the amount of the capital goods produced in period t, K_t is the input of physical capital services in period t, and the average stock of physical capital at the beginning of the same period, and L_t is the input of labor services in period t. The vector φ and the matrix ϕ consist of constant technological parameters that are defined in the Appendix.

We assume that function $F(\cdot)$ satisfies the standard neoclassical conditions on the aggregate technological possibilities. That is, F is homogeneous of degree one in (I_t, K_t, L_t) for all $(\theta_t^c, \overline{H}_t, \overline{K}_t)$ and F is concave in $(\ln I_t, \ln K_t, \ln L_t)$. The consumption production possibilities allow for exogenous technical progress if we assume that θ_t^c grows. It is possible to get endogenous technical progress due to the externality impact of $\overline{K}_t$ (Romer 1986) and endogenous technical progress due to the externality impact of $\overline{H}_t$ (Lucas 1988).

[2] The debates in the empirical literature about convergence (see Quah 1994; Mankiw, Romer, and Weil 1992; and Barrow and Sala-i-Martin 1992) can be formally evaluated in a single structural stochastic model that allows for conditional or/and unconditional convergence as well as exogenous and endogenous growth.

[3] Using a simple modification to the error processes, the model can be estimated using the panel data of countries. Hence, the model provides a parametric specification for the general class of stochastic models that are discussed by Quah (1994).

The traditional internal effect of human capital on production comes through the definition of labor services as

$$L_t = U_t H_t, \tag{2}$$

where U_t is the fraction of time each household devotes to market activities. Since all households are identical, we, trivially, have that:

$$\bar{K}_t = K_t \quad \text{and} \quad \bar{H}_t = H_t. \tag{3}$$

We shall continue to use the $(-)$ notation to distinguish between the external effect of physical and human capital from its internal effect. It is precisely due to the combination of these two effects that our formulation allows for increasing returns to scale in production with respect to physical or human capital. Human capital accumulates according to the Cobb-Douglas transition function:

$$H_{t+1} \le \theta_t^h \bar{H}_t^{\chi_{\bar{h}}} H_t^{\chi_h} U_t^{\chi_u} V_t^{\chi_v}, \tag{4}$$

where θ_t^h is a technological shock at time t, V_t is the fraction of time that each household devotes to direct human capital accumulation in period t. The vector of parameters in (4) satisfy: $0 \le \chi_{\bar{h}}, \chi_h < \infty$, $0 \le \chi_u, \chi_v \le 1$ and $(\chi_v, \chi_u) \ne (0, 0)$. In this formulation human capital accumulates through learning by doing in production due to the presence of U_t in the RHS of (4), and human capital accumulates by directly devoting time to it, due to the presence of V_t on the RHS of (4). Moreover, (4) allows for decreasing, constant as well as increasing returns to scale in the accumulation of human capital. The latter could be due to an "external" effect of human capital accumulation through $\bar{H}_t$, if $\chi_{\bar{h}} > 1$ (Azariadis and Drazen 1990), or due to an internal effect of human capital accumulation through $\bar{H}_t$ if $\chi_h > 1$, or due to some combination of external and internal effects of human capital accumulation, if $\chi_{\bar{h}} + \chi_h > 1$. The latter specification allows for decreasing returns-to-scale with respect to the accumulation of a household's own human capital. Furthermore, it allows for society's human capital as well as the individual households to play a role in the latter's accumulation process.

Physical capital accumulates according to a Cobb-Douglas transition function:

$$K_{t+1} \le \theta_t^k K_t^{\psi_k} I_t^{\psi_i}, \tag{5}$$

where θ_t^k is a technological shock in period t, and $\psi = (\psi_k, \psi_i)'$ are the constant technological parameters such that $0 \le \psi_k \le 1$, $0 \le \psi_i \le 1$ and $\psi_i + \psi_k = 1$. This capital accumulation function incorporates the hypothesis of adjustment costs in the physical capital accumulation process, if the transformation of current capital to next period capital is concave.

However, there are no adjustment costs in the model if we assume that $\theta_t^k \equiv 1$ for all t, $\psi_i = 1$, and $\psi_k = 0$, so that (5) is redundant and (1) may be thought of as a transformation function between consumption goods and all capital goods. That is, firms in the economy are using capital services and labor services to produce consumption goods, maintaining the old capital goods and producing new capital goods. If the maintenance of the old capital goods and the production of new capital goods are characterized by the usual neoclassical technological assumptions, our formulation applies.[4]

The representative household's lifetime utility is given by:

$$\sum_{t=\tau}^{\infty} \beta^{t-\tau} U(X_t^u), \tag{6}$$

where $U(X_t^u)$ is the concave temporal utility function, $X_t^u = (\ln \theta_t^u, \ln C_t, \ln U_t, \ln V_t)$, $\beta \in (0,1)$ is the household's pure time preference factor adjusted for population growth, and θ_t^u is a random shock to preference at time t.[5] The utility function is concave in $(\ln U_t, \ln V_t)$ and is assumed to have the translog specification such that:

$$U(\theta_t^u, C_t, U_t, V_t) \equiv \omega' X_t^u + \tfrac{1}{2} X_t^{u\prime} \Omega X_t^u, \tag{7}$$

where the vector ω and the symmetric matrix Ω are defined in the Appendix. The specification implies that the temporal utility is additively separable in the log of consumption and the remaining elements of the X_t^u vector. That is, the coefficient of relative risk aversion is equal to one. On the other hand, this temporal utility function allows for the disutility of work to be different from the disutility of direct human capital accumulation.

The stochastic element of the environment is defined by the stochastic process of the vector $\theta_t = (1, \ln \theta_t^c, \ln \theta_t^h, \ln \theta_t^k, \ln \theta_t^u)'$. These technological and preferences shocks represent aspects of the economy that are not fully modeled here and are treated as exogenous to the economy. The $\{\theta_t\}$ process evolves according to the law of motion:

$$\theta_{t+1} = Z\theta_t + \epsilon_{t+1}, \tag{8}$$

where ϵ_{t+1} is a vector white noise process with mean $\ln(\eta)$ and the eigenvalues of Z are in modulus less than $\beta^{-1/2}$, that is, we allow for geometric growth of order no greater than $\beta^{-1/2}$ and permits the introduction of constant exogenous technological progress. Note that this restriction, along with some additional restrictions on the preferences and technology parameters of the model, place a growth bound on the laws of motion of the endogenously determined variables.[6]

3. EQUILIBRIUM CHARACTERIZATION

Let P_t, R_t, W_t, Π_t denote the price of new capital goods, capital services, labor services, and per household profits, relative to the price of consumption goods in

[4] In (5) $I = 0$ implies that $K = 0$. This can be easily modified by adding a constant to I.

[5] Assuming a constant population growth rate, n, we have $\beta = (1+n)/(1+r)$ and if $n > 0$ both capital accumulation processes should be modified.

[6] To illustrate the usefulness of the specification, in Eckstein, Foulides, and Kollintzas (1993) we explicitly present how the specification nests several leading growth models, such as Solow (1956), Romer (1986) and Lucas (1988) models.

period t, respectively. The competitive economy is described by infinitely lived given number of agents who maximize their expected utility, (6), by the choice of $\{C_t^d, I_t^d, U_t, V_t\}_{t=\tau}^{\infty}$ subject to standard budget constraints. Furthermore, each firm maximizes per-period profits by the choice of $\{C_t^s, I_t^s, K_t^d, L_t^d\}_{t=\tau}^{\infty}$, where the superscript d and s represent demand and supply, respectively.[7]

Let lower-case letters denote the natural logarithms of the variables in the corresponding upper case letters, so that $u_t = \ln U_t$, $h_t = \ln H_t$, and so on. We show that if $\{C_t^d, I_t^d, U_t, V_t\}_{t=\tau}^{\infty}$ is an interior solution to the representative household's problem, then (u_t, h_{t+1}, i_{t+1}) must be a solution to:[8]

$$\max_{\{u_t, h_{t+1}, k_{t+1}\}} \Big\{ U\Big(\theta_t^u, \ln\big(e^{r_t+k_t} + e^{w_t+u_t+h_t} + e^{\pi_t} - e^{p_t+\psi_i^{-1}(k_{t+1}-\theta_t^k-\psi_k k_t)}\big),$$

$$u_t, \chi_v^{-1}\big(h_{t+1} - \theta_t^h - \chi_{\bar{h}}\bar{h}_t - \chi_h H_t - \chi_u u_t\big)\Big)$$

$$+ \beta E_t U\Big(\theta_{t+1}^u, \ln\big(e^{r_{t+1}+k_{t+1}} + e^{w_{t+1}+u_{t+1}+h_{t+1}} + e^{\pi_{t+1}}$$

$$- e^{p_{t+1}+\psi_i^{-1}(k_{t+2}-\theta_{t+1}^k-\psi_k k_{t+1})}\big),$$

$$u_{t+1}, \chi_v^{-1}\big(h_{t+2} - \theta_{t+1}^h - \chi_{\bar{h}} - \bar{h}_{t+1} - \chi_h h_{t+1} - \chi_u u_{t+1}\big)\Big)\Big\}.$$

Therefore, the following (Euler) conditions must hold in equilibrium:

(9)

$$\chi_v \omega_c s_t^1 + \chi_v(\omega_u + \omega_{u\theta}\theta_t^u + \omega_{uu}u_t + \omega_{uv}v_t) - \chi_u(\omega_v + \omega_{v\theta}\theta_t^u + \omega_{vu}u_t + \omega_{vv}v_t) = 0,$$

(10) $$(\omega_v + \omega_{v\theta}\theta_t^u + \omega_{vu}u_t + \omega_{vv}v_t)$$

$$+ \beta E_t\big(\chi_u \omega_c s_{t+1}^l - \chi_h(\omega_v + \omega_{v\theta}\theta_{t+1}^u + \omega_{vu}u_{t+1} + \omega_{vv}v_{t+1})\big) = 0$$

(11) $$-s_t^i + \beta E_t(\psi_i s_{t+1}^k + \psi_k s_{t+1}^i) = 0,$$

where,

$$s_t^i \equiv C_t^{-1}P_t I_t = -\big[\varphi_i + \varphi_{i\theta}\theta_t^c + \varphi_{ii}i_t + (\varphi_{i\bar{k}} + \varphi_{ik})k_t + \varphi_{il}u_t + (\varphi_{i\bar{h}} + \varphi_{il})h_t\big],$$

(12) $$s_t^k \equiv C_t^{-1}R_t K_t = \varphi_k + \varphi_{k\theta}\theta_t^c + (\varphi_{k\bar{k}} + \varphi_{kk})k_t + \varphi_{ki}i_t + \varphi_{kl}u_t + (\varphi_{k\bar{h}} + \varphi_{kl})h_t,$$

$$s_t^l \equiv C_t^{-1}W_t L_t = \varphi_l + \varphi_{l\theta}\theta_t^c + \varphi_{li}i_t + (\varphi_{l\bar{k}} + \varphi_{lk})k_t + \varphi_{ll}u_t + (\varphi_{l\bar{h}} + \varphi_{ll})h_t$$

are the investment, capital and labor shares of consumption expenditures.

[7] In Eckstein, Foulides, and Kollitzas (1993) we formally define the equilibrium.

[8] A proof of this is available on request. For simplicity and without loss of generality we assume that $\chi_v \neq 0$.

In order to obtain a bivariate dynamic system we use (12) to substitute for s_t^l (i.e. labor's share) in (9), and solve (9) for v_t, we then substitute the result in (4). Under the above substitution, equations (4) and (5) give the following system:

$$y_{t+1} = D_\theta \theta_t + D_y y_t + D_z z_t \tag{13}$$

where $y_t = (h_t, k_t)'$, $z_t = (u_t, i_t)'$, and D_θ, D_y, are matrices of constant parameters that are derived from the model and defined in the Appendix.[9]

In order to seek a solution to the dynamic problem we eliminate u_t and i_t by combining equations (9) through (12), and the result is the second-order dynamic-stochastic expectations equation:

$$E_t(A_2 y_{t+2} + A_1 y_{t+1} + A_0 y_t + B_1 \theta_{t+1} + B_0 \theta_t) = 0 \tag{14}$$

where A_i and B_j $(i = 0, 1, 2;\ j = 0, 1)$ are matrices of parameters defined in the Appendix. The solution for the law of motion for y_t, that satisfies (14) and the above assumptions, provides the equilibrium characterization of the model. The following proposition establishes the basic existence and characterization result of the paper.

PROPOSITION. (*Existence and Representation*). *Suppose the above model, and* (*i*) $\chi_v^2 \rho(\lambda) = (\chi_u \omega_{uv} - \chi_u \omega_{vv}) \neq 0$; (*ii*) $det(A_2) \neq 0$, (*iii*) $det(A_0) \neq 0$; (*iv*) $A_2^{-1} A_0 \neq diag\{\alpha, \alpha\}$, $\forall \alpha \in (-\beta^{-1}, \beta^{-1})$; (*v*) *the eigenvalues of* $\rho(\lambda) = A_2 \lambda^2 + A_1 \lambda + A_0$ *are distinct*; (*vi*) $\rho(\lambda)$ *admits a spectral factorization in the circle of radius* $\beta^{-1/2}$. *Then, there exists a unique recursive competitive equilibrium. That is, there exist matrices*: $\Lambda = diag\{\lambda_h, \lambda_k\}$, $M = diag\{\mu_h, \mu_k\}$, $N = [v_h, v_k]$, *and* $\Psi = [\xi_h, \xi_k]$, *where* $\lambda_h, \lambda_k (\mu_h, \mu_k)$ *are the smallest (largest) modulus eigenvalues of* $\rho(\lambda)$, *and* $v_h, v_k (\xi_h, \xi_k)$ *are linearly independent eigenvectors associated with* $\lambda_h, \lambda_k (\mu_h, \mu_k)$, *such that in this space the equilibrium satisfies the relationship*:

$$y_{t+1} = \Gamma y_t + \Delta \theta_t, \tag{15}$$

where, $y_t = (h_t, k_t)'$, $\Gamma = N \Lambda N^{-1}$, $\Delta = \Pi^{-1} \beta^{-1} A_2^{-1} B_0 + \Sigma_{j=1}^{\infty} \Pi^{-j} (A_2^{-1} B_1 + \beta^{-1} \Pi^{-1} A_2^{-1} B_0) Z^j$, $\Pi = (\Psi M \Psi^{-1} - \Gamma) \Psi M \Psi^{-1} (\Psi M \Psi^{-1} - \Gamma)^{-1}$.

PROOF. Available upon request.[10] The first restriction of the proposition requires that the elasticity

$$-(dV_t/dU_t)(U_t/V_t) = (\chi_u \omega_{vu} - \chi_v \omega_{uu}) / (\chi_v \omega_{uu} - \chi_u \omega_{vv})$$

is bounded away from infinity. This restriction, in turn, allows us to solve (10) for v_t. Conditions (ii) through (v) are regularity requirements and if these conditions are not satisfied the equilibrium equation (10) could be reduced to a single-order stochastic difference equation. The last restriction (vi) is the basic stability condition

[9] In general, all matrices that are not defined in the text can be found in the Appendix.

[10] A more general description of the proof and the restrictions is also available in Eckstein, Foulides and Kollintzas (1993).

that ensures that we may take the eigenvalues of Γ and Π to be less than $\beta^{-1/2}$ in modulus (Kollintzas 1986).

The main implication of the proposition is that under the restrictions above the equilibrium has an exact log-linear representation. It is straightforward to compute the aggregate levels of human capital, the capital stock, investment, employment, wages, interest rate, and the relative price of investment goods to consumption goods, (see Eckstein, Foulides, and Kollintzas 1993).

In order to characterize the equilibrium growth path of the economy, we combine the law of motion of the log of human and physical capital stocks, (15), with the law of motion of the vector of exogenous stochastic shocks, (8), to get:

$$\begin{bmatrix} y_{t+1} \\ \theta_{t+1} \end{bmatrix} = \begin{bmatrix} \Gamma & \Delta \\ 0 & Z \end{bmatrix} \begin{bmatrix} y_t \\ \theta_t \end{bmatrix} + \begin{bmatrix} 0 \\ \epsilon_{t+1} \end{bmatrix}. \tag{16}$$

It is clear from (16) that the law of motion for the endogenous stocks of the economy (y) simply combine linearly the past values of y and θ, while the exogenous shocks follow an independent path. The linearity allows us to combine all possible cases of exogenous and endogenous growth to characterize the growth properties of the economy. It it also clear from (16) that there is no need to assume that growth is balanced between the different stocks and flows of consumption and investment in the economy.

An important aspect of the closed form solution of the model economy is our ability to characterize the transitional dynamics of the variables to their long-run paths. The transitional dynamics are different according to the specification of the economy, but they are especially important when convergence is slow (i.e. $\|\Gamma\|$ is less than but close to one). Furthermore, from the solution to equation (13) it is possible to show that stability rules out complex eigenvalues if and only if $A_1' = A_1$ and $A_0 = B^{-1}A_2$ in (13). However, in our case these conditions do not hold and it is possible to get endogenous cycles. Monotone transitions to the long run are, certainly, also possible.

4. CONCLUDING REMARK

The existence of closed-form solutions for the economy enables us to describe the many kinds of growth patterns that are consistent with the single setup. Hence, the reduced form laws of motion of the stocks of human and physical capital from models of endogenous and exogenous growth could have exactly the same form. That is, log-linear models of capital stocks can be consistent with any existing model of growth and the reduced form provide an observationally equivalent representation of these models. However, given the closed-form solution, it is possible to use the entire assumed structure of the economy to provide a complete interpretation of observed patterns of growth and business cycle fluctuation via, for example, simulation experiments or formal econometric analysis. The next step is clear—use the data to reexamine the literature debates on the sources of growth and the long-run properties of world income distribution.

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APPENDIX

(i) *Equation* (1) *Definitions.*

$$\varphi = (\varphi_\theta, \varphi_{\bar{h}}, \varphi_{\bar{k}}, \varphi_i, \varphi_k, \varphi_l)' \quad \text{and} \quad \phi = \begin{bmatrix} \varphi_{\theta\theta} & \varphi_{\theta\bar{h}} & \varphi_{\theta\bar{k}} & \varphi_{\theta i} & \varphi_{\theta k} & \varphi_{\theta l} \\ & \varphi_{\bar{h}\bar{h}} & \varphi_{\bar{h}\bar{k}} & \varphi_{\bar{h}i} & \varphi_{\bar{h}k} & \varphi_{\bar{h}l} \\ & & \varphi_{\bar{k}\bar{k}} & \varphi_{\bar{k}i} & \varphi_{\bar{k}k} & \varphi_{\bar{k}l} \\ & & & \varphi_{ii} & \varphi_{ik} & \varphi_{il} \\ & & & & \varphi_{kk} & \varphi_{kl} \\ & & & & & \varphi_{ll} \end{bmatrix}.$$

(ii) *Equation* (7) *Definitions.*

$$\omega = (\omega_\theta, \omega_c, \omega_u, \omega_v)' \quad \text{and} \quad \Omega = \begin{bmatrix} \omega_{\theta\theta} & 0 & \omega_{\theta u} & \omega_{\theta v} \\ & 0 & 0 & 0 \\ & & \omega_{uu} & \omega_{uv} \\ & & & \omega_{vv} \end{bmatrix}.$$

(iii) *Equation* (13) *Definitions.*

$$D_z = \begin{bmatrix} \rho^{-1} & -\rho^{-1} & \varphi_{il} \\ 0 & & \psi_i \end{bmatrix}, \quad D_y = \begin{bmatrix} \chi_{\bar{h}} + \chi_h - \rho^{-1}(\varphi_{\bar{h}l} + \varphi_{ll}) & -\rho^{-1}\varphi_{kl} \\ 0 & \psi_k \end{bmatrix},$$

where $\rho = \chi_v^{-2}(\chi_v \omega_{uv} - \chi_u \omega_v)$.

$$D_\theta = \begin{bmatrix} \chi_v \dfrac{\chi_v(\varphi_l + \omega_u) - \chi_u \omega_v}{\chi_u \omega_{vv} - \chi_v \omega_{uv}} & 1 & 0 & \chi_v \dfrac{\chi_v \omega_{u\theta} - \chi_u \omega_{\theta v}}{\chi_u \omega_{vv} - \chi_v \omega_{uv}} & -\rho^{-1}\varphi_{\theta l} \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

(iv) *Equation* (14) *Definitions.*

$$A_2 = \begin{bmatrix} \rho\varphi_{ll} + \chi_h & \psi_i^{-1}\varphi_{il}(1 + \varphi_{ll} - \rho x_h) \\ \rho(\psi_i \varphi_{kl} - \psi_k \varphi_{il}) & \psi_i^{-1}\{-\psi_k(\varphi_{ii} + \varphi_{il}^2) + \psi_i(\varphi_{ik} + \varphi_{il}\varphi_{kl})\} \end{bmatrix}.$$

$$A_1 = \begin{bmatrix} (\varphi_{\bar{h}l} + \varphi_{ll})(1 + \varphi_{ll} - \rho\chi_h) & (\psi_i^{-1}\psi_k \varphi_{il} - \varphi_{kl})(\rho\chi_h - \varphi_{ll} - 1) + \beta^{-1}\rho\psi_i^{-1}\varphi \\ \quad -(\chi_{\bar{h}} + \chi_h)(\chi_h + \rho\varphi_{ll}) - \beta^{-1} & \\ \psi_i(\varphi_{\bar{h}k} + \varphi_{kl}) - \psi_k(\varphi_{\bar{h}i} + \varphi_{il}) & -\psi_i(\varphi_{\bar{k}k} + \varphi_{kk}) - \varphi_k(\varphi_{\bar{k}i} + \varphi_{ik}) + (\varphi_{ii} + \varphi_{il}^2) \\ \beta^{-1}\rho\varphi_{il} - \rho\{\chi_{\bar{h}} + \chi_h - e^{-1}(\varphi_{\bar{h}l} + \varphi_{ll})\} & (\psi_k^2 + \beta^{-1}\psi_i^{-1}) + \varphi_{kl}(\psi_i \varphi_{kl} - 2\psi_k \varphi_{il}) - \psi_k \varphi_i \\ (\psi_i \varphi_{kl} - \psi_k \varphi_{il}) & \end{bmatrix}$$

$$A_0 = \begin{bmatrix} \rho(\varphi_{\bar{h}l} + \varphi_{ll}) + (\chi_{\bar{h}} + \chi_h) & \rho\left(\varphi_{kl} - \psi_i^{-1}\psi_k\varphi_{il}\right) \\ \varphi_{il} + \varphi_{\bar{h}i} - \varphi_{il}\{\rho(\chi_{\bar{h}} + \chi_{\bar{h}} + \chi_h) - (\varphi_{\bar{h}l} + \varphi_{ll})\} & \varphi_{\bar{k}i} + \varphi_{ik} + \varphi_{il}\left(\varphi_{kl} - \psi_i^{-1}\psi_k\varphi_{il}\right) - \psi_i^{-1}\psi_k\varphi_{ii} \end{bmatrix}.$$

$$B_1 = \left[\begin{array}{c:c:c} \varphi_l - \chi_h \dfrac{\omega_u\omega_{vv} - \omega_v\omega_{uv} + \varphi_l\omega_{vv}}{\chi_u\omega_{vv} - \chi_v\omega_{uv}} & \varphi_{\theta l}(1 - \rho\chi_h + \varphi_{ll}) & -\rho\left(\varphi_{ll} + \rho^{-1}\chi_h\right) \\ \quad + (\varphi_l + \omega_u - \omega_v)\left(\varphi_{ll} + \chi_h\rho^{-1}\right) & & \\ \hdashline \psi_i\varphi_k - \psi_k\varphi_i + (\varphi_l + \omega_u - \omega_v) & (\psi_i\varphi_{\theta k} - \psi_k\varphi_{\theta i}) & -\rho(\psi_i\varphi_{kl} - \psi_k\varphi_{il}) \\ \quad (\psi_i\varphi_{kl} - \psi_k\varphi_{il}) & \quad + \varphi_{\theta l}(\psi_i\varphi_{kl} - \psi_k\varphi_{il}) & \end{array}\right.$$

$$\left.\begin{array}{c:c} \psi_i^{-1}\varphi_{il}(\rho\chi_h - \varphi_{ll} - 1) & -\chi_h\dfrac{\omega_{\theta u}\omega_{vv} - \omega_{uv}\omega_{\theta v}}{\chi_u\omega_{vv} - \chi_v\omega_{uv}} + \left(\omega_{\theta u} - \chi_u\chi_{uv}^{-1}\omega_{\theta v}\right) \\ & \left(\varphi_{ll} + \chi_h e^{-1}\right) \\ \hdashline -\varphi_{il}\left(\psi_i^{-1}\psi_k - \varphi_{kl}\right) + \psi_i^{-1}\psi_k\varphi_{ii} - \varphi_{ik} & \left(\omega_{\theta u} - \chi_u\chi_v^{-1}\omega_{\theta v}\right)(\psi_i\varphi_{kl} - \psi_k\varphi_{il}) \end{array}\right].$$

$$B_0 = \left[\begin{array}{c:c:c:c} \dfrac{\omega_u\omega_{vv} - \omega_v\omega_{uv} + \omega_{vv}\varphi_l}{\chi_u\omega_{vv} - \chi_v\omega_{uv}} & \rho\varphi_{\theta l} & 1 & \rho\psi_i^{-1}\varphi_{il} \\ \quad -\rho^{-1}(\varphi_{ll} + \omega_u - \omega_v) & & & \\ \hdashline \rho_i + \varphi_{il}(\varphi_{ll} + \omega_u - \omega_v) & \varphi_{\theta i} + \varphi_{\theta l}\varphi_{il} & \rho\varphi_{il} & -\psi_i^{-1}\left(\varphi_{il}^2 + \varphi_{ii}\right) \end{array}\right.$$

$$\left.\begin{array}{:c} \dfrac{\omega_{\theta u}\omega_{vv} - \omega_{uv}\omega_{\theta v}}{\chi_u\omega_{vv} - \chi_v\omega_{uv}} - \rho^{-1}\left(\omega_{\theta u} - \chi_u\chi_v^{-1}\omega_{\theta v}\right) \\ \hdashline \varphi_{il}\left(\omega_{\theta u} - \chi_v^{-1}\chi_u\omega_{\theta v}\right) \end{array}\right].$$

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