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# THE DYNAMICS OF INFLATION WITH CONSTANT DEFICIT UNDER EXPECTED REGIME CHANGE*

*Benjamin Bental and Zvi Eckstein*

The economic theory of high or hyper-inflation was motivated by observations on major historical inflationary episodes. These episodes tend to display some important common features. The first and most dramatic feature is the behaviour of the monthly inflation rates. These rates have a clear upward trend for a while and finally drop abruptly to zero when a sustained stabilisation programme is enacted. This behaviour is described in detail by Sargent (1982) for the cases of Germany, Austria, Hungary and Poland after World War I. Other well-documented cases which fit the same pattern are those of Hungary and Greece in the aftermath of World War II (see Bomberger and Makinen, 1983; Makinen, 1984). More recently, Israel and Bolivia have also experienced a period of increasing inflation rates which ended with a successful stabilisation programme (Bruno, 1986; Bental and Eckstein, 1988).

Another well-documented common feature is the decline of real balances of high-powered money during the inflationary episode. After stabilisation occurs, real balances increase for a short while, but stop increasing at a value which falls short of their pre-inflation value. This feature is present in all of the episodes mentioned above, and can be seen in Figs. 1–4. There is some indication that the post stabilisation increases in real balances is matched by increases in the real value of assets of the cental bank or by decreases in the value of the national debt (Sargent, 1982; Bental and Eckstein, 1988).

The third feature which seems to characterise inflationary episodes pertains to the behaviour of the government's revenue from increasing the quantity of high-powered money (seignorage). It seems that no clear trend exists in these series over the inflationary period, as can be seen in Figs. 1–4. Sargent and Wallace (1973) computed the seignorage series for all the cases mentioned above as well as for the Russian episode of the twenties, and comment that 'these data are generally without noticeable trends'. Bruno (1986) and Bruno and Fischer (1986) as well as Bental and Eckstein (1988) document that the Israel case fits the same pattern.

The real revenue of the inflation tax seems to be quite small in most cases. Cagan (1956), Barro (1972), and Sargent (1982) conclude that in many of the post World War I inflationary episodes the seignorage collected by the government fell short of the maximal amount sustainable in steady state.

Finally, the stabilisation policies adopted by the governments in the diverse

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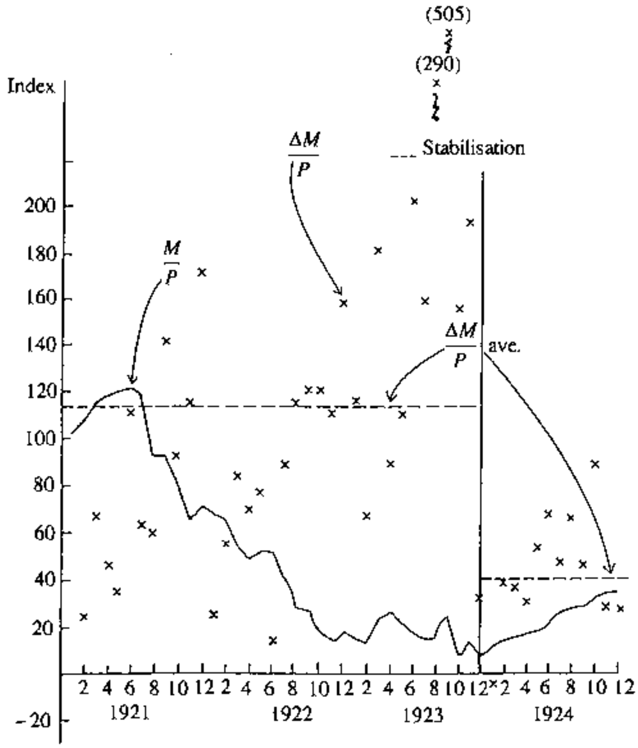


Fig. 1. German data (source: Sargent, 1982). ——, Real balances; ×, seignorage; ---, average seignorage.

countries and historical periods mentioned above seem also to be strikingly similar. In all cases the government budget deficit was eliminated. Furthermore, it seems that the budget was balanced mainly by cutting subsidies to consumers and by increasing direct and consumption taxes (Sargent, 1982). It is not always easy to calculate the government consumption because of the large share of the subsidies in the total expenditure, but at least for the case of Israel it can be documented that government consumption did not change substantially after the stabilisation (Bruno, 1986).

There have been many attempts to address these facts in formal analyses. However, most attempts address only some of the facts, and ignore others. A recent attempt to explain the behaviour of prices, real balances and the inflation tax revenue during the inflationary period has been done by a multi-steady-state theory (for example, Sargent and Wallace, 1987; Quah, 1987; Bruno, 1989). In this theory there are two steady-state inflation rates that correspond to the same government deficit. The lower of the two is dynamically unstable while the higher is stable. Accordingly, an economy in which the government runs a constant deficit is likely to be on the path of increasing inflation rates that converge to the higher steady state. However, this theory ignores the end of the inflationary process and does not incorporate the stabilisation policy.

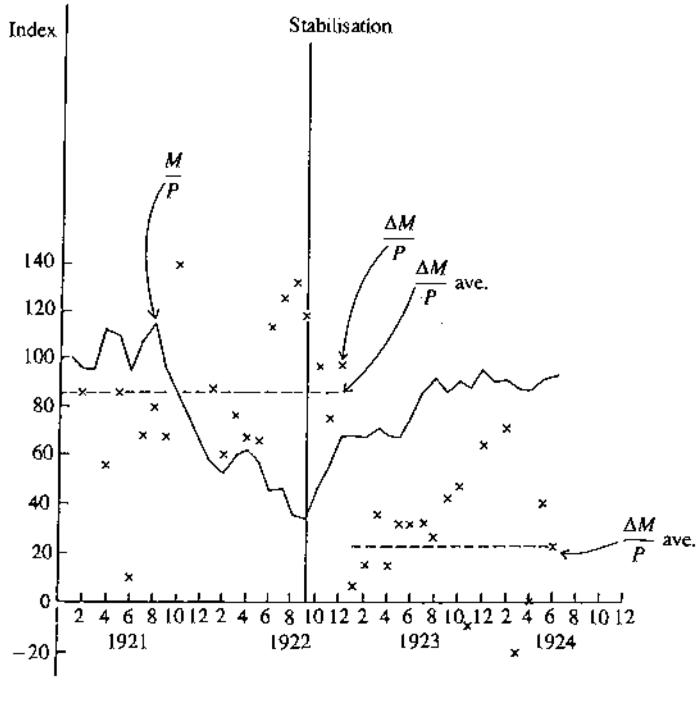


Fig. 2. Austrian data (source: Sargent, 1982). ——, Real balances; ×, seignorage; – – –, average seignorage.

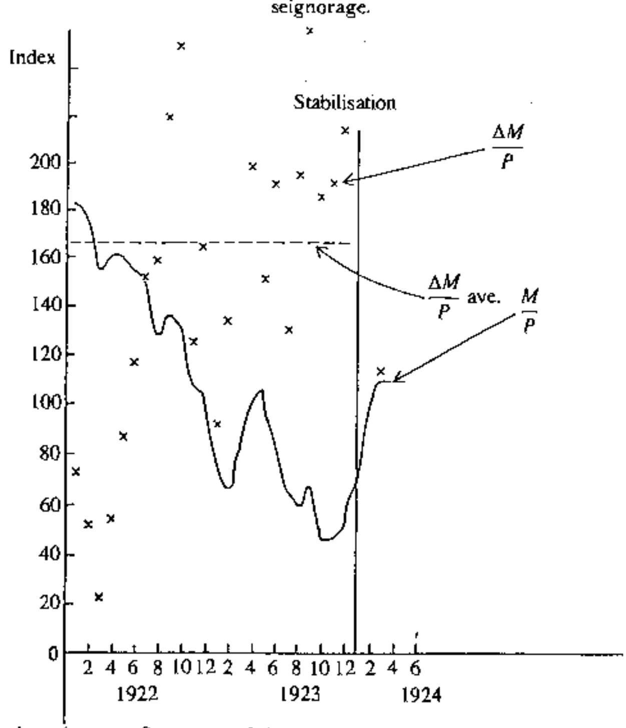


Fig. 3. Polish data (source: Sargent, 1982). ——, Real balances; ×, seignorage; – – –, average seignorage.

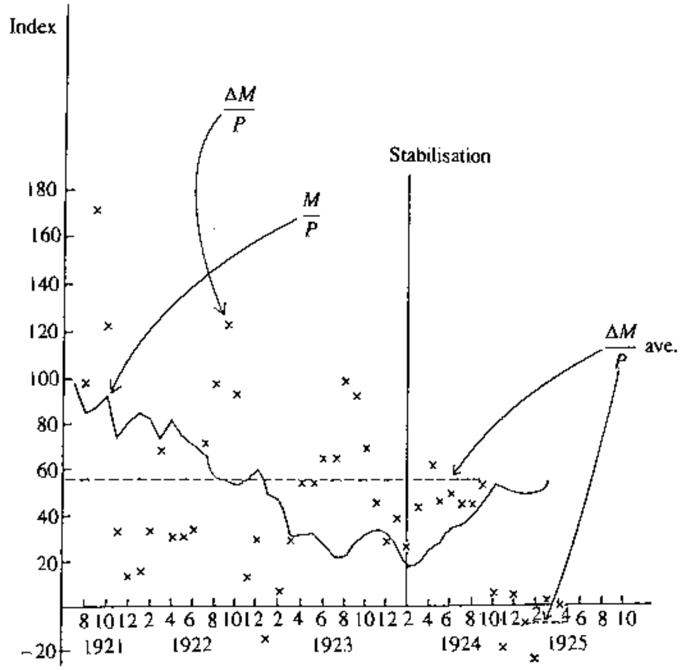


Fig. 4. Hungarian data (source: Sargent, 1982). ——, Real balances; ×, seignorage; ---, average seignorage.

On the other hand, there is a class of models which explain increasing inflation rates while allowing agents to anticipate a change in the government's policies. However, none of these models attempt to generate the dynamics of inflation jointly with the above observations on seignorage before and after the stabilisation. For example, Flood and Garber (1980), La Haye (1985) as well as Drazen and Helpman (1985) imply increasing seignorage during the inflationary period. Sargent and Wallace (1981) imply no inflation tax prior to the policy change and a constant inflation tax revenue after the change.

The goal of our paper is to provide a framework that is compatible with all the common characteristics of the inflationary episodes under the discipline of rational expectations. We want to concentrate on the trends of the different variables and choose accordingly to construct a deterministic environment. We assume that money demand is stable, and that it increases in disposable income and decreases in expected inflation. Agents know in advance the whole future path of government policies. In particular, they know the date and composition of any stabilisation package.[1] After stabilisation the government's budget is balanced and the price level remains fixed.[2]

[1] A more general model would incorporate uncertainty concerning both the date and composition of the stabilisation programme, as suggested by Drazen and Helpman (1985). However, we believe that our main conclusions are robust with respect to the introduction of uncertainty as long as the probability of stabilisation increases in time.

[2] This specification is not common to all studies of hyperinflation. In Sargent and Wallace (1987), Dornbusch and Fischer (1986), as well as Bruno (1986), inflation may occur even when there are no government deficits and the nominal money supply is held constant. We believe that the evidence from the behaviour of economies with zero monetary growth favours strongly the view that the price level remains fixed in such circumstances.

The rationally expected stabilisation programme rules out the explanations offered by the multi-steady state approach. With expected stabilisation, the economy has a unique equilibrium path which is associated with the low steady state inflation. However, the properties of the equilibrium path prior to stabilisation depend on the size of the (constant) pre-stabilisation deficit as well as on the particulars of the stabilisation programme. Specifically, we show that high deficits (higher than the maximal deficit sustainable in steady state) require a future stabilisation, and as a result inflation rates decrease towards the stabilisation date. Increasing inflation is, therefore, compatible only with low pre-stabilisation deficits. However, low deficits are just a necessary condition for inflation to be increasing. An additional requirement is that demand for real balances after stabilisation falls. Given our assumptions on the money demand, a stabilisation programme which balances the budget by increasing taxes rather than by decreasing government consumption will lead to a fall in the demand for real balances. Thus, our model is compatible with all the common features of actual inflationary episodes described above. Moreover, it establishes a strict correspondence between the set of policies followed in these episodes and the behaviour of the inflation rates.

We discuss the properties of the model in Section I. In particular, we make some assumptions on the shape of the inflation-tax curve. In Section II, we show that these assumptions can be justified by several money-demand models. A money demand equation that is similar to that suggested by Cagan (1956), is one possible approach. In addition we show that money demand equations that are generated explicitly in an overlapping generations model or derived from cash in advance constraints as in Lucas and Stokey (1987), are also compatible with our specification. In Section III we characterise the pre-stabilisation equilibrium path and derive the conditions that yield increasing inflation rates. Section IV summarises our findings.

## I. A GENERAL CLASS OF MONETARY MODELS

We analyse a discrete time, constant population, single good model in which the *per-capita* government consumption each period is $g_t$ units of the good. The government finances its consumption by *per-capita* taxes, $\tau_t$, and money creation. Accordingly, the government's budget constraint is given by

$$d_t = g_t - \tau_t = \frac{M_t - M_{t-1}}{p_t}, \tag{1}$$

where $d_t$ is the *per-capita* deficit, $M_t$ is the *per-capita* nominal balances at the 'end' of the period $t$ and $p_t$ is the price level.

We postulate that the *per-capita* equilibrium demand for real balances, $m_t$, is given by

$$m_t = f(y - \tau_t, \pi^e_{t+1}), \tag{2}$$

where $y$ represents *per capita* income (constant over time) and $\pi^e_{t+1}$ is the expected gross inflation rate between periods $t$ and $t+1$, given by $p_{t+1}/p_t$. We assume that the money demand is increasing in disposable income so that $f_1 > 0$ and is decreasing in expected inflation, that is, $f_2 < 0$, where subscripts denote the appropriate partial derivatives. In the following section we discuss some models that yield a money demand equation of this form.[3]

An equilibrium consists of sequences $\{m_t, \pi_{t+1}\}_{t=1}^{\infty}$ such that for given policy sequences $\{g_t, \tau_t, M_t\}_{t=1}^{\infty}$ and an initial $M_0$ (i) the money market clears for every $t$, and (ii) perfect foresight prevails, that is

$$\pi_{t+1}^e = \frac{p_{t+1}}{p_t} = \pi_{t+1}.$$

The equality of supply and demand in the money market implies:

$$m_t = \frac{M_t}{p_t}. \tag{3}$$

Using (3) in (1) and the definition of $\pi_t$ yields:

$$\pi_t = \frac{m_{t-1}}{m_t - d_t}. \tag{4}$$

Accordingly, the equilibrium inflation sequence must satisfy (4) and (2), and is given by

$$\pi_t = \frac{f(y-\tau_{t-1}, \pi_t)}{f(y-\tau_t, \pi_{t+1}) - d_t}. \tag{5}$$

Since $f_2$ is assumed to be negative everywhere, we can use the implicit function theorem to obtain

$$\pi_t = h(y-\tau_{t-1}, y-\tau_t, d_t, \pi_{t+1}). \tag{6}$$

The function $h(\cdot)$ has several properties which can be derived from the assumptions made so far. First, the signs of the derivatives of $f(\cdot)$ imply $h_1 > 0$, $h_2 > 0$, $h_3 > 0$ and $h_4 > 0$. In addition, equation (5) implies that if $\tau_t = \tau$ and $d_t = 0$ for all $t$, $\pi_t = 1$ for all $t$ solves (6), i.e. $1 = h(y-\tau, y-\tau, 0, 1)$. Thus, in the absence of a deficit, a steady state exists in which prices never change.

Next we note that if the money demand function $f(\cdot)$ is derived from a model in which individual budget constraints are respected, then, given the taxes $\tau$, there must exist a finite upper bound on the amount of seignorage the government can raise in steady state, $\bar{d}$. Moreover, we assume that given a deficit which falls short of the maximal deficit $\bar{d}$, there exist two steady state inflation rates which solve (6), so that the inflation-tax 'Laffer curve' has a bell shape.

The assumption on the behaviour of the inflation tax curve, together with $h_3 > 0$ require that $h_{44} > 0$ (see Fig. 5). This curvature determines the dynamics of the system for $d < \bar{d}$. In particular, the higher steady state inflation

[3] This setup is consistent with an environment in which the *per capita* capital stock is fixed. With this assumption the real interest rate is constant, and *per capita* income is fixed.

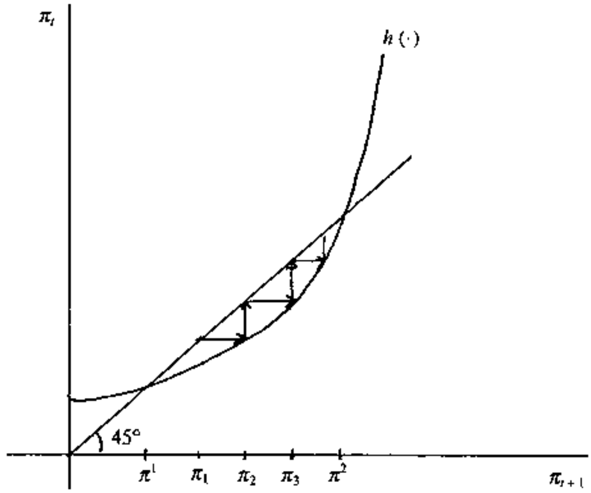


Fig. 5. The dynamic stability of $\pi^2$.

rate, denoted by $\pi^2$, is dynamically stable while the lower, denoted by $\pi^1$, is not. In other words, inflation paths which start in the neighbourhood of $\pi^2$ converge to that rate, while paths which start in the neighbourhood of $\pi^1$ diverge from it.

Standard macroeconomic theory provides no guidance related to the shape of the inflation tax curve. Nevertheless, the bell-shaped inflation tax curve is often assumed, and many of the models used in theoretical and empirical analyses of high inflation economies possess such a curve. In fact, the dynamic stability of the higher steady state inflation rate provided the basis of the analyses of runaway inflations in Sargent and Wallace (1987), Quah (1987) and Bruno (1989) among others.

The instability of the lower steady state inflation rate has some unpleasant implications for economies with no government deficit. In such economies the equilibrium in which the value of money is fixed and positive is unstable, while the stable steady state equilibrium is the one in which money has no value. This problem no longer exists if the value of money is bounded from below by some device at a strictly positive number $\mu$ (see Wallace (1981) for an example of a scheme which provides such a lower bound). Given this lower bound, the only equilibrium is the one in which the value of money remains fixed at a positive value. Any other equilibrium would entail increasing inflation rates and decreasing real money balances tending to zero. Therefore, the value of money must reach $\mu$ at some finite date $T$. However, this value cannot decrease below $\mu$ and therefore the inflation rate cannot continue to rise after $T$. But if inflation stops at $T$, from a repeated application of (6) we obtain that the inflation cannot rise prior to $T$ either.

The introduction of a lower bound $\mu$ on the value of money does not affect the analysis of economies in which the government collects an inflation tax as long as $d_t > \mu$ for all $t$. Since $\mu$ can be chosen to be arbitrarily small, the additional assumption about its existence does not interfere with the dynamics of the inflationary path while it serves to guarantee the uniqueness of the monetary equilibrium for a zero deficit economy.

The next section provides three monetary models which fit the general framework discussed above. The reader may proceed directly to Section IV without loss of continuity.

## II. THREE UNDERLYING MODELS

To support our claim that the features of the general model are compatible with existing monetary theories, we discuss three examples of monetary environments that fit the framework of Section I.

### *Cagan Demand for Money*

This classical demand for money specification is of particular importance in empirical studies. The demand for money is given by

$$m_t = A\, e^{-b(\pi_{t+1}-1)}(y-\tau_t), \tag{7}$$

with $b > 0$, which clearly satisfies the assumptions on $f(\cdot)$. Accordingly the $h(\cdot)$ function (equation (6)) satisfies all conditions concerning the first derivatives.

For this model, it is convenient to express $\pi_{t+1}$ as a function of $\pi_t$, $y-\tau_{t-1}$, $y-\tau_t$ and $d_t$. At constant values of $\tau_t$ and $d_t$ the inverse function of equation (6) is given by

$$\pi_{t+1} = -\frac{1}{b}\ln\left[\frac{1}{\pi_t}e^{-b(\pi_t-1)} + \frac{d}{A(y-\tau)}\right] + 1. \tag{8}$$

Obviously, when $d = 0$, $\pi_t = \pi_{t+1} = 1$ solves equation (8). Further, for sufficiently small values of $b$ (in particular $0 < b < 1$), the r.h.s. of (8) is a concave function of $\pi_t$, so that the equivalent to $h_{44} > 0$ is satisfied. Hence, the Cagan equation satisfies the conditions of the general model.

### *An Overlapping Generations Model*

In this setting the economy is postulated by two-period lived overlapping generations of equal size. Agents are endowed with $y_1 > 0$ units of a nonstorable consumption good in the first period of their lives and with $y_2 \geqslant 0$ units of the good in the second period. Every member of the initial old generation is endowed with $M_0$ units of money (in addition to $y_2$). The young agents have identical, strictly concave utility functions defined over their lifetime consumption bundle, given by $u[c_1(t), c_2(t)]$, with $c_i(t)$ denoting the consumption of an agent born at period $t$ at age $i$, $i = 1, 2$. The initial old want to maximise their consumption.

Assuming that only the young are taxed, demand for real balances is derived from the maximisation of $u[c_1(t), c_2(t)]$, subject to

$$c_1(t) + \frac{M_t}{p_t} = y_1 - \tau_t \tag{9}$$

and
$$c_2(t) = y_2 + \frac{M_t}{p_{t+1}}. \tag{10}$$

The demand for real balances $m_t$ satisfies

$$\pi_{t+1} = \frac{u_2[y_1 - \tau_t - m_t, y_2 + (1/\pi_{t+1})\, m_t]}{u_1[y_1 - \tau_t - m_t, y_2 + (1/\pi_{t+1})\, m_t]}. \tag{11}$$

Under certain conditions on $u(\cdot)$, equation (11) can be solved for $m_t$, yielding a money-demand equation that is similar in form to equation (2) (see Wallace, 1981). The differences between (2) and the money demand equation resulting from (11) are due to the overlapping generation structure. For example, the real balances ($m$) are measured per member of the younger generation. Likewise, $y$ in (2) corresponds to $y_1$ here, and $y_2$ could be viewed as a shift parameter of the function $f(\cdot)$. If we assume that $c_1(t)$ and $c_2(t)$ are both normal goods and gross-substitutes, we obtain the desired signs for the derivatives of this money-demand function, namely

$$\frac{\partial m_t}{\partial(y_1 - \tau_t)} > 0 \quad \text{and} \quad \frac{\partial m_t}{\partial \pi_{t+1}} < 0.^4$$

Accordingly, the first derivatives of $h(\cdot)$ (equation (6)) have the correct signs. The sign of $h_{44}$ depends on third derivatives of the utility function so that different examples may be established. For the popular example of $u(c_1, c_2) = \ln c_1 + \ln c_2$, equation (2) is linear, given by $m_t = \frac{1}{2}(y_1 - \tau_t - \pi_{t+1} y_2)$ and $f_{22} = 0$. The function $h(\cdot)$ takes the form

$$\pi_t = \frac{y_1 - \tau_{t-1}}{y_1 - \tau_t - 2d_t - (\pi_{t+1} - 1)\, y_2}$$

so that
$$\frac{\partial^2 \pi_t}{\partial \pi_{t+1}^2} > 0, \quad \text{i.e. } h_{44} > 0.$$

*A Cash in Advance Model*

Here we use a framework similar to that suggested by Lucas and Stokey (1987). The economy is populated by an infinitely lived representative consumer. The consumption goods are classified into two subsets. First, there are goods that can be obtained by credit, called the 'credit goods'. The second subset consists of goods that have to be bought with cash, called the 'cash goods'. Each subset is represented by a single good. The monetary utility derived from consuming $c_{1t}$ units of the cash good and $c_{2t}$ units of the credit good at period $t$ is given by $u(c_{1t}, c_{2t})$.

Each period, agents are endowed with $y$ units of a resource. Every unit of this resource can be costlessly converted to one unit of the cash good or the credit

[4] The derivatives of the money demand equation are obtained from the first-order condition for money holding. Accordingly, the properties of the bordered Hessian matrix for the utility function $u(\cdot)$ are crucial for the analysis. The normalcy and gross-substitution assumptions provide the necessary information about these properties.

good. In addition, the government consumes $g$ units *per capita* of the resource. Therefore, the *per capita* resource constraint of the economy is given by

$$c_{1t}+c_{2t}+g \leqslant y. \tag{12}$$

The *per capita* government's budget constraint at period $t$ is given by

$$g-\tau = \frac{M_t - M_{t-1}}{p_t}, \tag{13}$$

where $m_t$ is the *per capita* amount of money at the end of period $t$.

Individual budget constraints take the following form. First, the overall expenditure has to satisfy

$$p_t(c_{1t}+c_{2t})+M_t+B_t \leqslant p_t(y-\tau)+(1+R_t)\,B_{t-1}+M_{t-1}, \tag{14}$$

when $B_t$ is the amount of net loans taken by the agent at period $t$, and $R_t$ is the nominal interest rate.

In addition, the cash good at period $t$ must be purchased with money accumulated at period $t-1$ so that

$$p_t\,c_{1t} \leqslant M_{t-1}. \tag{15}$$

Given $M_0$ and $B_0$, the representative consumer maximises $\sum_{t=0}^{\infty}\beta^t u(c_{1t}, c_{2t})$ subject to (14) and (15), where $\beta < 1$ is the subjective discount factor. We assume that in equilibrium the cash in advance constraint (15) is binding, as well as the budget constraint (14). In addition, we know that in equilibrium $B_t = 0$ for all $t$. Using (12), the equilibrium first-order conditions for an optimum are

$$-\pi_{t+1}\,u_2(m_t, y-g-m_t)+\beta u_1(m_{t+1}, y-g-m_{t+1}) = 0 \tag{16}$$

and

$$-\pi_{t+1}\,u_2(m_t, y-g-m_t)+(1+R_{t+1})\,\beta u_2(m_{t+1}, y-g-m_{t+1}) = 0, \tag{17}$$

where

$$m_t = \frac{M_{t-1}}{p_t}.$$

We rewrite (13) as

$$g-\tau = \pi_{t+1}\,m_{t+1}-m_t. \tag{18}$$

Equations (16), (17), and (18) describe the dynamics of the system. The equivalent of equation (2) can be obtained by eliminating $m_{t+1}$ from (16) and (18). The equivalent to equation (6) can then be obtained by substituting the result of the aforementioned procedure in (18) for both $m_t$ and $m_{t+1}$, and solving for $\pi_{t+1}$ in terms of $\pi_{t+2}$.

The complexity of the model precludes a meaningful general analysis. However, for a special case where $u(c_{1t}, c_{2t}) = \alpha c_{1t}+\ln c_{2t}$ we obtain

$$m_t = y-\tau-d-\frac{\pi_{t+1}}{\alpha\beta}, \tag{19}$$

and

$$\pi_{t+1} = \frac{y-\tau}{y-\tau-d-\dfrac{\pi_{t+2}}{\alpha\beta}+\dfrac{1}{\alpha\beta}}, \tag{20}$$

where $d = g - \tau$. Equations (19) and (20) have the required properties (in particular $h_{44} > 0$).

## III. STABILISATION POLICIES

We consider equilibria that result from a specific class of government policies. In particular, we assume that there exists a date $T$, with $1 < T < \infty$, after which the government conducts a stabilisation policy that generates a stable price path. Accordingly, $\pi_{t+1} = 1$ for $t \geqslant T$. The date $T$ is called the stabilisation date. There are several possible stabilisation policies that would generate a constant price level. That is, any sequence of $d_t$ and $\tau_t$ that satisfies (5) given that $\pi_{t+1} = 1$, for all $t \geqslant T$ is a stabilisation programme. Obviously, there are many such programmes and proposition 1 below holds for any stabilisation programme. Here we are particularly interested in the zero deficit programme. To guarantee that $\pi_{t+1} = 1$ for $t \geqslant T$, we assume that there exists a scheme which guarantees that the value of money never falls below some $\mu > 0$. We can now establish the following result.

PROPOSITION 1. *If there exists a date $T$ such that $\pi_{t+1} = 1$ for $t \geqslant T$, then the inflation path for $t < T$ is determined uniquely.*

*Proof.* Since $\pi_{T+1} = 1$ and $h_4 > 0$, (6) determines $\pi_T$ uniquely. Repeated application of all this procedure determines the inflation path for all $t < T$.

Notice that the uniqueness of the equilibrium depends only on the fact that $h_4 > 0$, which implies that the demand for money must be strictly decreasing with respect to expected inflation. Further, Proposition 1 holds regardless of the particular nature of the policy sequence after the stabilisation date, as long as the policy generates price level stability in a credible way. Conditional on the assumption that stabilisation is expected, the proposition rules out theories which explain hyperinflations as being a result of the multiplicity of rational expectations equilibria mentioned above.

We turn next to the analysis of the pre-stabilisation inflation rates. In particular, we restrict the class of government policy sequences at the pre-stabilisation date. To mimic our general observation that the deficits were approximately constant during the inflationary phase, we set $d_t = d$ and $\tau_t = \tau$ for $t \leqslant T-1$. Under these restrictions our aim is to identify conditions under which the pre-stabilisation inflation rates are increasing towards the stabilisation date.

PROPOSITION 2. *Let $d_t = d$ and $\tau_t = \tau$, for $t \leqslant T-1$. Then, a necessary condition for the inflation rates to increase is that the pre-stabilisation deficit satisfies $d < \bar{d}$.*

*Proof.* Using the assumption that $h_{44} > 0$, we have that for any $d > \bar{d}$, $\pi_t = h(y-\tau, y-\tau, d, \pi_{t+1}) > \pi_{t+1}$ for all $t \leqslant T-1$. Therefore, inflation must be decreasing towards stabilisation.

Proposition 2 establishes the fact that if the pre-stabilisation government deficit is bigger than the maximal steady state deficit, inflation cannot be increasing regardless of the tax/transfers related to the stabilisation policy. Furthermore, it is not feasible to have such deficits unless the economy is to be stabilised. Therefore, the government can finance a deficit in the 'short-run' which is not maintainable in the 'long-run', and inflation rates must be decreasing towards the stabilisation date. This result has the same flavour as that of Sargent and Wallace's (1981) 'unpleasant monetarist arithmetic'. There the government uses bonds to finance its deficit, a policy which is not feasible in the long run. Eventually the government reverts to money printing. As a result, inflation rates may be increasing during the initial phase in which no money is printed in anticipation of the high future steady-state inflation.

As a result of Proposition 2, we further restrict the class of pre-stabilisation policies that may be consistent with increasing inflation rates. Specifically, the deficit is now assumed to be at a level that could be maintained in steady state. We first show that the unique equilibrium path established in Proposition 1 converges (backwards) to the lower steady-state inflation rate. This we do by delaying the stabilisation date $T$ indefinitely in order to evaluate the effect of the delay on the initial inflation rate, $\pi_1$.

PROPOSITION 3. *Let* $d_t = d$ *and* $\tau_t = \tau$ *for* $t = 0, 1, 2, \ldots, T-1$, *with* $0 < d < \bar{d}$. *Then, for any* $\pi_T < \pi^2$, $\lim_{T\to\infty} \pi_1 = \pi^1$ (*where* $\pi_1$ *is the inflation at the initial date, and* $\pi^1$ *and* $\pi^2$ *are the roots of* $h(y-\tau, y-\tau, d, \pi) = \pi$, *with* $\pi^1 < \pi^2$).

*Proof.* The shape of h$(\cdot)$ ($h_4 > 0$ and $h_{44} > 0$), implies that given $\pi_T < \pi^2$ the sequences $\{\pi_{T-i}\}_{i=0}^{T-1}$ are strictly monotone and satisfy $|\pi_{T-j-2} - \pi_{T-j-1}| < |\pi_{T-j-1} - \pi_{T-j}|$ for $j = 0, \ldots, T-3$. Therefore, given any $\epsilon > 0$, there exists a $T$ sufficiently large such that $|\pi_1 - \pi^1| < \epsilon$. By continuity of $h(\cdot)$ $\lim_{T\to\infty} \pi_1 = \pi^1$ (a graphical exposition is given in Fig. 6).

The expected future stabilisation not only rules out multiple equilibria (Proposition 1) but also reverses the classification of the 'stable' and the

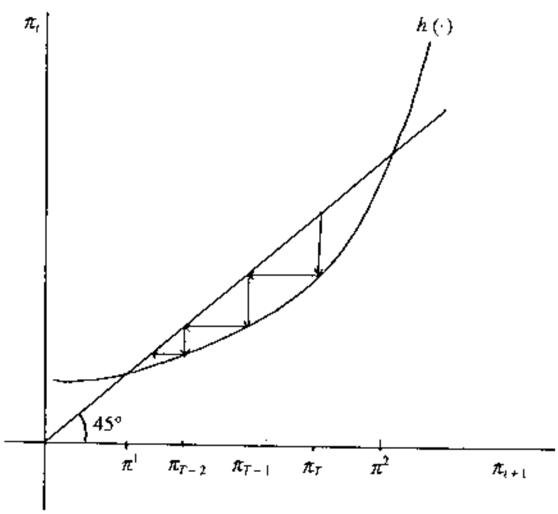


Fig. 6. The backwards stability of $\pi^1$.

'unstable' inflation paths. That is, the only equilibrium path converges to the low inflation if the stabilisation date is delayed indefinitely. Therefore, the explanation of runaway inflations based on the stability of the path converging to the high steady-state inflation ($\pi^2$) is inconsistent with expected stabilisation.

Notice that if the stabilisation policy implies $\pi_T > \pi^2$, then the stabilisation date cannot be postponed indefinitely because inflation rates become arbitrarily large as time progresses. In this case, a stabilisation must occur within a finite time and inflation must be decreasing towards stabilisation. Another case in which inflation is decreasing towards stabilisation is characterised next:

COROLLARY 1. *If* $d_t = d$ *and* $\tau_t = \tau$ *for* $t \geq 0, 1, \ldots, T-1$ *with* $0 < d < \bar{d}$ *and* $\pi_T = 1$, *then* $\pi_{T-i} > \pi_{T-i+1}$ *for* $i = 1, \ldots, T-1$.

*Proof.* The low steady-stage root $\pi^1$ for $0 < d < \bar{d}$ is greater than 1. Therefore, $\pi_{T-1} = h(y-\tau, y-\tau, d, 1) > 1$ and

$$\pi_{T-i} = h(y-\tau, y-\tau, d, \pi_{T-i+1}) > \pi_{T-i+1} \quad \text{for} \quad i = 2, \ldots, T-1.$$

The corollary argues that if $\pi_T = 1$, a hyperinflation is ruled out. A sufficient condition to obtain $\pi_T = 1$ is to maintain $\tau_t = \tau$ for all $t$ with $d_t = 0$ for $t \geq T$. In this case $\pi_T = h(y-\tau, y-\tau, 0, 1)$, and by the assumption made on $h(\cdot)$, $\pi_T = 1$. Consequently, increasing inflation rates towards stabilisation are not consistent with a policy that keeps post-stabilisation taxes fixed at their pre-stabilisation levels and reduces the deficit solely by decreasing government consumption. This conclusion can be reformulated as follows.

PROPOSITION 4. *Let* $\tau_t = \tau_1$ *and* $d_t = d$ *with* $0 < d < \bar{d}$ *for* $t \leq T-1$. *Further, let* $d_t = 0$ *and* $\tau_t = \tau_2$ *for* $t \geq T$. *Then* $\tau_2 > \tau_1$ *is a necessary condition for inflation rates to be rising towards stabilisation.*

*Proof.* To get $\pi_t < \pi_{t+1}$ for $t \leq T-1$ it is necessary and sufficient to have $\pi^1 < \pi_T < \pi^2$. But $\pi_T = h(y-\tau_1, y-\tau_2, 0, 1)$. As $\pi^1 > 1$, we must have $\pi_T > 1$. Since $h(y-\tau_1, y-\tau_1, 0, 1) = 1$ and $h_2 < 0$, $\tau_2 > \tau_1$ is necessary to get $\pi_T > 1$. Fig. 7 shows an appropriate configuration of $\tau_1$ and $\tau_2$.

Proposition 4 states that a positive inflation at the stabilisation date is necessary if inflation is to increase prior to stabilisation. This inflation must be due to reduced demand for real balances because the government deficit has been eliminated. The required reduction in real balances is, in this model, a result of increases in post-stabilisation taxes.[5] As we argued above, the historical stabilisation programmes have indeed involved tax increases and subsidy increases (rather than cuts in government consumption). Furthermore, post-stabilisation real balances have not returned to their original levels. These observations are consistent with our model.

[5] The importance of the tax scheme at a regime-change is demonstrated in a different context in Aiyagari (1985). There it is shown that Sargent and Wallace's (1981) monetarist arithmetics depend crucially on the tax policy at the end of the bond-financing phase.

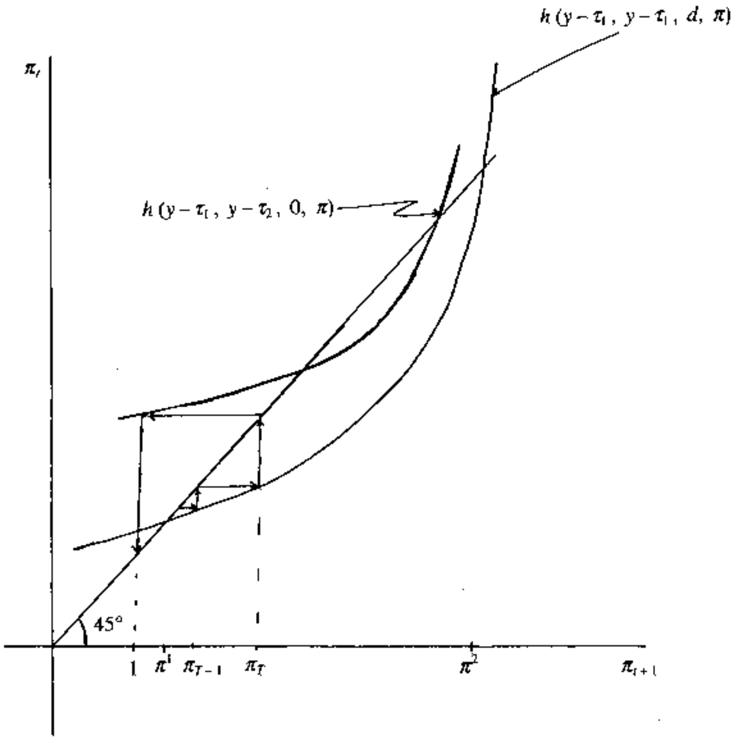


Fig. 7. Increasing inflation.

## IV. CONCLUDING REMARKS

The theoretical structure analysed in this paper succeeds in providing a unified framework within which all major phenomena common to high inflation episodes are present. Specifically, we show that a period of increasing inflation rates that ends abruptly, with falling real balances and (more-or-less) constant inflation tax revenues can be a result of the expected stabilisation programme. We identified two conditions that must be present in order to generate the inflationary period. First, the inflation tax revenues should be smaller than the maximum sustainable in steady state, and second, the expected stabilisation programme must reduce the demand for real balances by increasing taxes. We argue that both conditions were fulfilled in the actual historical episodes. In this way we show that Sargent's (1982) view about the end of big inflations is consistent with the inflationary process itself when the end is anticipated by the population.

The post stabilisation period in all the historical episodes is characterised by an increase in the supply of high powered money that did not affect the price level. This phenomenon is explained by Sargent (1982) as a result of the change in the asset composition of the central banks that accompanied the increased money supply. Essentially, the central banks exchanged money for private and public debt. Several authors have shown that such open market operations are neutral (Wallace, 1981; Chamley and Polemarchakis, 1984; Peled, 1985). All of these models are compatible with our second example, in which the demand for money is derived from an overlapping generations structure, and in which money is essentially undominated in return. It is clear that the assumption on the lack of rate of return dominance is reasonable for the post stabilisation period, whereas during the inflation money was clearly dominated by other assets (such as foreign currency). Accordingly, Sargent's (1982) explanation of this particular phenomenon is consistent with our description of the pre-stabilisation path of the economy. In fact, the post stabilisation open market operations are neutral not only with respect to the price path which follows these operations, but also with respect to the price path which precedes them, provided that the open market operations are fully anticipated.

The expected stabilisation induces a unique equilibrium in our model. If the stabilisation is expected to occur in the distant future, the inflation rate in the economy will be close to the lower of the two inflation rates that generate the required seignorage. In models that do not include the anticipated stabilisation feature, this low inflation is dynamically unstable and therefore sustainable only under very special specification of the policy parameters (see Sargent and Wallace (1987)). Our result that the low inflation rate is the stable one conforms with similar conclusions derived by Marcet and Sargent (1987). The fact that the latter models are quite different from ours makes the conclusion more robust and calls into question previous explanations of high inflation that depended on the multiple-steady-state theory.

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## References

Aiyagari, S. Rao (1985). 'Deficits, interest rates and the tax distribution.' *Federal Reserve Bank of Minneapolis Quarterly Review*, Winter, pp. 5–14.

Barro, Robert, J. (1972). 'Inflationary finance and the welfare cost of inflation.' *Journal of Political Economy*, vol. 80, September/October, pp. 978–1001.

Bental, B. and Eckstein, Z. (1988). 'Inflation, deficit and seignorage with expected stabilisation.' In (E. Helpman, A. Razin and E. Sadka, eds.). *Economic Effects of the Government Budget*. M.I.T. Press.

Bomberger, William A. and Makinen, Gail E. (1983). 'The Hungarian hyperinflation and stabilisation of 1945–46.' *Journal of Political Economy*, vol. 91, October, pp. 801–24.

Bruno, Michael (1986). 'Israel's stabilisation, the end of the "lost decade"?' unpublished, The Hebrew University, June.

—— (1989). 'Econometrics and the design of economic reform.' *Econometrica*, vol. 57, pp. 275–306.

—— and Fischer, Stanley (1986). 'The inflationary process: shocks and accommodation.' (ed. Yoram Ben-Porath). In *The Israeli Economy: Maturing through Crisis*. Harvard University Press, pp. 347–71.

Cagan, Phillip (1956). 'The Monetary Dynamics of Inflation.' (ed. Milton Friedman). In *Studies in the Quantity Theory of Money*. Chicago University Press.

Chamley, C. and Polemarchakis, H. (1984). 'Assets, general equilibrium and the neutrality of money.' *Review of Economic Studies*, vol. 51, pp. 129–30.

Drazen, Alan and Helpman, Elhanan (1985). 'Inflationary consequences of uncertain macroeconomic policies.' Foerder Institute for Economic Research, Working Paper no. 21 (July).

Dornbusch, Rudiger and Fischer, Stanley (1986). 'Stopping hyperinflations past and present.' NBER Working Paper no. 1810.

Flood, Robert P. and Garber, Peter M. (1980). 'An economic theory of monetary reform.' *Journal of Political Economy*, vol. 88 (February), pp. 24–58.

La Haye, Laura (1985). 'Inflation and currency reform.' *Journal of Political Economy*, vol. 93, June, pp. 537–60.

Lucas, R. E. and Stokey, N. L. (1987). 'Money and interest in a cash in advance economy.' *Econometrica*, vol. 55(3), May, pp. 491–514.

Makinen, Gail, E. (1984). 'The Greek stabilisation of 1944–46.' IER 74, December, pp. 1067–74.

Marcet, A. and Sargent, T. J. (1987). 'Least squares learning and the dynamics of hyperinflation.' manuscript.

Peled, D. (1985). 'Stochastic inflation and government provision of indexed bonds.' *Journal of Monetary Economics*, pp. 291–307.

Quah, D. (1987). 'Stable and unstable inflationary dynamics and exogenous seignorage financing.' MIT unpublished.

Sargent, Thomas, J. (1977). 'The demand for money during hyperinflation under rational expectations: I.' *International Economic Review*, vol. 18, February, pp. 59–82.

—— (1982). 'The ends of four big inflations.' (ed. Robert E. Hall). In *Inflation: Causes and Effects*. Chicago University Press.

—— and Wallace, Neil (1973). 'Rational expectations and the dynamics of hyperinflation.' *International Economic Review*, vol. 14, pp. 328–50.

—— —— (1981). 'Some unpleasant monetarist arithmetics.' *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall, pp. 1–15.

—— —— (1987). 'Inflation and the government constraint.' (ed. A. Razin and E. Sadka). In *Economics in Theory and Practice*. Macmillan.

Wallace, Neil (1981). 'A Modigliani-Miller theorem for open market operations.' *American Economic Review*, vol. 71, pp. 267–74.