International Capital Markets and Wealth Transfers*

Magnus Dahlquist† Christian Heyerdahl-Larsen§
Anna Pavlova‡ Julien Pénasse¶

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†Stockholm School of Economics and CEPR, magnus.dahlquist@hhs.se.
‡London Business School and CEPR, apavlova@london.edu.
§Kelley School of Business, Indiana University, chheyer@indiana.edu.
¶University of Luxembourg, julien.penasse@uni.lu.

Abstract

In times of global stress, there are large movements in exchange rates and asset prices. Currencies of developed economies appreciate, with the US dollar appreciating the most. Global stock markets fall, but the US market falls by less. While the external balance sheet of the US is riskier and its net foreign assets fall, this effect is overturned by the dollar appreciation, resulting in a wealth transfer to the US. To rationalize these facts, we build a general equilibrium model with time-varying risk appetites that produces asymmetric portfolios. Richer countries have more appetite for risk, levering up their external portfolios by borrowing from poorer countries. Consequently, their net foreign assets fall in times of stress, yet there is a wealth transfer from poor to rich countries due to currency appreciations. The model delivers time-varying currency risk premia, matches key asset pricing moments, and produces realistic external portfolios.

Keywords: Currency risk premium, habit formation, net foreign assets, wealth transfers.
JEL Classification: E43, F31, G12, G15.

1 Introduction

As capital markets around the world are linked, a shock originating in one market propagates to the rest of the world. During times of global stress, financial markets worldwide suffer losses. However, not all markets are affected alike – some fall more than others. Times of global stress are also accompanied by large movements in the exchange rates and the US dollar (USD) tends to appreciate during these times. The turmoil in the financial markets accompanied by the USD appreciation is typically associated with alarmingly large drops in the US net foreign assets (NFA) position. This is because the US foreign assets consist mostly of equities while US foreign liabilities consist mostly of USD denominated debt. During normal times, the US exploits an "exorbitant privilege" and earns a risk premium on its NFA position, as documented by Gourinchas and Rey (2007) in their influential work. However, as argued by Gourinchas, Rey, and Govillot (2017), the flip side of the exorbitant privilege is that the US carries an "exorbitant duty" to provide insurance to the rest of the world that pays off during times of global stress. Arguably, the US then transfers wealth to the rest of the world in bad times.

In this paper we revisit the influential hypothesis of Gourinchas et al. and argue that the wealth transfers may go the opposite way, i.e., the world wealth distribution may shift in favor of the US in times of stress. While the NFA indeed fall, US investors benefit from the USD appreciation. This is because they exhibit a home bias in their portfolios[1] and US asset values fall less than asset values in the rest of the world because of the USD appreciation. This helps us resolve the "reserve currency paradox" of Maggiori (2017). We develop a general equilibrium model that generates these patterns. Specifically, we consider a model with time-varying risk appetites that produces asymmetric portfolios. Richer countries effectively have more appetite for risk, levering up their external portfolios by borrowing from poorer countries. This leads to asymmetric responses of macroeconomic variables to underlying shocks. The model is solved in closed form, which allows us to illustrate the economic mechanisms in a transparent way. In

[1] A large literature documents a portfolio home bias (see, e.g., Coeurdacier and Rey, 2013).

addition, the model is able to jointly match key asset pricing moments and deliver time-varying currency risk premia. Notably, the model captures the reversal in currency risk premia over different horizons, which has been challenging to explain within equilibrium models as argued by Engel (2016).

In the economy there are multiple countries, each producing their own good. All goods are tradable and consumers derive utility from consuming all of the goods produced in the economy, albeit with a preference for their domestically produced good. Risk appetites are time varying, which we capture with preferences that exhibit deep habits.[2] A higher consumption of a particular good in the current period makes consumers more willing to buy that good in the future due to the force of habit. In contrast to standard habit formation, deep habits give rise to more realistic exchange rates properties; we show that they are able to generate both exchange rate appreciation in rich countries during times of stress and time-varying currency risk premia.[3]

We solve for equilibrium in our dynamic model in closed form and show that a rich country like the US behaves as more risk tolerant and holds a riskier portfolio than the rest of the world. This is implemented by borrowing from poorer countries to invest in stocks (i.e., the richer countries hold a leveraged position in the stock market). Yet, because of a greater appetite for the domestically produced consumption good,[4] consumers in a rich country do not scale back consumption of the domestic good as much as the rest of the world. They are able to achieve this by buying insurance from the rest of the world, which pays off during times of stress in the domestic market. This channel is new to our model, and it creates a more nuanced response to shocks – rich countries can act as more averse than the rest of the world to domestic output shocks but at the same time be more tolerant in response to other shocks.

The asymmetric responses of richer and poorer countries to shocks drive the dynamics of

[2]In contrast to standard habit formation, as in for instance Campbell and Cochrane (1999), deep habits assume that that the agents have habit formation with respect to each good rather than over the aggregate consumption basket.

[3]Using deep habits is not essential for our main results. In Appendix F we consider a simple model with demand shocks and no habits that qualitatively delivers our main economic mechanism. However, to match currency risk premia and asset prices we need the richer model.

[4]This is an outcome of consumption home bias and deep habits.

net exports (NX), NFA, and cross-country wealth transfers. Consider a rich country like the US. Because US consumers have stronger habits with respect to US-produced goods, they are reluctant to reduce their consumption in those goods and therefore the US exports less of them in response to a negative domestic output shock. At the same time, the lower supply of the US goods pushes up their relative price and makes imports relatively cheaper for US consumers. Consequently, their expenditure on imported goods declines. In response to adverse domestic output shocks, the US then effectively becomes a more closed economy. Following the decline in foreign good prices, foreign stock markets fall. However, as the relative price of US-produced goods rises, the US enjoys a USD appreciation and its stock market falls less than foreign stock markets. This benefits the US investors as most of their wealth is invested in US assets due to their consumption home bias. Their wealth share therefore *increases* following adverse domestic output shocks. In other words, they receive a wealth transfer from the rest of the world. This result is in contrast to Gourinchas et al. (2017), who argue that the US has an exorbitant duty and transfers wealth to the rest of world (provides insurance) at times of stress. Yet, this does not contradict the finding that the NFA of the US fall during times of stress. This is because the US external portfolio is a leveraged position in foreign stock markets. Hence, our model simultaneously implies lower NFA and a wealth transfer to the US, i.e., the US's share of world wealth rises in times of stress. We graphically illustrate this mechanism in Appendix A.

The above mechanism helps resolve what Maggiori (2017) terms the "reserve currency paradox." In the presence of consumption home bias, a wealth transfer from the US to the rest of the world in times of stress should lead to a USD depreciation. The paradox is that we observe the opposite in the data (i.e., a USD appreciation). We argue that there is no paradox, and show empirically and theoretically that the direction of the wealth transfer in times of stress is to the US, not from the US.

We calibrate our model to the G10 countries and show that the model matches standard moments of the stock and foreign exchange (FX) markets.[5] For example, the model delivers

[5]G10 consists of Australia, Canada, Switzerland, Germany, the United Kingdom, Japan, Norway, New

high equity premia and volatile returns. To examine the model's ability to capture the dynamics during times of stress, we compare the model-implied moments to the data using NBER recession dates. We show that the model reproduces the facts that in times of stress: (i) the USD appreciates; (ii) the US stock market falls less than foreign stock markets; (iii) the US NFA fall; and yet (iv) the wealth of the US relative to the rest of the world increases.

Finally, we investigate the currency risk premium implied by the model in more detail. We verify that our model can reproduce the well-established deviation from UIP and the ability of the real exchange rate to predict currency returns. Specifically, the model is rich enough to reproduce the predictability reversal documented by Bacchetta and Van Wincoop (2010) and Engel (2016), which is challenging for leading models in the literature. Engel (2016) argues that this is difficult to match in a frictionless model and suggests an explanation based on liquidity risk. Our model is frictionless and the additional feature we rely on is stochastic volatility of the output growth process (calibrated to the real GDP growth of the G10 countries). The variation in risk appetite together with the stochastic output volatility are enough to resolve the predictability reversal. In conclusion, our model can match the properties of the NFA, create rich wealth dynamics, and jointly match the dynamics of stocks and currencies.

Related literature

Our paper is related to the literature documenting the importance of valuation changes in cross-country portfolio holdings, starting from Lane and Milesi-Ferretti (2001) and Gourinchas and Rey (2007). They highlight the asymmetric nature of the US external portfolio, which consists of a levered position in foreign stock markets with USD denominated liabilities. Gourinchas and Rey (2007) argue that the US earns a risk premium on its external portfolio. The flip side is that the US provides insurance to the rest of the world that pays off during times of global stress, resulting in a wealth transfer from the US to the rest of the world in those times (Gourinchas et al., 2017, and more recently Sauzet, 2022). We argue that the USD appreciation that coincides with times of stress is a boon to the US stock market. This effect can be large

Zealand, Sweden, and the United States.

enough to reverse the direction of the wealth transfer.

Other related papers studying global imbalances, NFA, and the exorbitant privilege include Dou and Verdelhan (2015), Maggiori (2017), and Jiang, Richmond, and Zhang (2020b). Like we do, these papers study the effects of portfolio asymmetries.[6] One of our contributions to this strand of literature is to resolve the "reserve currency paradox" of Maggiori (2017). Several other papers resolve this paradox based on a convenience yield of US bonds, which rises in times of global stress, leading to a USD appreciation and a wealth transfer to the US. This mechanism is complementary to ours. The first paper in this strand is Jiang, Krishnamurthy, and Lustig (2020a), who model explicitly the convenience yield of USD-denominated bonds. Kekre and Lenel (2021) extend their work and embed the convenience yield in a workhorse New Keynesian model. Devereux, Engel, and Wu (2022) microfound the convenience yield as superior collateral properties of US government bonds on intermediaries' balance sheets, also within a New Keynesian model. With Jiang et al. (2020a), we also share an empirical result with that in times of global stress relative wealth of the US vis-à-vis the rest of the world rises, although the details of our wealth computation differ.

Our paper also relates to the macro-finance literature studying asset prices and exchange rates. The closest to our paper are Moore and Roche (2008), Verdelhan (2010), Evans (2014), Heyerdahl-Larsen (2014), and Stathopoulos (2017), who use time-varying risk appetite driven by habit formation to generate realistic asset pricing and exchange rate moments. Other approaches include models with long-run risk (e.g., Colacito and Croce, 2011, Bansal and Shaliastovich, 2013) and rare disasters (Farhi and Gabaix, 2016). Similar to these papers, our model matches asset prices and exchange rate moments, but our primary focus is on asymmetric portfolio positions, NFA, and cross-country wealth transfers.

Finally, our paper also relates to the literature on the currency risk premium and its properties. It is well established that the UIP does not hold and that the interest rate differential predicts currency excess returns (high interest rate currencies appreciate over time, resulting in high returns). Fama (1984) shows that under rational expectations this implies tight con-

[6] Another important paper studying cross-country asymmetries is Hassan (2013).

ditions on the currency risk premium. More recently, Bacchetta and Van Wincoop (2010) and Engel (2016) show that UIP deviations reverse over the horizon. Dahlquist and Pénasse (2022) argue that this reversal relates to the predictability of currency returns by the real exchange rate. Chernov and Creal (2021) study this in a no-arbitrage model of the real exchange rate. Our model reproduces the reversal in general equilibrium.

2 Empirical motivation

In this section, we highlight empirical patterns that help us formulate the theoretical model. First, we show that the USD appreciates during times of stress and that both the USD and the US equity market are less vulnerable to volatility shocks. Next, we discuss that the US has a riskier external balance sheet and that its NFA fall during times of stress. Finally, we argue that in spite of NFA falling the US becomes relatively richer than the rest of the world during times of stress. Appendix B contains a more detailed description of the data.

Figure 1 shows the behavior of FX and equity markets during times of stress. We measure times of stress by NBER recessions and equity volatility. The top panel shows the log real USD against a basket of nineteen currencies (in blue) together with the monthly S&P 500 realized volatility (in red). The exchange rate is expressed in USD per unit of foreign currency basket, which means that an increase in the exchange rate is a USD depreciation against the foreign currencies. Shaded regions are NBER recessions. During recessions, the USD appreciates on average by about 3.1% per year. We view this as a lower bound as the USD tends to revert before a recession ends. Stock market volatility also captures times of stress. Increases in equity realized volatility coincide with USD appreciations. We next test for the statistical significance of such an association.

The bottom left panel shows regressions results for changes in the real exchange rate on realized volatility (in blue) and the VIX (in red). We report volatility beta coefficients and 90% confidence bands from panel regressions with currency fixed effects. All real exchange rates are expressed in domestic currency per unit of foreign currency basket. The figure shows results for the USD, G10 currencies (excluding the USD), and emerging market currencies. The USD has a significantly negative volatility beta, confirming that the USD appreciates in times of stress. The volatility betas for non-USD G10 and emerging market currencies are not statistically significantly different from zero.

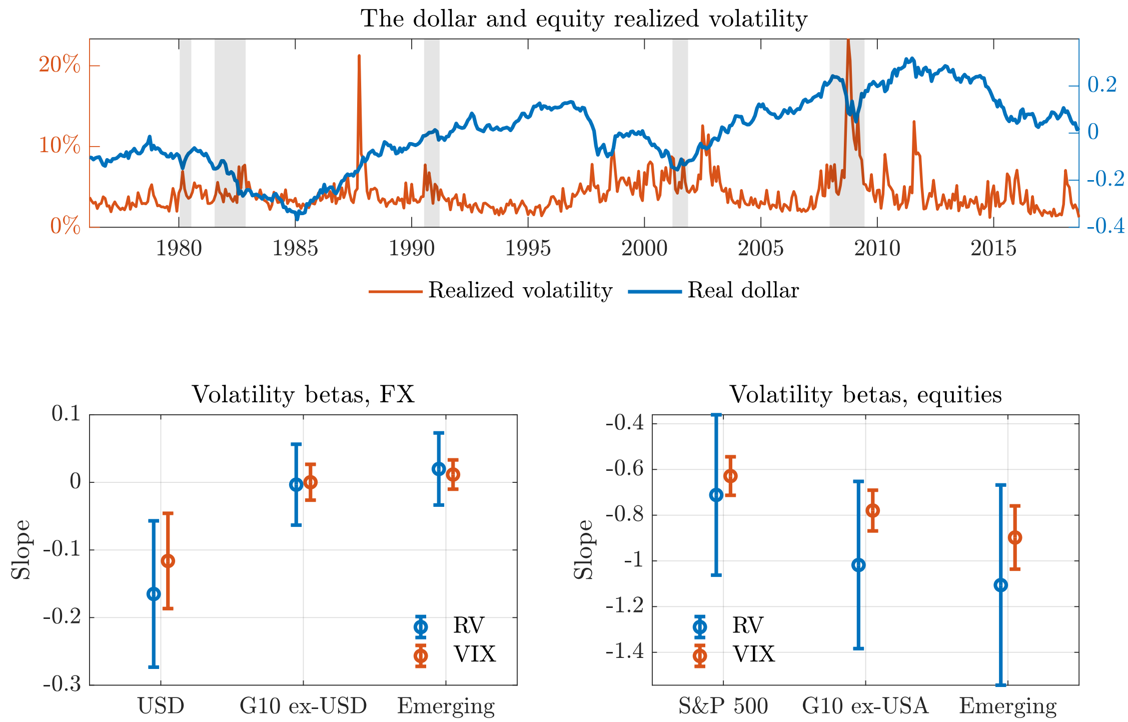

Figure 1: US dollar and equity volatility
The top figure shows the log real USD against a basket of nineteen currencies (in blue) together with the monthly S&P 500 realized volatility (in red). Shaded regions are NBER recessions. The bottom left figure shows panel regression results for changes in the real exchange rate on changes in realized volatility (in blue) and changes in the VIX (in red); the bottom right figure shows the panel regression results for equity excess returns (expressed in USD). Volatility beta coefficients and 90% confidence bands are depicted by circles and error bars. The bottom figures show results for the US, the G10 countries (excluding the US) and ten emerging currencies. All real exchange rates are expressed in domestic currency per unit of foreign currency basket.

It is well established that the US stock return volatility increases in times of stress (see, e.g., Brandt and Kang, 2004; Mele, 2007). The bottom right panel extends the evidence to international equity markets. The figure shows volatility betas on excess returns for the US, G10 markets (excluding the US), and emerging markets. All equity markets exhibit negative volatility betas (i.e., they fall in times of stress as measured by US equity volatility increases). Notably, the US equity market falls less than the G10 markets, which in turn fall less than emerging markets. We present the results for excess returns expressed in USD, but the monotonic pattern is also present for excess returns in local currencies. In sum, the figure shows that the USD appreciates during times of stress, and that both the USD and the US equity market are less exposed to increases in volatility.

We reproduce in Figure 2 important facts about US external positions as documented by Gourinchas and Rey (2015) and Gourinchas et al. (2017). The left panels show the gross equity and debt positions as fractions of GDP. US foreign assets predominantly consist of equities, while US foreign liabilities mostly consist of debt. It means that NFA are disproportionally exposed to equities and earn an equity premium. This results in the exorbitant privilege to the US. The right panel shows NFA over GDP together with the equity volatility. NFA are defined as US net equity positions plus US net debt positions. The NFA fall during times of stress, a consequence of the US net equity position and of the USD appreciation. This results in the exorbitant duty to the US. The great financial crisis (GFC) was a case in point – between 2007:Q4 and 2009:Q1 NFA fell by 14.5 percentage points. Gourinchas et al. (2017) find a similar decline (19 percentage points) using a broader definition of NFA.

The fall in NFA during times of stress reflects a wealth transfer from the US to the rest of the world. However, this transfer does not necessarily imply that the US is relatively worse off in times of stress, as the US relative position depends not only on NFA but also on domestic wealth. Indeed, the results presented in Figure 1 exert an opposing force on US relative wealth. While the USD appreciation tends to have a negative impact on US NFA, it has a positive impact on US domestic wealth. The lower vulnerability of the US equity market also makes the US relatively richer in times of stress. Which force dominates thus depends on the relative change of NFA and domestic wealth.

To illustrate the effect of the dollar appreciation, we consider estimates of total wealth for each of the G10 markets. This is not a straightforward exercise. The previous literature has provided estimates of wealth that vary dramatically. On the one hand, Piketty and Zucman (2014) aggregate wealth and find that the US wealth-income ratio equals around 4. In their

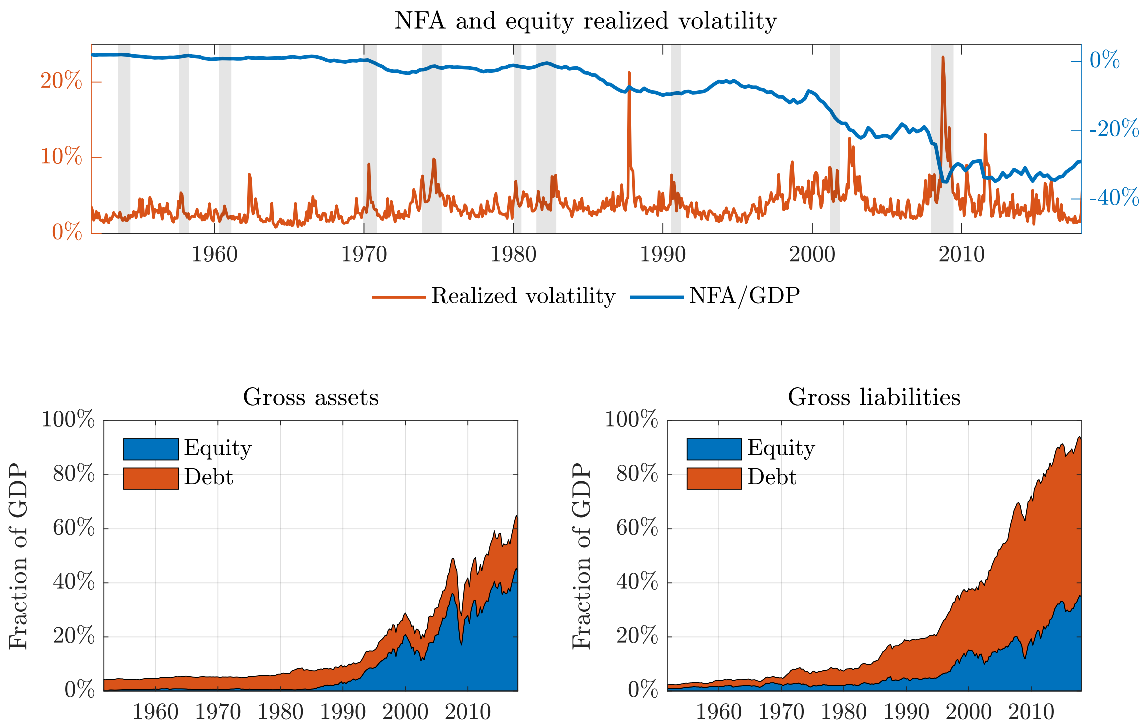

Figure 2: US external positions and equity volatility
The top panel shows net foreign assets (defined as gross equity and debt assets minus gross equity and debt liabilities) as a fraction of GDP (in blue) together with the monthly S&P 500 realized volatility (in red). Shaded regions are NBER recessions. The bottom panels show gross external equity and debt positions as fractions of US GDP.

analysis, wealth is measured based on observables and it excludes, for example, human wealth. On the other hand, the finance literature typically measures wealth as the present value of future consumption. Taking this route, Lustig, Van Nieuwerburgh, and Verdelhan (2013) find that the US wealth-consumption ratio equals around 80 (see also Shiller, 1995). Even if both estimates are very different, the more conservative estimate is more than ten times larger than US NFA, which today equal around 30% of GDP (in absolute value). In what follows, we assume that the domestic wealth-GDP ratio in a given country equals the price-dividend ratio of domestic equities. This assumption implies that total wealth equals the domestic price-dividend ratio times the domestic GDP plus NFA.[7] (Over our sample period, the average US price-dividend ratio equals 41.) We consider our estimate of total wealth being conservative.

Figure 3 shows the share of US wealth over the combined wealth of G10 markets over the 2004–2014 period. The US wealth share increased markedly during the GFC, consistently with our reasoning above. The NFA fell by 14.5 percentage points of GDP during the global financial crisis (a 70% decline). While substantial, the NFA decline coincides with a 17.3% USD appreciation over the same period (a drop in the black plots indicates a USD appreciation). Notice that the USD appreciation peaked at the end of 2008, after which the Global Financial Crisis (GFC) subsided and the VIX also returned to its pre-crisis level. Consistent with our model, the US wealth share is rising when USD is appreciating up until the end of 2008.

The pattern illustrated in Figure 3 is a common pattern across recessions. Given that NFA are relatively small relative to domestic wealth, a USD appreciation is generally sufficient to overcome the fall of NFA. In other words, the US becomes relatively richer in times of stress. In Appendix D, we also present figures for the GDP and consumption shares of the US for the same period. We find that both increase during the GFC, which is consistent with the increase in the US wealth share. In Section 5, we show that this pattern is common across US recessions.

Finally, Appendix E presents an alternative computation of the US wealth share, using

[7] We obtain similar results when defining total wealth in a given country as the domestic price-dividend ratio times consumption.

data from the World Inequality Database (WID), originally assembled by Piketty and Zucman (2014). We again focus on the GFC. We find that up until the end of 2008, the US wealth share was flat and even slightly increasing, primarily thanks to the USD appreciation, but it decreased in 2009. Overall, the US wealth share decreased slightly during the GFC. However, as we remarked previously, Piketty and Zucman (2014)'s wealth estimate does not include human wealth. We follow the literature and think of human capital as a bond (Lustig et al., 2013). Specifically, human capital of the US is close to a bond whose cash flows are denominated in USD. Fluctuations in the USD would to a large extent drive fluctuations in the value of a bond. If we enhance WID wealth with our estimate of human capital, we get a well-pronounced rise in the US wealth share up until the end of the recession in June 2009.

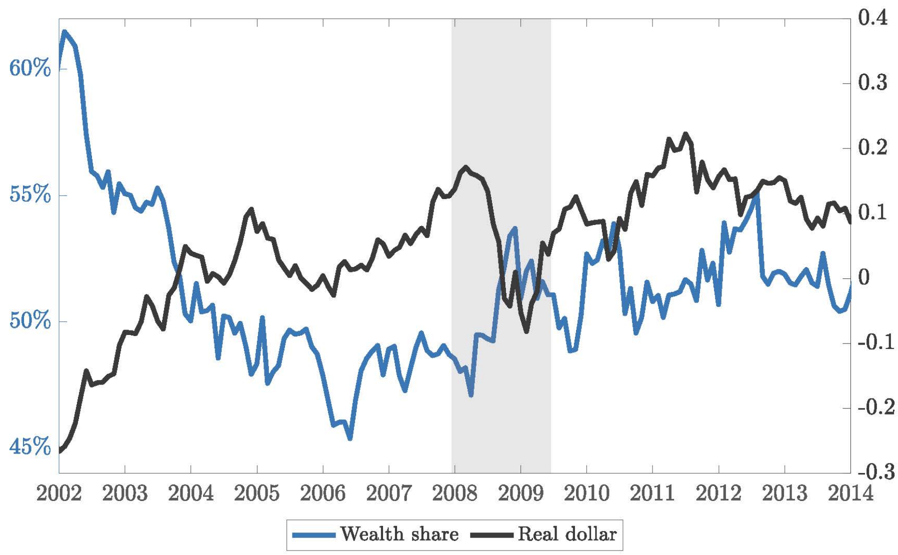

Figure 3: US wealth during the Great Financial Crisis and the real USD
This figure shows the US wealth as a fraction of the combined wealth of economies corresponding to the G10 currencies (in blue) and the log real USD against a basket of G10 currencies (in black). Wealth in a given economy is given by Wealth = PD×GDP + NFA.

3 Model

In this section, we present an N-country economy, which rationalizes the empirical facts in Section 2 and provides additional predictions. The key ingredient of the model is time-varying risk appetites, as in Campbell and Cochrane (1999), which are high during good times but reduce drastically during times of stress. Another important feature is that we allow countries to be asymmetric in terms of their size, which leads to asymmetric external portfolios and wealth transfers across countries in good and bad times. In Section 5.5, we show further that the model delivers time-varying currency risk premia and matches key asset pricing moments. Appendix C contains all proofs.

It is important to note that time-varying risk appetites, which we model using deep habit preferences, are not essential for our main mechanism. Appendix F presents a simple two-country, two-good model with a home bias in consumption that generates the same mechanism. The simple model, however, is not designed to match the currency risk premia and asset pricing moments. We therefore rely on our main model for the quantitative predictions.

3.1 Output

There are N countries in the world economy, each producing its own perishable consumption good. We refer to country one as the home country, the US, and the other $N-1$ countries as foreign. The output of country i is given by

$$Y_{i,t} = Y_t X_{i,t}, \tag{1}$$

where Y_t is a level factor and $X_{i,t}$ is a country-specific factor. The dynamics of the level factor are

$$\frac{dY_t}{Y_t} = \mu_Y dt + \sigma_Y dw_{Y,t}, \tag{2}$$

where $w_{Y,t}$ is a 1-dimensional standard Brownian motion. To model the country-specific factor, we define $x_{i,t} = \log(X_{i,t})$, where

$$dx_{i,t} = \kappa_{x,i} \left(\bar{x}_i - x_{i,t} \right) dt + \sigma'_{x_{i,t}} dw_{X,t}. \tag{3}$$

The shock, $w_{X,t}$, is an N-dimensional standard Brownian motion. The country-specific factors are mean-reverting and consequently the quantities of each good, $Y_{i,t}$, are co-integrated with each other. We stack together the $N+1$ shocks in the vector $w_t = (w_{Y,t}, w_{X,t})$. We assume that the volatility in Equation (3) is $\sigma_{x_{i,t}} = \nu_t \sigma_{x_i}$, where ν_t is a common stochastic volatility component, which can be driven by other shocks than w_t. We specify the dynamics of ν_t when calibrating the model in Section 5, as it is not critical for the derivation of the equilibrium.

The role of the level factor is primarily to aid our calibration, and specifically to help us match the equity premium. Shocks to this factor leave exchange rates unaffected. Most of the interesting dynamics in our model is driven by shocks to the country-specific factors, which we heareafter refer to as domestic output shocks. These shocks are global in nature since they are propagated to other countries through trade.

3.2 Preferences

We introduce time-varying risk appetites of investors by adopting habit-based preferences, which is one of the workhorse preferences in finance (e.g., Campbell and Cochrane, 1999). Risk appetites are high when investors' consumption significantly exceeds their habits; the opposite is true when consumption is close to the habit. Specifically, each country is populated by a representative investor with preferences given by

$$U^j = E \left[\int_0^\infty e^{-\rho t} \sum_{i=1}^{N} \log \left(C_{i,t}^j - H_{i,t}^j \right) dt \right], \tag{4}$$

where $C_{i,t}^j$ and $H_{i,t}^j$ are the consumption and habit of good i for the investor in country j, respectively, and ρ is the time preference parameter. We assume that the habit for each good is external (i.e., the investor does not take into account how consumption today affects the habit level in the future). Moreover, we model habit as good-specific, that is, "deep habits" as proposed by Ravn, Schmitt-Grohé, and Uribe (2006).[8] Hence, there is a separate habit for each good. With deep habits, as opposed to habit defined over the aggregate consumption basket, the relative prices are volatile even with smooth consumption of the individual goods.

Assumption 1. *We assume that the habit of good i for the investor in country j is $H^j_{i,t} = \phi^j_i H_{i,t}$ with $\sum_{i=1}^N \phi^j_i = 1$, $\sum_{j=1}^N \phi^j_i = 1$ and $\phi^j_i \geq 0$ for $i, j = 1, \ldots, N$, where $H_{i,t} < Y_{i,t}$ for all t.*

Assumption 1 implies that all investors benchmark their consumption against the same habit level for each good, $H_{i,t}$, but attach different importance of the habit governed by ϕ^j_i. Note that except from the importance attached to the habit level, ϕ^j_i, there is no heterogeneity in the preferences.

Another important feature of agents' preferences is consumption home bias.

Assumption 2. *Agents exhibit consumption home bias, that is, $\phi^j_j > \phi^j_i$ for $i \neq j$.*

Assumption 2 implies that the weights on the domestically produced good in the consumption basket is higher than foreign produced consumption goods. This common assumption is a robust feature of the data and can be microfounded by explicitly modeling non-traded goods.

Generalizing Campbell and Cochrane (1999) to an environment with multiple goods, we define the aggregate surplus consumption ratio for good i as

$$s_{i,t} = \frac{Y_{i,t} - H_{i,t}}{Y_{i,t}}. \tag{5}$$

A low surplus consumption ratio for good i corresponds to bad times for consumers of good i and more so for consumers of country i as they exhibit a home bias in consumption. As suggested by Menzly, Santos, and Veronesi (2004), it is convenient to work with the inverse surplus consumption ratio

$$\mathcal{R}_{i,t} \equiv \frac{1}{s_{i,t}}. \tag{6}$$

[8] van Binsbergen (2016) also considers preferences with deep habits to study asset prices in a production economy.

We make the following assumption about its dynamics.

Assumption 3. *The inverse surplus consumption ratio of good i, $\mathcal{R}_{i,t}$, has a two-factor structure and can be decomposed as $\mathcal{R}_{i,t} = G_t G_{i,t}$ where*

$$dG_t = \kappa \left(\bar{G} - G_t\right) dt - \alpha \left(G_t - \lambda\right) \sigma_Y dw_{Y,t} \quad (7)$$

$$dG_{i,t} = \kappa_i \left(\bar{G}_i - G_{i,t}\right) dt - \alpha_i \left(G_{i,t} - \lambda_i\right) \sigma^{'}_{x_{i,t}} dw_{X,t}, \quad (8)$$

where $\alpha > 0$, $\alpha_i > 0$, $\lambda \geq 1$, $\lambda_i \geq 1$, $\bar{G} > \lambda$, and $\bar{G}_i > \lambda_i$.

The inverse surplus consumption ratio $\mathcal{R}_{i,t}$ is driven by two independent processes G_t and $G_{i,t}$. Both of these processes have the same form as the inverse surplus consumption ratio in Menzly et al. (2004), where investors have standard habit formation. We assume a two-factor structure to parallel our specification of the countries' output processes. The habit level factor, G_t, is driven by the same Brownian motion $w_{Y,t}$ as the level factor Y_t in the output processes, while the second, good-specific factor $G_{i,t}$ by the same process driving the domestic output component $X_{i,t}$. Note that, as in Menzly et al. (2004), the inverse surplus consumption ratio of each country is locally perfectly negatively correlated with the country's output. This structure helps us jointly match asset prices and exchange rate moments. One can show that that the two-factor structure also emerges if investors have deep habits and standard habits over an aggregate good-specific habit-adjusted consumption basket. In this case, G_t represents the inverse surplus consumption ratio of the standard habit and $G_{i,t}$ represents the inverse surplus consumption ratios of the good-specific habits. Alternatively, one can interpret the two-factor structure as factors impacting the curvature of the utility for a specific good, where one component is due to a level factor and the other component is unique to that good. The restrictions in Assumption 3 imply that the inverse surplus consumption ratio is high after a series of negative shocks to the level factor Y_t in the output processes or the country-specific factor $X_{i,t}$ and therefore the curvature of the utility function is high in these states. The variations in the inverse surplus consumption ratios drive the variation in the risk appetite in our model. One can think of innovations to these factors as demand shocks (as in, e.g., Pavlova and Rigobon, 2007 and Gabaix and Maggiori, 2015). Equation (8) then implies that an adverse domestic output shock coincides with a demand shock for the home good, as domestic consumers get closer to their habit.

3.3 Goods prices and exchange rates

We denote the prices of each good i in units of the numeraire as $p_{i,t}$. The dynamics of $p_{i,t}$ are

$$\frac{dp_{i,t}}{p_{i,t}} = \mu_{p_{i,t}} dt + \sigma'_{p_{i,t}} dw_t, \tag{9}$$

where $\mu_{p_{i,t}}$ and $\sigma_{p_{i,t}}$ are determined in equilibrium.

Defining the real exchange rate requires a definition of the countries' consumption price indexes. In a model with deep habits there is no natural concept of an aggregate consumption price index or aggregate consumption. We follow Ravn, Schmitt-Grohé, and Uribe (2012) and use an arithmetic average consumption basket, which mimics the construction of price indices in most developed countries. Specifically, we define the price of the consumption basket in country j as

$$P_t^j = \sum_{i=1}^{N} h_i^j p_{i,t}, \tag{10}$$

where h_i^j represents the importance of good i in the basket of country j. We discuss how we set the weights when we calibrate the model in Section 5.

In what follows, we are primarily concerned with exchange rates faced by investors in the the home country. For each foreign country j, we can define the real exchange rate as the ratio of the price of country j's consumption basket to the price of the home consumption basket, that is,

$$Q_t^j = \frac{P_t^j}{P_t^1}. \tag{11}$$

3.4 Investment opportunities

In the model there are $N+1$ priced shocks and therefore we need $N+2$ assets to complete the market. To do so, we introduce N stocks, a locally risk-free bond paying out in the numeraire basket of country one, and a "global insurance contract." All stock markets are in fixed net supply of one share each and the bond and the global insurance contract are in zero net supply.

Each country's stock market is the claim to the aggregate output produced in that country. The value of the aggregate stock market of country $i = 1, \ldots, N$ in units of numeraire is $S_{i,t}$, with instantanous returns given by

$$dR_{i,t} = \frac{dS_{i,t} + p_{i,t} Y_{i,t} dt}{S_{i,t}} = \mu_{R_{i,t}} dt + \sigma'_{R_{i,t}} dw_t. \tag{12}$$

The dynamics of the bond are

$$\frac{dB_t}{B_t} = r_t dt. \tag{13}$$

The stock and bond dynamics are determined in equilibrium.

The global insurance contract hedges the level shock to Y_t (common for all countries). The dynamic of the global insurance contract is

$$dR_{0,t} = \mu_{0,t} dt - \sigma'_0 dw_t, \tag{14}$$

where $\sigma_0 = (1, 0)$ with 0 being an N-vector of zeros. The parameter σ_0 is exogenously specified and $\mu_{0,t}$ is determined in equilibrium. The global insurance contract is introduced for purely technical reasons to complete the market.

Since we study the violation of the UIP and the currency risk premium it is also convenient to introduce N bonds, each paying out in the respective country's numeraire basket. The bond of country one corresponds to Equation (13) (i.e., $B_t^1 = B_t$ and $r_t^1 = r_t$). The rest of the bonds are redundant, but we price them using no-arbitrage. The dynamics of the bonds are

$$\frac{dB_t^j}{B_t^j} = r_t^j dt. \tag{15}$$

3.5 Individual optimization

Investors maximize their lifetime utility in Equation (4), subject to the dynamic budget constraint

$$dW_t^j = \varphi_t^j \frac{dB_t^1}{B_t^1} + \pi_{0,t}^j dR_{0,t} + \sum_{i=1}^{N} \pi_i^j dR_{i,t} - \sum_{i=1}^{N} p_{i,t} C_{i,t}^j dt, \tag{16}$$

for $j = 1, \ldots, N$, where W_t^j denotes the wealth of investors in country j at time t, π_i^j represents the amount held in stock i by investor j, φ_t^j is the amount held in the money market account (bond) of country one, and $\pi_{0,t}^j$ is the amount held in the global insurance contract. Since investors only have financial wealth, total wealth at any point in time is the sum of the dollar positions, i.e., $W_t^j = \sum_{i=1}^N \pi_{i,t}^j + \varphi_t^j + \pi_{0,t}^j$.

We assume that $W_0^j = \sum_{i=1}^N \pi_{i,0}^j$ (i.e., the investors are endowed with shares in the stocks). We set the initial wealth of all investors to be sufficiently high to support consumption levels that exceed the habit levels.[9]

4 Equilibrium

In this section we solve and characterize the equilibrium. Specifically, we solve for the consumption, wealth, and asset prices in closed form. We show how different shocks impact the NFA and wealth shares of the countries, and relate the results to Gourinchas et al. (2017). We start with the definition of the equilibrium.

Definition 1. *Equilibrium is a collection of allocations* $\left(C_{i,t}^j, \varphi_t^j, \pi_0^j, \pi_i^j\right)$ *for* $i, j = 1 \ldots, N$, *and a price system* $\left(\mu_{R_{i,t}}, \mu_{p_{i,t}}, \mu_{0,t}, r_t^1, \sigma_{R_{i,t}}, \sigma_{p_{i,t}}\right)$ *such that the allocations solve the investors'*

[9]Given that the habit formation is external, the initial wealth has to be high enough to support a consumption that exceeds the habit. Hence, the initial allocations cannot be chosen independently from the habit parameters ϕ_i^j. A sufficient condition for this is to choose $\pi_i^j = \phi_i^j S_{i,0}$.

optimization problems and all markets clear, i.e., $\sum_{j=1}^{N} C_{i,t}^{j} = Y_{i,t}$, $\sum_{j=1}^{N} \pi_{i,t}^{j} = S_{i,t}$, $\sum_{j=1}^{N} \pi_{0,t}^{j} = 0$ *and* $\sum_{j=1}^{N} \varphi_{t}^{j} = 0$ *for all* t *and* $i = 1, \ldots, N$.

Since financial markets are complete in our economy, the equilibrium consumption allocations coincide with those in the central planner's problem. We therefore consider the planner's problem first and then solve for the prices that prevail in the decentralized equilibrium. Let a^j denote the Pareto weight of investor j normalized such that $\sum_{j=1}^{N} a^j = 1$, then the planner's problem is

$$U\left(Y_t; H_t, a\right) = \max_{C_t^j} \sum_{j=1}^{N} a^j \sum_{i=1}^{N} \log\left(C_{i,t}^{j} - H_{i,t}^{j}\right) \quad s.t. \quad \sum_{j=1}^{N} C_{i,t}^{j} = Y_{i,t} \quad i = 1, \ldots, N, \tag{17}$$

where $Y_t = (Y_{1,t}, \ldots, Y_{N,t})$, $H_t = (H_{1,t}, \ldots, H_{N,t})$ and $a = \left(a^1, \ldots, a^N\right)$ for all t. The higher the initial wealth of a country, the higher is its Pareto weight.[10] In the calibration that follows, we assume that the home country, the US, is a rich country, which is captured by a high Pareto weight a^1. We also refer to the home country's weight a^1 as a^{US}.

4.1 Risk sharing and time-varying risk appetites

Key to our understanding of cross-country wealth transfers in times of stress is the risk sharing and how investors optimally choose different exposures to the shocks in the economy. To address these issues we characterize risk sharing in our model by solving the central planner's problem in (17), which is easy to solve as it reduces to a sequence of simple state-by-state optimizations. The first-order conditions (FOC) are

$$\frac{a^j}{C_{i,t}^{j} - H_{i,t}^{j}} = \eta_{i,t}, \qquad i, j = 1, \ldots, N, \tag{18}$$

where $\eta_t = (\eta_{1,t}, \ldots, \eta_{N,t})$ are the Lagrange multipliers on the resource constraint in (17). Hence, we can relate the consumption of investor j and k through $\frac{a^j}{C_i^j - H_i^j} = \frac{a^k}{C_i^k - H_i^k}$. Imposing

[10] The Pareto weights are determined by matching the consumption allocations in the competitive equilibrium to those in the planner's problem.

the market clearing in each consumption good and using $\sum_{j=1}^{N} \phi_i^j = 1$, we have that

$$\left(C_{i,t}^j - H_{i,t}^j\right) = a^j \left(Y_{i,t} - H_{i,t}\right). \tag{19}$$

Hence, the optimal habit-adjusted consumption of every investor in each good, $C_{i,t}^j - H_{i,t}^j$, is proportional to the aggregate habit-adjusted output, $Y_{i,t} - H_{i,t}$. In other words, the habit-adjusted consumption does not depend on the individual habit directly, but only the aggregate habit in the economy.[11] However, the actual level of consumption, $C_{i,t}^j$, depends on individual habit dynamics, as from Assumption 1, $C_{i,t}^j = \phi_i^j H_{i,t} + a^j \left(Y_{i,t} - H_{i,t}\right)$. Given the above, we have the following:

Proposition 1. *The optimal consumption of investor j of good i is*

$$C_{i,t}^j = f_{i,t}^j Y_{i,t} \tag{20}$$

where $f_{i,t}^j \equiv \frac{C_{i,t}^j}{Y_{i,t}}$ is the consumption share given by

$$f_{i,t}^j = \phi_i^j + \left(a^j - \phi_i^j\right) s_{i,t}, \tag{21}$$

where $s_{i,t}$ is the aggregate surplus-consumption ratio for good i defined in Equation (5).

From Proposition 1, we see that the consumption share is linear in the surplus consumption ratio. We assume that the aggregate habit levels are never negative and never exceeds the aggregate output of the same good. Specifically, we have that for good i, $s_{i,t} \in (0, s_i^{max}]$. Given the above, we have the following proposition.

Proposition 2. *The consumption share of investor j of good i is bounded in the intervals $\left(a^j s_i^{max} + \phi_i^j \left(1 - s_i^{max}\right), \phi_i^j\right)$ if $\phi_i^j > a^j$ and $\left(\phi_i^j, a^j s_i^{max} + \phi_i^j \left(1 - s_i^{max}\right)\right)$ if $\phi_i^j < a^j$. Moreover, $\frac{\partial f_i^j}{\partial s_i} = a^j - \phi_i^j$.*

[11] See Heyerdahl-Larsen (2014) for a discussion.

From Proposition 2 we see that the response of the consumption share to the surplus consumption ratio depends on the sign of the difference between country j's Pareto weight and its habit sensitivity, $a^j - \phi_i^j$. As a higher surplus consumption ratio implies a better state of the world, the consumption share is pro-cyclical for countries with a low sensitivity to the habit, ϕ_i^j, or high wealth as measured by the Pareto weight, a^j. In other words, these countries provide insurance to the rest of the world.

Habit formation models with single goods such as Campbell and Cochrane (1999) produce a countercyclical risk aversion that depend on the aggregate surplus consumption ratio. In a setting with multiple goods the concept of risk aversion is not well defined. Still, we can make a comparison to Campbell and Cochrane (1999). To do so, consider the marginal utility of investor j with respect to consumption of good i is

$$\frac{1}{C_{i,t}^j - H_{i,t}^j} = \frac{1}{C_{i,t}^j s_{i,t}^j}, \tag{22}$$

where $s_{i,t}^j = \frac{C_{i,t}^j - H_{i,t}^j}{C_{i,t}^j}$ is the surplus consumption ratio of good i for investor j.[12] The aggregate surplus consumption ratio for good i, $s_{i,t}$, defined in Equation (5) is in general different to the individual-specific surplus consumption ratio, $s_{i,t}^j$, as different agents have different sensitivities to the aggregate habit represented by the weights ϕ^j. As in Campbell and Cochrane (1999), risk premia depends on the surplus consumption ratios, $s_{i,t}^j$. The surplus consumption ratio, $s_{i,t}^j$, decreases in bad times when the consumption approaches the habit. In Campbell and Cochrane (1999), the (effective) risk aversion increases when the surplus consumption ratio decreases. In our paper, the mechanism is similar except that there is a surplus consumption ratio for each good. We will refer to the states in which the surplus consumption ratio for good i is high as times when investors have high *risk appetite* towards risk originating in country i.

[12]Equation (22) can also be written in terms of the the local curvature of investor j's utility with respect to consumption of good i: $-\frac{C_{i,t}^j u_i^j(C)}{u_{ii}^j(C)} = \frac{C_{i,t}^j}{C_{i,t}^j - H_{i,t}^j} = \frac{1}{s_{i,t}^j}$, where u is logarithmic and where u_i^j and u_{ii}^j are the first and second order derivatives of the instantaneous utility of agent j for good i. This expression is similar to the local risk-aversion coefficient in Campbell and Cochrane (1999), except that "risk aversion" is defined with respect to each good.

To further interpret how international investors share risk, it is convenient to consider the inverse of the surplus consumption ratio $\mathcal{R}_{i,t}^{j} \equiv \frac{1}{s_{i,t}^{j}}$, where a high value of $\mathcal{R}_{i,t}^{j}$ corresponds to bad times. Recall that the inverse surplus consumption ratio, the dynamic of which is specified in Equations (7) and (8), is driven by the same shocks that drive output. Therefore bad times defined in terms of the high inverse surplus consumption ratios also correspond to times of adverse output shocks, which is why in the quantitative analysis that follows we associate them with recessions. These are times when the marginal utility of the consumption of good i by investor j is high. We will refer to $\mathcal{R}_{i,t}^{j}$ as the local curvature of investor j's utility for good i (see also footnote 12). Using Equations (19)–(21), we have that

$$\mathcal{R}_{i,t}^{j} = 1 + (\mathcal{R}_{i,t} - 1)\,\frac{\phi_i^j}{a^j}. \tag{23}$$

From Equation (23) we see that the local curvature of the utility function depends on the ratio $\frac{\phi_i^j}{a^j}$. Investors with a high risk appetite provide insurance to those with low risk appetite in equilibrium, leading to a pro-cyclical consumption share. Investors with a high risk appetite take larger losses in bad times and gain more in good times.

A procyclical consumption share of investors with higher risk appetites would also obtain in models in which investors have heterogeneous risk appetites. However, the non-homothetic preferences in our model imply a relationship between wealth and risk appetite. Wealthier investors effectively have a stronger risk appetite. Moreover, investors in our model all have less appetite for risk in bad states. Finally, and maybe most importantly, given the multiple good setting with different habit sensitivities, ϕ_i^j, for each good, the investors have different utility curvatures for different goods. However, the initial wealth, represented by the Pareto weight a^j, directly impacts the overall risk appetite as it changes the local utility curvature of all goods.

4.2 Relative prices, exchange rates, net exports, and stock prices

In this subsection we solve for the relative prices, exchange rates, and net exports. We link the relative prices and the exchange rates to the the stochastic discount factor (SDF). We then use the relative prices to characterize net exports and show how net exports relate to the risk sharing in the economy.

In our setting markets are complete and thus the SDF is unique, but it is useful to express it in different units: in terms of the numeraire basket of country j, M^j, and in terms of goods produced by country i, M_i, for countries $i = 1, \ldots, N$. The next proposition shows that country j's SDF in units of the numeraire basket is a weighted average of the SDF in units of goods produced by all countries.

Proposition 3. *The stochastic discount factor in country j is*

$$M_t^j = \sum_{i=1}^{N} h_i^j M_{i,t} \tag{24}$$

where

$$M_{i,t} = e^{-\rho t} \frac{1}{Y_{i,t} - H_{i,t}} \tag{25}$$

where h_i^j is the weight of good i in the basket of country j.

The stochastic discount factor in Proposition 3 is a multi-good analog of the stochastic discount factor with external habit. Equation (25) is the familiar expression from Campbell and Cochrane (1999). The stochastic discount factor in Equation (24) takes into account that all payoffs are defined in terms of the basket we use as numeraire.

The marginal utility in Equation (22) is proportional to the discount factor, $M_{i,t}$, that prices a cash flow in units of good i. It follows from the no-arbitrage condition that the price at time $t < s$ of a claim to an asset with payoff $D_{i,s}$ in terms of good i at time s is $E_t\left(\frac{M_{i,s}}{M_{i,t}} D_{i,s}\right) = E_t\left(\frac{M_s^1}{M_t^1} \frac{p_{i,s}}{p_{i,t}} D_{i,s}\right)$. In complete market it follows that

$$p_{i,t} = \frac{M_{i,t}}{M_t^1} \tag{26}$$

From Equation (11), the exchange rate between country j and 1 is the ratio of the price of the basket of country j to country 1. Using Equation (24), we have the following

$$Q_t^j = \frac{M_t^j}{M_t^1} \tag{27}$$

Given the expression for the relative prices, the equilibrium characterization are given in the next proposition.

Proposition 4. *The equilibrium price of good i in terms of the consumption basket of country 1 is*

$$p_{i,t} = \left(\frac{G_{i,t}}{Y_{i,t}}\right)\left(\frac{1}{\sum_{l=1}^{N} h_l^1 \frac{G_{l,t}}{Y_{l,t}}}\right). \tag{28}$$

The exchange rate of country j is

$$Q_t^j = \frac{\sum_{l=1}^{N} h_l^j \frac{G_{l,t}}{Y_{l,t}}}{\sum_{l=1}^{N} h_l^1 \frac{G_{l,t}}{Y_{l,t}}}. \tag{29}$$

From Equation (29), we see that the real exchange rate not only depends on the relative output but also on the good specific risk appetite through the inverse surplus consumption ratios. As we show below, most of the variation in the real exchange rates come from the variation in the risk appetite due to changes in the surplus consumption ratios. Also, note that the exchange rate only depends on good-specific habits, $G_{i,t}$. Acting similarly to a demand shock for good i, a positive shock to the good-specific habit $G_{i,t}$ (i.e., when output of that good falls closer to the habit) increases the demand for that good and boosts its relative price. Accordingly, the real exchange rate of country i appreciates. The habit (or risk appetite) level factor G_t, does not generate any price asymmetries and therefore does not impact the exchange rate. Nevertheless, the level factor is important for understanding risk sharing and hence the NFA.

As we show below, the NFA of a country are tightly linked to its net exports. To do so, note that the net exports of country 1, NX_t, is given by

$$NX_t = \underbrace{p_{1,t}\left(Y_{1,t} - C^1_{1,t}\right)}_{\text{Exports}} - \underbrace{\sum_{i=2}^{N} p_{i,t} C^1_{i,t}}_{\text{Imports}} \tag{30}$$

Using the optimal consumption from Proposition 1 and the relative price in Proposition 4 we get the following.

Proposition 5. *The net export, NX_t, of country one is*

$$NX_t = p_{1,t} Y_{1,t} \left(1 - \sum_{i=1}^{N} \left(\frac{G_{i,t}}{G_{1,t}}\right)\left(\phi_i^1 + \left(a^1 - \phi_i^1\right) s_{i,t}\right)\right), \tag{31}$$

where $p_{1,t}$ is given in Equation (28).

It is useful to consider the net exports as a fraction of the home production value

$$\frac{NX_t}{p_{1,t} Y_t} = \underbrace{1 - \left(\phi_1^1 + \left(a^1 - \phi_1^1\right) s_{1,t}\right)}_{\text{Exports fraction}} - \underbrace{\sum_{i=2}^{N} \left(\frac{G_{i,t}}{G_{1,t}}\right)\left(\phi_i^1 + \left(a^1 - \phi_i^1\right) s_{i,t}\right)}_{\text{Imports fraction}} \tag{32}$$

As we see from Equation (32), the net exports as a fraction of the home production value only depends on the risk appetite and not the level of the output. However, in contrast to the real exchange rate, the relative net exports also depend on the risk appetite level factor, G_t (through $s_{1,t}$) .

In bad times, both US and foreign stock markets fall. The stock markets in each country are the claim to the aggregate output of the good in that country. We solve for the stock prices in closed form and report them in the proposition below.

Proposition 6. *Then the stock price of country $i = 1, \ldots, N$ is*

$$S_{i,t} = p_{i,t} Y_{i,t} PD_{i,t}, \tag{33}$$

where $PD_{i,t}$ is the price-dividend ratio and is given by

$$PD_{i,t} = \frac{1}{\mathcal{R}_{i,t}} F_{i,t}, \tag{34}$$

where

$$\begin{aligned} F_{i,t} \quad = \quad & \frac{\bar{\mathcal{R}}_i}{\rho} + \frac{\bar{G}\left(G_{i,t} - \bar{G}_i\right)}{\rho + \kappa_i} + \frac{\bar{G}_i\left(G_t - \bar{G}\right)}{\rho + \kappa} \\ & + \frac{\left(G_t - \bar{G}\right)\left(G_{i,t} - \bar{G}_i\right)}{\rho + \kappa + \kappa_i} \end{aligned} \tag{35}$$

where $\bar{\mathcal{R}}_i = \bar{G}\bar{G}_i$.

Consider a negative shock to the US output. The direct effect of this shock is that the US stock market falls. The shock impacts the foreign stock markets in two ways. The first channel is the USD appreciation, which we have discussed above. Hence, measured in USD, all foreign stock markets fall. Formally, the term $p_{i,t}Y_{i,t}$ in Equation (33) decreases. The second channel is through the positive correlation of output and habit shocks. Investors worldwide have lower risk appetites as their consumption of all goods gets closer to the habit. This lowers the valuation ratios, $PD_{i,t}$, in Equation (33). We show these formally in the proof of Proposition 6. The right hand panel of Figure 4 illustrates this mechanism. On the horizontal axis, we measure the US risk appetite, which in our model positively comoves with shocks to output.

Figure 4 plots additionally the import, export and net exports as a fraction of the home production value as we change the US risk appetite. The figure shows that both import and exports are increasing in the US risk appetite, $1/G_{US}$, but at different rates. Specifically, imports increase at a faster rate than exports, leading to a decrease in net exports. The reason for this is the consumption home bias. Due to consumption home bias, the US has a higher utility curvature with respect to the domestic good relative to the rest of the world. Hence, when the US surplus consumption ratio is high, the US consumer exports more of the domestically produced good. Note that since we measure all quantities relative to the home production value, the exchange rate has no effect on the exports in the figure. In contrast, the imports are affected by the exchange rate, and since the USD is depreciating in good times (i.e., when the US surplus consumption ratio is high), the imports become more expensive. Still, the US consumer does not change the amount imported (the consumption share stays the same), and therefore the value of the imports relative to the home production value increases. The exchange rate effect on the imports is large and therefore net exports are decreasing as the US surplus consumption ratio increases. In other words, the US becomes more of a closed economy in bad times.

For completeness, we plot the same quantities presented in Figure 4 as a function of the level factor driving risk appetites, $1/G$, in Figure 8 in Appendix D. The level factor does not affect the exchange rates and therefore fluctuations in this factor do not produce asymmetric effects on the countries' stock markets.

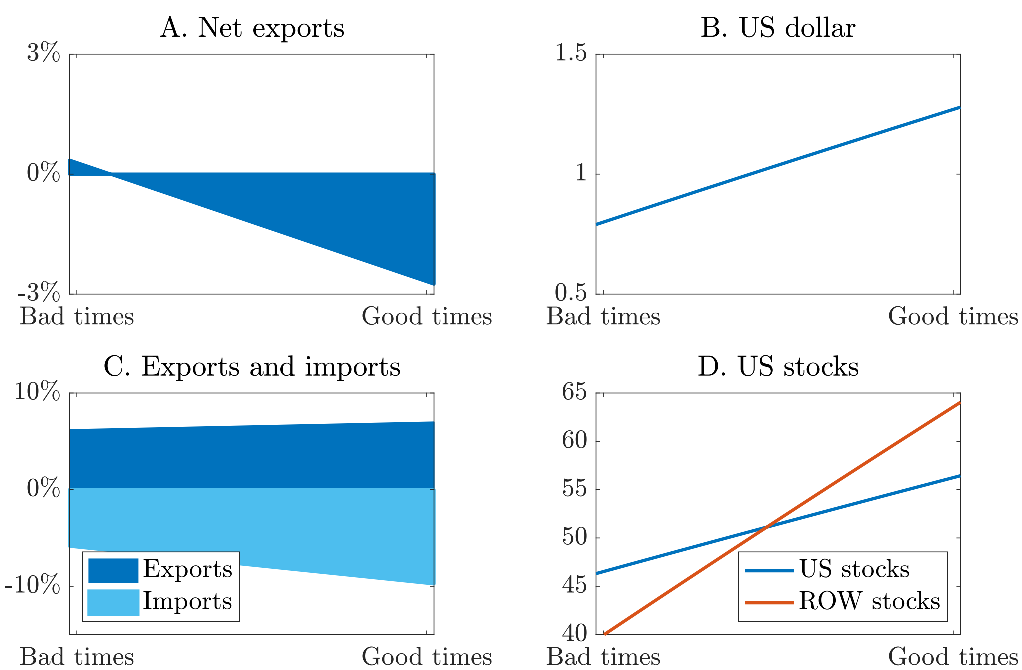

Figure 4: Exchange rates, stock markets, and net exports
The figure plots the net exports (Panel A), the real exchange rate (USD against ROW currencies, Panel B), exports and imports (Panel C) and US stock prices (Panel D). For all panels the habit level factor is at its long-run mean ($G = \bar{G}$). The parameters are the same as the parameters used in the calibration (see Tables 1 and 3).

4.3 Net foreign assets and countries' wealth

In this subsection we solve for each country's NFA, stock market, and wealth. We show how the NFA relate to a country's wealth and its stock market value, and link the NFA to future net exports.

The NFA are defined as the difference between the country j's ownership of foreign assets assets minus the ownership of foreign investors of assets in country j. Stated formally,

$$NFA_t^j = \sum_{i=1,\, i\neq j}^{N} \pi_{i,t}^j - \sum_{k=1,\, k\neq j}^{N} \pi_{j,t}^k + \varphi_t^j + \pi_{0,t}^j, \tag{36}$$

where the first term is country j's ownership of foreign stocks, the second term is the amount of country j's stock market held by foreign countries, and the last two terms reflect country j's positions in the bond and global insurance contract. Recall that country j's time-t wealth is $W_t^j = \sum_{i=1}^N \pi_{i,t}^j + \varphi_t^j + \pi_{0,t}^j$. Together with equation (36) and market clearing (see Definition 1), this implies that $NFA_t^j = W_t^j - S_{j,t}$. We have the following immediate relation between net exports and NFA

$$NFA_t^j = W_t^j - S_{j,t} = -E_t\left(\int_t^\infty \frac{M_s^1}{M_t^1} NX_s^j ds\right). \tag{37}$$

If we endow the country with the claim to their own output and no bonds at time zero ($W_0^j = S_{j,0}$), then we trivially have that $NFA_0^j = 0$ and the budget condition implies that the net present value of future net exports are zero.

To calculate the NFA, we need to know the value of the stock markets in each country and the wealth of each country. We have computed the former in Proposition 6 and we now use the optimal consumption reported in Proposition 1 to can calculate the wealth of each investor in equilibrium.

Proposition 7. *The wealth of investor $j = 1, \dots, N$ is*

$$W_t^j = \sum_{i=1}^{N} \phi_i^j S_{i,t} + \frac{b^j}{e^{\rho t} M_t^1}, \tag{38}$$

where

$$b^j = \frac{1}{\rho}\left(Na^j - \sum_{i=1}^{N} \phi_i^j\right). \tag{39}$$

From Proposition 7, we see that the wealth can be decomposed in the following way:

$$W_t^j = \underbrace{\sum_{i=1}^{N} \phi_i^j S_{i,t}}_{\text{Buy and hold portfolio}} + \underbrace{\frac{b^j}{e^{\rho t} M_t^1}}_{\text{Dynamic portfolio}} \tag{40}$$

Hence, key for understanding the portfolio positions is the coefficients b^j. Note that b^j is high when the Pareto weights a^j is high or if the habit coefficients ϕ_i^j are low (i.e., if the investor is wealthy or risk tolerant). The next Proposition considers a special case where there is no trade in risky assets in equilibrium with the exception of an initial trade at the beginning of the economy.

Corollary 1. *If $b^j = 0$ for all $j = 1, ..., N$ then the optimal portfolio of investor j is a buy-and-hold portfolio with weights ϕ_i^j and there is no trade is the risk-free asset.*

Corollary 1 highlights the case when there is no lending or borrowing in equilibrium. For instance, this would be the case if we consider a symmetric economy (i.e., $\phi_j^j = \phi_k^k$, $\phi_i^j = \phi_i^k$ for $i \neq j, k$ and $a^j = \frac{1}{N}$). To decentralize the economy, the Pareto weights has to be mapped into initial wealth. Assume that the all surplus consumption ratios are in the steady state at time zero and that each country is initially endowed with the claim to their own endowment stream (the stock market), then we have the following.

Proposition 8. *If each country is initially endowed with the claim to their own output, i.e., $W_0^j = S_{j,t}$, and the surplus consumption ratios are in the steady state at time 0, i.e., $\mathcal{R}_{i,0} = \bar{\mathcal{R}}_i$ for $i = 1, \ldots, N$, then the Pareto weights, a^j, are*

$$a^j = \frac{1}{N}\left(\sum_{i=1}^{N} \phi_i^j + \left(\bar{\mathcal{R}}_j - \sum_{i=1}^{N} \phi_i^j \bar{\mathcal{R}}_i\right)\right). \tag{41}$$

There are several things to note from Proposition 8. First, if $\bar{\mathcal{R}}_i = \bar{\mathcal{R}}$ for $i = 1, \ldots, N$, then $a^j = \frac{1}{N}\left(\bar{\mathcal{R}} - \left(\bar{\mathcal{R}} - 1\right)\sum_{i=1}^{N}\phi_i^j\right) = \frac{1}{N}$.[13] Moreover, the Pareto weight is decreasing in the average risk aversion. This follows from the fact that a higher average risk aversion implies a higher marginal utility and therefore the central planner puts less weight on that investor through a lower Pareto weight. Second, a higher steady state level of the inverse surplus consumption ratio, $\bar{\mathcal{R}}_i$, implies a higher wealth for that country. The reason is that a good with a high steady state inverse surplus consumption ratio, $\bar{\mathcal{R}}_i$, is a valuable good as investors are very reluctant to scale back on it. Therefore the country that is endowed with such a good is also a rich country. In our calibration, we set the Pareto weights, a, directly instead of a specific initial value for the inverse surplus consumption ratios.

4.4 Is there an exorbitant duty?

In this subsection, we examine wealth transfers and the connection to the exorbitant duty. We show that the model reproduces the exorbitant duty of Gourinchas et al. (2017), but also produces an important offsetting effect that dominates in times of stress.

It is convenient to split the expression for the NFA of country j into two components

$$NFA_t^j = W_t^j - S_{j,t} = \underbrace{\sum_{i=1, i\neq j}^{N} \phi_i^j S_{i,t} - \left(1 - \phi_j^j\right) S_{j,t}}_{\text{portfolio home bias intensity}} + \underbrace{\frac{b^j}{e^{\rho t} M_t^1}}_{\text{exorbitant duty}}. \tag{42}$$

We refer to the first component as the portfolio home bias intensity and the second as the exorbitant duty. The conventional wisdom is that the rich country takes a leveraged position in the external portfolio and therefore experience higher losses during bad times (exorbitant duty), but in good times earns a higher return (exorbitant privilege) due to the risk premium in risky assets. This mechanism is also in our model as the next proposition shows.

Proposition 9. *Let $ED_t^j = \frac{b^j}{e^{\rho t} M_t^1}$ be the exorbitant duty term for country j. Consider a*

[13] The fact that $a^j = \frac{1}{N}$ follows from Assumption 1. It is possible to relax the assumption that $\sum_{i=1}^{N}\phi_i^j = 1$ as long as the "average risk aversion" measured by $\sum_{i=1}^{N}\phi_i^j$ of any investor is not too big so the investor cannot afford a consumption that exceed the habit level.

wealthy country, i.e., $a^j > \frac{1}{N}$*. Then we have the following:*

$$\frac{\partial ED_t^j}{\partial \mathcal{R}_{i,t}} < 0 \tag{43}$$

for all $i = 1, \ldots, N$*.*

From Proposition 9 we see that the exorbitant duty component of the NFA of rich countries is decreasing in the inverse surplus consumption ratios, $\mathcal{R}_i$; consequently, in bad times the contribution of the exorbitant duty term to the NFA is negative. Effectively, rich countries behave as less risk averse investors and provide insurance to the rest of the world.

Rather than working with the NFA in levels, it is convenient to normalize the NFA by total world wealth. Hence, the NFA can be expressed as the difference between the wealth share, $f_{j,t}^W$, and the stock market share, $f_{j,t}^S$,

$$nfa_t^j = f_{j,t}^W - f_{j,t}^S. \tag{44}$$

Figure 5 shows nfa^{US} as function of the US surplus consumption ratio $s_{US} = \frac{1}{G_{US}}$. Consistent with the discussion above, the figure shows that the NFA of the US are decreasing in bad states (i.e., for low levels of the US surplus consumption ratio or, equivalently, following adverse US output shocks). In bad states, all stock markets fall but the US stock market falls by less. The reason is that the effect of the USD appreciation mitigates the fall in the US stock market, while exacerbating the fall in the foreign stock markets when measured in USD. The wealth share of the US, f_{US}^W, is the sum of the normalized NFA and the stock market share. However, since most of the wealth of the US investors is in US domestic assets, the relative increase in the US stock market dominates the decrease in the wealth share due to the NFA. In conclusion, the response of the NFA to a bad times as measured by a high G_{US} is negative. Still, the wealth share of the US is increasing. We graphically illustrate this mechanism in Appendix A.

Our mechanism helps resolve the "reserve currency paradox" of Maggiori (2017). The

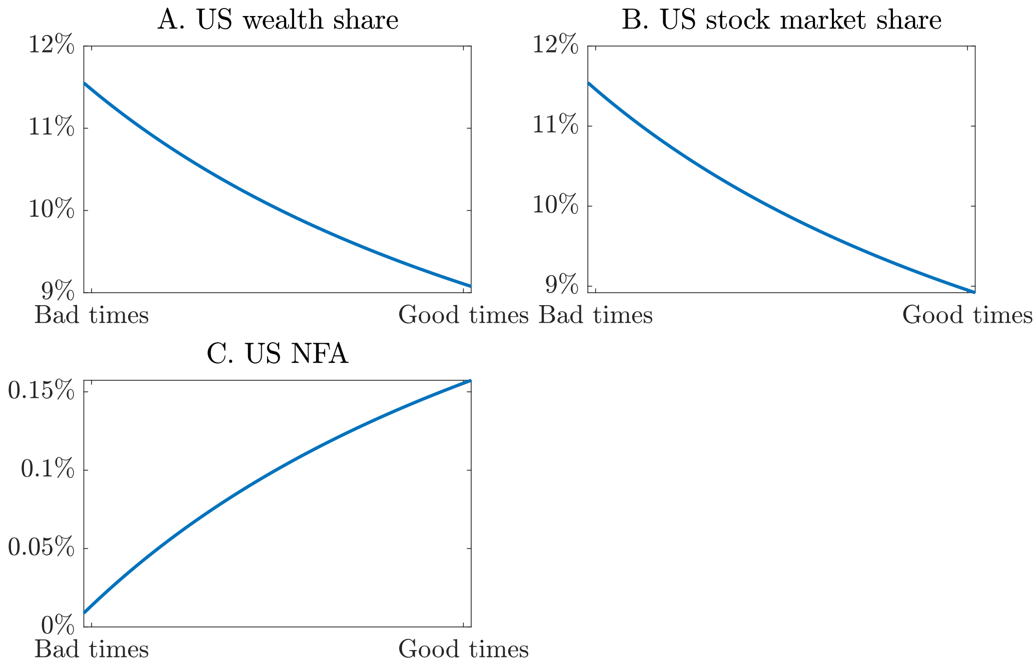

Figure 5: US wealth share, US stock market share, and net foreign assets
The figure plots the US wealth scaled by world wealth (Panel A), the US stock market share in world stock market (Panel B), and the US NFA scaled by world wealth (Panel C) as functions of the US risk appetite ($1/G_{US}$). For all panels the habit level factor is at its long-run mean ($G = \bar{G}$). The parameters are the same as the parameters used in the calibration (see Tables 1 and 3).

paradox is that, as a global insurer, the US transfers wealth to the rest of the world during times of stress, and yet the USD appreciates. However, in the presence of consumption home bias, wealth transfers from the US to the rest of the world should bring about USD depreciations, not appreciations. The latter result echoes the classical "transfer problem," which dates back to a debate between Keynes and Ohlin (Keynes 1929a,b; Ohlin 1929a,b). According to Keynes, an income (wealth) transfer from a domestic to a foreign country should boost demand for the foreign good more than that for the domestic good (because of the income (wealth) effect coupled with consumption home bias), leading to the foreign country's exchange rate appreciation.[14] In our model, there is no paradox. In times of stress, the US's share of world wealth increases. In other words, the US *receives* a wealth transfer from the rest of the world and the USD appreciates, which is fully consistent with the Keynesian view. In our analysis

[14]See Pavlova and Rigobon (2008) for an application of the transfer problem in the context of a continuous-time multi-country economy.

of Section 2, we provide empirical evidence in support of our theoretical prediction that the wealth share of the US increases in times of stress.

The mechanism is different if we instead consider the effect of the level factor, G_t. First, consistent with Proposition 9, when the habit level increases ($1/G_t$ decreases), the NFA of US drop. Second, there is no appreciation of the domestic currency and therefore the wealth share of the US decreases. The reason for this is that the habit level factor does not impact relative prices and hence exchange rates. We illustrate this mechanism in Figure 9 in Appendix D.

To summarize, the discussion above illustrates that the *US NFA decrease in bad times*, regardless of the nature of the shock. However, the US wealth share is sensitive to the source of the shock. If the shock is affecting the domestic risk appetite for the US good, the USD appreciates resulting in a *wealth transfer to the US*. If instead it is due to a shock to the habit level factor G_t, there is no effect on the exchange rate and the wealth share of the US decreases.

4.5 Dynamics

The previous section focused on the "level" of the equilibrium quantities such as the exchange rate, optimal consumption, net exports, wealth, stock market valuations and NFA. However, we did not derive the dynamics of these quantities. In this section we focus on the dynamics.

We start with the dynamics of the SDF from Proposition 10. This pins down the equilibrium market prices of risk and the real risk-free rates.

Proposition 10. *The dynamics of the state price density, M_t^j, is*

$$\frac{dM_t^j}{M_t^j} = -r_t^j dt - \left(\theta_t^j\right)' dw_t, \tag{45}$$

where

$$r_t^j = \sum_{i=1}^{N} \omega_{i,t}^j r_{i,t} \tag{46}$$

and

$$\theta_t^j = \sum_{i=1}^{N} \omega_{i,t}^j \theta_{i,t} \tag{47}$$

where

$$\omega_{i,t}^{j} = \frac{h_i^j M_{i,t}}{\sum_{l=1}^{N} h_l^j M_{l,t}}, \tag{48}$$

and where

$$\begin{aligned} r_{i,t} &= \rho + \mu_{Y_{i,t}} - \sigma_{Y_{i,t}}^{\top}\sigma_{Y_{i,t}} \\ &+\kappa\left(1 - \frac{\bar{G}}{G_t}\right) - \alpha\left(1 - \frac{\lambda}{G_t}\right)\sigma_Y^2 \\ &+\kappa_i\left(1 - \frac{\bar{G}_i}{G_{i,t}}\right) - \alpha_i\left(1 - \frac{\lambda_i}{G_{i,t}}\right)\sigma_{x_{i,t}}'\sigma_{x_{i,t}} \end{aligned} \tag{49}$$

and and the market prices of risk are

$$\theta_{i,t} = \left(\theta_{Y,t}, \theta_{x_i,t}\right), \tag{50}$$

where

$$\theta_{Y,t} = \sigma_Y\left(1 + \alpha\left(1 - \frac{\lambda}{G_t}\right)\right) \tag{51}$$

$$\theta_{x_i,t} = \sigma_{x_{i,t}}\left(1 + \alpha_i\left(1 - \frac{\lambda_i}{G_{i,t}}\right)\right) \tag{52}$$

where $\theta_{Y,t} \in \mathbb{R}$ *and* $\theta_{x_i,t} \in \mathbb{R}^N$*.*

Proposition 10 highlights several features of the model. First, as illustrated by Proposition 10, country $j's$ SDF is a weighted average of the good-specific stochastic discount factors. This is inherited by both the risk-free rate and the market prices of risk. They are, just as the SDFs, weighted average of the corresponding risk-free rates and market prices of risk of the good specific interest rates and market prices of risk. However, in contrast to the level of the SDFs, the risk-free rates and market prices of risk have weights that are stochastic themselves as illustrated by the weights in Equation (48).

Given the state price densities in each country, we can derive the dynamics of the real exchange rate (taking the home country/country one) as the numeraire.

Proposition 11. *The dynamics of the real exchange rate of country j relative to country 1 is*

$$\frac{dQ_t^j}{Q_t^j} = \mu_{Q_t^j} dt + \sigma'_{Q_t^j} dw_t, \tag{53}$$

where

$$\sigma_{Q_t^j} = \theta_t - \theta_t^j, \tag{54}$$

and

$$\mu_{Q_t^j} = r_t - r_t^j + \theta_t' \sigma_{Q_t^j} \tag{55}$$

where we drop the superscript on home country quantities, i.e., $r_t = r_t^1$ and $\theta_t = \theta_t^1$.

Note that the drift, $\mu_{Q_t^j}$, is equal to instantaneous expected depreciation rate, i.e., $E_t\left(\frac{dQ_t^j}{Q_t^j}\right)/dt = \mu_{Q_t^j}$. In other words, the expected excess return (i.e., the currency risk premium), is equal to

$$E_t\left(\frac{dQ_t^j}{Q_t^j}/dt\right) + r_t^j - r_t = \theta_t' \sigma_{Q_t^j}. \tag{56}$$

Consequently, the drift of the real exchange rate, $\mu_{Q_t^j}$, depends on the interest differential and the risk premium. The uncovered interest rate parity (UIP) would hold whenever the risk premium, $\theta_t' \sigma_{Q_t^j}$, is zero.

As we will show in Section 5.5, the model can replicate the failure of the UIP in the data. In addition, the model also matches the predictability reversal as shown by Bacchetta and Van Wincoop (2010) and Engel (2016). To analyze the mechanism, it is useful to consider a special case when $h_j^j = 1$, that is each country's price index is putting all weight on the domestically produced consumption good.[15] In addition, we will assume for expositional reasons that the parameters for the two countries are the same and that $x_{i,t}$ and $x_{j,t}$ are uncorrelated. Given the above special case, we will drop subscripts and write $\alpha_i = \alpha$, $\sigma_{x_i,t} = \sigma_{x,t}$ etc.

[15] This implies a PPI based real exchange rate.

In this case we can write the drift of the real exchange rate as

$$\mu_{Q_t^j} = \underbrace{\left(\alpha\lambda\sigma_{x,t}^2 - \kappa\bar{G}\right)(s_{1,t} - s_{j,t})}_{\text{interest rate differential}} + \underbrace{\theta_{x_1,t}^2}_{\text{risk premium}} \tag{57}$$

It is easy to show that $\theta_{x_1,t}$ is decreasing the the surplus consumption ratio, and it immediately follows that the currency risk premium is also decreasing in the surplus consumption ratio. As shown by Fama (1984), to match a negative UIP slope, the risk premium has to be negatively correlated with the interest rate differential and the risk premium must be more volatile than the expected depreciation rate. Therefore, to match the UIP, the interest rate differential has to be increasing in the surplus consumption ratio.

From (57), one can see that the interest rate differential, $r_t^1 - r_t^j$, is increasing in the surplus consumption ratio of country 1, $s_{1,t}$, if $\left(\alpha\lambda\sigma_{x,t}^2 - \kappa\bar{G}\right) > 0$. Note that this condition implies that a positive shock to country-specific factor, $X_{1,t}$, increases the interest rate differential, which we refer to as a pro-cyclical interest rate differential. This corresponds to the condition in Verdelhan (2010) where a pro-cyclical interest rate differential is necessary to match the failure of the UIP. Note that due to the common level habit factor driving interest rates, the pro-cyclical interest rate differential does not imply a negative slope of the real term structure.

So how can the model explain a predictability reversal? To examine this, note that unlike Verdelhan (2010), the coefficient on the difference in the surplus consumption ratios, $\left(\alpha\lambda\sigma_{x,t}^2 - \kappa\bar{G}\right)$, is not constant as it depends on the volatility of the output processes. In times of high volatility, the pro-cyclicality of the interest rate differential is stronger. As the volatility drops, the cyclicality changes from pro- to counter-cyclical. In addition, higher output volatility implies a higher risk premium and therefore the cyclicality of the interest rate differential and the currency risk premium are positively correlated. In our calibration in Section 5.5, the volatility is less persistent than the domestic habits and this implies different long-run and short run predicability.

Given the stock prices in Proposition 6, an application of Ito's lemma gives the following.

Proposition 12. *The instantaneous return on the stock market in country i is*

$$dR_{i,t} = \mu_{R_{i,t}} dt + \sigma'_{R_{i,t}} dw_t, \tag{58}$$

where the expected return is

$$\mu_{R_{i,t}} = r_t + \sigma'_{R_{i,t}} \theta_t, \tag{59}$$

with $\sigma_{R_{i,t}} = \left(\sigma^Y_{R_{i,t}}, \sigma^X_{R_{i,t}}\right)$ *and*

$$\sigma^Y_{R_{i,t}} = \sigma_Y + \theta_{Y,t} - \frac{1}{PD_{i,t}} \left(\frac{\bar{G}_i}{\rho + \kappa} + \frac{G_{i,t} - \bar{G}_i}{\rho + \kappa + \kappa_i} \right) \alpha \left(G_t - \lambda\right) \sigma_Y \tag{60}$$

$$\sigma^x_{R_{i,t}} = \sigma_{x,t} + \theta_{x,t} - \frac{1}{PD_{i,t}} \left(\frac{\bar{G}}{\rho + \kappa_i} + \frac{G_t - \bar{G}}{\rho + \kappa + \kappa_i} \right) \alpha_i \left(G_{i,t} - \lambda_n\right) \sigma_{x,t}, \tag{61}$$

where $\theta_t = (\theta_{Y,t}, \theta_{x,t})$ *contains the market prices of risk with respect to* $w_{Y,t}$ *and* $w_{X,t}$ *as in Proposition 10.*

Given the dynamic budget condition in Equation (16), applying Ito's lemma to the optimal wealth in Proposition 7 we can solve for the optimal portfolio positions.

Proposition 13. *Let* $\pi^j_t = \left(\pi^j_{0,t}, \pi^j_{1,t}, \ldots, \pi^j_{N,t}\right)$ *and define the return diffusion matrix as* $\sigma_{R,t} \in \mathbb{R}^{N+1 \times N+1}$ *where element* (i,j) *is the loading on asset* i *on Brownian motion* j*. Then the optimal dollar amount invested in each of the risky assets by investor* j *is*

$$\pi^j_t = I_{S_t} \phi^j + \frac{b^j}{e^{\rho t} M^1_t} \left(\sigma'_{R,t}\right)^{-1} \theta_t. \tag{62}$$

The optimal dollar amount invested in the risk-free asset is

$$\varphi^j_t = \frac{b^j}{e^{\rho t} M^1_t} \left(1 - \mathbf{1}' \left(\sigma'_{R,t}\right)^{-1} \theta_t\right) \tag{63}$$

where I_{S_t} *is a matrix with* $S_{i,t}$ *in element* (i,i) *and zero everywhere else and* $\mathbf{1}$ *is a* $N+1$ *dimensional vector of ones.*

The optimal amount invested in the risky assets can be decomposed into two parts. The first, $I_{S_t}\phi^j$, is the static buy-and-hold composition of the portfolio. In a symmetric economy with $a^j = \frac{1}{N}$ this is the only term. This give us the following Corollary.

Corollary 1. *Consider a symmetric economy with $\phi^j_j = \phi^H > \phi^F = \phi^j_i$ for $i \neq j$. Let the surplus consumption ratios be in their steady states, then there is both consumption and portfolio home bias in equilibrium.*

The second part, $\frac{b^j}{e^{\rho t} M^1_t}\left(\sigma'_{R,t}\right)^{-1}\theta_t$, is the dynamic component. First note that this part is proportional to the growth optimal portfolio, $\left(\sigma'_{R,t}\right)^{-1}\theta_t$ (i.e., the optimal portfolio of a log utility investor). The sign of the proportionality factor, $\frac{b^j}{e^{\rho t} M^1_t}$, is determined by b^j. As b^j is positive for countries that are rich or have a low average risk aversion, these are also the countries that increase their exposure to the risky assets in equilibrium relative to the the buy-and-hold component. However, this part of their portfolio does not have any portfolio home bias, but is simply based on the growth optimal portfolio. Countries that have a high risk aversion or are poor as measured by a negative b^j reduces their exposure to the risky assets relative to the static buy-and-hold component. From the optimal position in the bond in Proposition 13 we see that the the rich or risk tolerant countries fund their increased exposure to the risky assets by borrowing.

5 Calibration

In this section we calibrate the model to the G10 countries. We consider an asymmetric calibration with USA being wealthier than the average country. For the rest of countries, we assume symmetry.[16]

[16]Relatedly, Hassan (2013) studies the connection between country size and the level of interest rates.

5.1 Output

We calibrate the output processes to real GDP data. For our theoretical results we did not need to specify the dynamics of the volatilities for the country specific part of the endowment process, $\sigma_{x_{i,t}}$. However, to simulate the model we need the dynamics. We assume that the shocks to the US impact all other countries, but not the other way around. Specifically, for US we assume that $\sigma_{x_{1,t},1} = \nu_t$ and $\sigma_{x_{1,t},k} = 0$ for $k = 2, \ldots, 10$. For the other countries we assume that $\sigma_{x_{i,t},1} = \psi \nu_t$ and $\sigma_{x_{i,t},i} = \sqrt{1-\psi^2}\nu_t$ and zero for the rest. We define $V_t = \log(\nu_t)$ and assume the following dynamics

$$dV_t = \kappa_V \left(\bar{V} - V_t\right) + \sigma_V dz_{V,t} \tag{64}$$

where $z_{V,t}$ is a Brownian motion independent of all other shocks. Table 1 presents the parameter values for the output processes.

Table 1: Output parameters

Expected GDP growth rate	μ_Y	0.0200
Volatility of world trend growth	σ_Y	0.0080
Speed of mean reversion in GDP growth	κ_x	0.0326
Long run mean country specific factor	$\bar{x}$	0.0
Speed of mean reversion in volatility	κ_V	0.1583
Long run mean of volatility	$\bar{V}$	−4.1366
Volatility of volatility	σ_V	0.1842
US and ROW correlation	ψ	0.1

The table shows the parameters for the output processes; see Equations (1)–(3) and (64).

Table 2 reports the moments of the GDP in the data and the model. We consider real GDP data for G10 countries, with ROW referring to the average excluding the US. The annualized average real GDP growth is 3% in the US and 2.7% in the ROW, which is higher than comparable numbers reported in the literature. Therefore we use a GDP growth of 2.0% in the model. The standard deviation of the GDP growth in the US is lower than the ROW in the data. In the model we set the standard deviation of the real GDP growth to be 2% in

Table 2: Output moments

	Data	Model
Mean GDP growth	3.0	2.0
Mean GDP growth (ROW)	2.7	2.0
Stdev GDP growth	1.6	2.0
Stdev GDP growth (ROW)	2.0	2.0
Average GDP correlation	0.255	0.250
Average GDP correlation (ROW)	0.194	0.183
GDP growth mean reversion	−0.08	−0.08

The table compares the population moments from the model with the data. The population moments are based on 10,000 years of monthly observations. The quarterly real GDP growth data were downloaded from the OECD. The rest of the world consists of G10 countries (excluding the US). Means and standard deviations are annualized. The GDP growth mean reversion is the regression coefficient of real GDP shares on future GDP growth.

both the US and the ROW. US real GDP has a higher correlation with the real GDP growth than the ROW with a value of 25.5% compared to an average correlation among the ROW of 19.4%. We match the correlation in the data closely with values of 25.0% and 18.3%, respectively. We use the parameter ψ, to match the average correlation between US vs the ROW and within the ROW. In our model, GDP growth is predictable as $x_{i,t}$ is a mean reverting process. To calibrate the speed of mean reversion in GDP growth we calculate the real GDP shares, $s_{i,t}^{GDP} = \frac{Y_{i,t}}{\sum_{l=1}^{N} Y_{l,t}}$, and use the GDP shares to predict GDP growth. As Table 2 illustrates, the GDP predictability is similar in the data and the model.

5.2 Endowments and preferences

In addition to the output processes, we need to specify the parameters of the endowments and preferences. Table 3 shows the parameters. We set the US weight, $a^1 = a^{US} = 0.15$, to capture that the endowment of the US is higher than that of the average country. The remaining countries have equal endowments (i.e., we set $a^j = 0.85/9$ for $j = 2, \dots, 10$). The preference parameter that govern the degree on consumption home bias, ϕ_j^j, is set to 0.95 and we set the weight on all other goods to be homogeneous (i.e., $\phi_i^j = 0.05/9$ for $j \neq i$). These are also the weights used for the consumption baskets (i.e., $h_i^j = \phi_i^j$). The remaining parameters are set to match asset pricing and exchange rate moments.

Specifically, the parameters that govern the level habit factor, $\left(\kappa, \bar{G}, \alpha, \lambda\right)$, are mainly impacting the moments of asset prices and the risk-free rates. They do not affect the exchange rate moments. Hence, we set the parameters of the level habit factor to target a realistic equity premia and low risk-free rates. Note that the level habit factor moves all stocks market symmetrically, which will lead to a higher correlation among stocks relative to that in the data.

The parameters that govern the domestic habits, $\left(\kappa_i, \bar{G}_i, \alpha_i, \lambda_i\right)$, are set to target the exchange rate dynamics. Unlike the level habit factor, the domestic habits impact both asset pricing and exchange rate moments. We maintain as much symmetry as possible with the only parameter that differs across countries is that long-run mean, $\bar{G}_i$, which is higher for the US. A higher long-run mean leads to a higher price of the US good relative to goods produced in other countries and therefore making the US stock market a larger than the stock markets in other countries on average. Second, we set the parameters to match interest rate dynamics and exchange rate volatility in such a way that the model replicates the failure of the uncovered interest rate parity (UIP). We discuss the UIP deviations in detail in a later section.

5.3 Unconditional moments

Table 4 summarizes the key unconditional moments. The model generates an equity premium of 3.1% in the US and 3.7% in the ROW which are somewhat lower than the corresponding values in the data. The higher equity premium of the ROW is due to the fact the returns are measured in USD and therefore impacted by the exchange rate volatility. The model matches closely the volatility of both the US stock market and the ROW, where the ROW is more volatile than the US stock market. The model implies an average correlation between the stock markets of 84%, which is higher than in the data. As the table illustrates the model generates a reasonable level for the risk-free rates with a slightly high volatility. The exchange rate has a volatility of 12.3% which is slightly higher than the data equivalent of 9.1%.

Table 3: Endowment and preference parameters

Description	Parameter	Value	Primary target	Secondary target
Weights and time preference				
US	a^{US}	0.15	NFA dynamics	US is net borrower
Other countries	$\frac{1-a^{US}}{N-1}$	0.85/9	NFA dynamics	US is net borrower
Home good	ϕ_j^j	0.95	consumption home bias	portfolio home bias
Foreign goods	ϕ_i^j	0.05/9	consumption home bias	portfolio home bias
Time preference	ρ	0.02	risk-free rates	price-dividend ratios
Habit level factor, G_t				
Speed of mean reversion	κ	0.015	equity premium	risk-free rates
Long-run mean	$\bar{G}$	8	equity premium	risk-free rates
Sensitivity	α	60	equity premium	risk-free rates
Minimum	λ	5	equity premium	risk-free rates
Domestic habit, $G_{i,t}$				
Speed of mean reversion	κ_i	0.015	exchange rate dynamics	risk-free rates
Long-run mean, US	$\bar{G}_{US}$	6.7	exchange rate dynamics	risk-free rates
Long-run mean, other countries	$\bar{G}_i$	6.5	exchange rate dynamics	risk-free rates
Sensitivity	α_i	81.6	exchange rate dynamics	risk-free rates
Minimum	λ_i	5	exchange rate dynamics	risk-free rates

The table shows the preference parameters and the initial allocations for the US and ROW. Primary and secondary target columns show the data moments we try to match.

5.4 Good versus bad times

In this section we discuss the business cycle variation implications of the model. In the data we calculate averages based on the NBER recession indicator in the US. During the sample period, 88.8% of the months are expansions and 11.2% are recessions. To create a similar measure in the model, we calculate the 88.8% percentile of the inverse surplus consumption ratio for the US, $\mathcal{R}_{US}$. We report the averages conditional on being below or above the recession threshold. Table 5 presents the results. From table 5 we see that the realized returns are lower in recessions for the ROW compared to the US both in the data and the model. Moreover, the model captures the increase in volatilities and correlations from expansions to recessions. The level of the real exchange rate drops in recessions both in the data and the model, implying a USD appreciation in recessions. The net exports increase, consistent with the analysis in section 4.2.

Table 4: Unconditional moments

	Data	Model
Mean excess return	6.28	3.05
Mean excess return (ROW)	5.90	3.65
Return volatility	14.34	14.46
Return volatility (ROW)	18.55	18.82
Mean return correlation	0.65	0.84
Risk-free rate	1.00	1.90
Risk-free rate (ROW)	1.62	1.95
Risk-free rate volatility	2.41	3.61
Risk-free rate volatility (ROW)	1.35	3.61
Mean price-dividend ratio	42.62	61.38
Mean price-dividend ratio (ROW)	33.89	60.54
Price-dividend ratio volatility	17.40	12.52
Price-dividend ratio volatility	8.40	12.30
Exchange rate volatility	9.17	12.31
GDP share	41.23	10.03
Consumption share	44.88	10.03

The table compares the population moments from the model with the data. The population moments are based on 10,000 years of of monthly observations. All variables are sampled monthly from GFD. We present data for for the US and averaged across rest of the world (ROW) countries. Means and standard deviations are expressed in percent and annualized. GDP share is defined as a fraction of US output in world output, where output is measured in USD. For the consumption share we calculate the USD value of the consumption expenditure for each country and take the ratio of US consumption expenditure to total consumption expenditure.

We can see that our main mechanism plays out both in the model and in the data. In recessions, stock markets in the US and ROW fall and the USD appreciates. Consequently, the NFA of the US drops. Importantly, the wealth share of the US goes up in recessions, both in the data and in the model. That is, the US receives a wealth transfer from the rest of the world in recessions. Additionally, stock returns and exchange rates become more volatile in recessions, as in the data.

Additionally, we compute how the US GDP and consumption shares behave in the model and in the data. Both go up in recessions. This is consistent with the wealth share and NFA results. Since wealth is the present value of future consumption, we would expect wealth and consumption shares to generally go in the same direction. Moreover, the GDP share drops more than the consumption share. This is consistent with a decrease in the NFA during recessions and, from Equation (44), we know that the NFA is the difference is the net present value of consumption and GDP.

Notice that the unconditional value of the US NFA/GDP ratio is positive in the model, while it has been negative in the last few decades. On average, US net exports are negative in the model, as in the data. It is difficult to rationalize both negative NFA and negative net exports on average within a standard model. One way to obtain this result is for the US dollar to exhibit special benefits, for instance, by modeling a positive convenience yield on US bonds (e.g., Jiang et al., 2020a).

Table 5: Business cycle properties

	Unconditional moments		Recessions (relative to unconditional)	
	Data	Model	Data	Model
Mean excess return	6.28	3.05	−15.05	−15.90
Mean excess return (ROW)	5.90	3.65	−26.60	−18.10
Return volatility	14.34	14.46	8.38	9.00
Return volatility (ROW)	18.55	18.82	8.26	14.40
Mean return correlation	0.65	0.84	0.18	0.08
Mean price-dividend ratio	42.62	61.38	−18.47	−25.47
Mean price-dividend ratio (ROW)	33.89	60.54	−18.50	−24.57
Mean real exchange rate	100.00	100.00	−15.81	−2.16
Exchange rate volatility	9.17	12.31	2.73	9.10
NFA/GDP	−9.09	70.00	−11.43	−40.10
NX/GDP	−1.56	−1.60	4.38	0.90
Wealth share	43.65	10.25	7.57	0.61
GDP share	41.23	10.03	1.39	0.95
Consumption share	44.88	10.03	0.29	0.88

The table shows unconditional moments and the relative changes in recessions. For excess return, return volatility, and correlation moments, a positive number in the Recession column indicates that the conditional moment is higher in recessions relative to its unconditional value. For price-dividend ratios, the real exchange rate, NFA/GDP, NX/GDP, and the wealth share, a positive number in the Recession column indicates the increase of the variable during a recession. All variables are sampled monthly with the exception of NFA and NX that are sampled quarterly. Means and standard deviations are expressed in percent and annualized. We present data for for the US and averaged across rest of the world (ROW) countries. In the data, recessions are defined by the NBER recessions indicator. In the model, recessions are the periods with the 11.2% highest values for the US inverse surplus consumption ratio ($\mathcal{R}$) in simulations. GDP share is defined as a fraction of US output in world output, where output is measured in USD. For the consumption share we calculate the USD value of the consumption expenditure for each country and take the ratio of US consumption expenditure to total consumption expenditure.

5.5 Currency risk premia

In this section we take a closer look at the currency risk premium. We verify that our model can reproduce the well-established deviation from UIP and the ability of the real exchange rate to predict currency returns. Under rational expectation, currency return predictability implies that UIP does not hold and that there is a time-varying currency risk premium. Table 6 shows predictability regressions of the currency excess return on the interest rate differential, often referred to as a Fama (1984) regression. The table presents results using data for both nominal and real interest rate differentials, and the population equivalent in real terms from the model. The regressions use currency fixed effects and standard errors are clustered by month.

In the regressions of the currency return on the interest rate differential (the foreign minus the US interest rate), the coefficient estimates are positive in the data and in the model. A nonzero coefficient indicates that currency returns are predictable (i.e., UIP does not hold and the currency risk premium is time-varying). Moreover, consistent with the literature, the estimates are above one, which means that a foreign currency tends to appreciate when its interest rate is higher than the US interest rate. The model reproduces these patterns.

Dahlquist and Pénasse (2022) argue in a present-value model under purchasing power parity that the Fama regression should be augmented by the real exchange rate. Table 6 shows in such augmented regressions that the coefficient associated with the real exchange rate is negative in the data and in the model. This means that when the USD price of a foreign currency is high, the subsequent currency return tends to be low, and vice versa. The model reproduces this fact as the currency risk premium is lower when the USD is appreciated.

We next ask if the model can reproduce the predictability reversal documented by Bacchetta and Van Wincoop (2010) and Engel (2016). In particular, Engel (2016) shows that the positive relationship between the expected currency return and the interest rate differential reverses over the horizon, a feature that rational expectation models of the currency risk premium have difficulty reproducing and considered to be a puzzle. Following Dahlquist and Pénasse (2022), we regress one month expected excess currency returns j periods ahead on the interest rate differential and real exchange rate. Expectations for horizons above one month are based on a VAR model of the currency return, the interest rate differential, and the real exchange rate. Figure 6 shows that the model can replicate the pattern in the data. In the short-run, the coefficient exceeds one, consistent with the Fama regression results. However, in the medium-run, the coefficient becomes negative. In the long-run, the coefficient converges to zero (i.e., there is no relation between the currency risk premium and the interest rate differential).

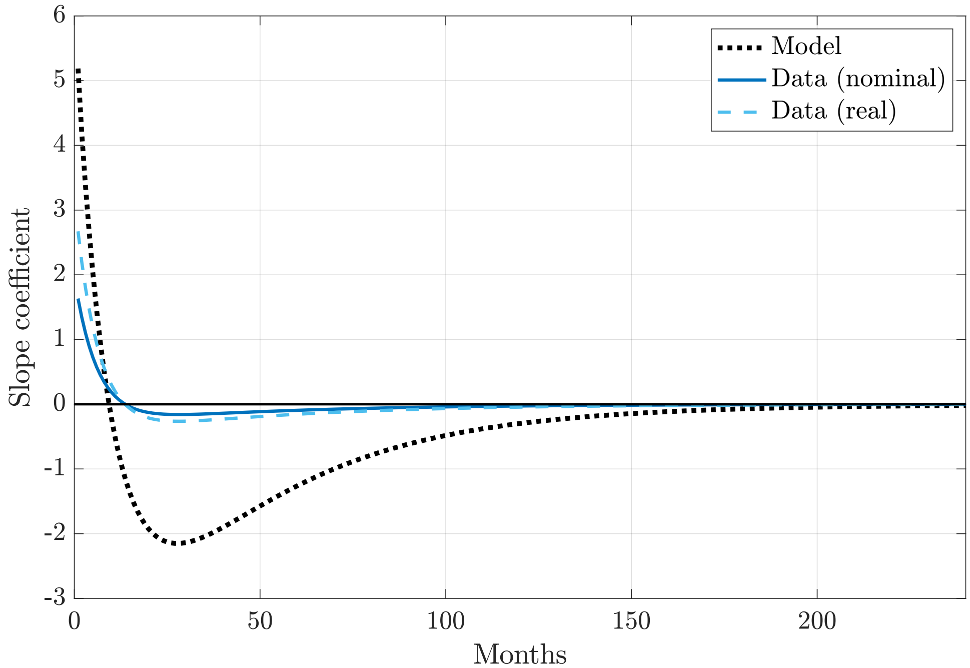

Figure 6: Predictability reversal puzzle
This figure shows the slope coefficient in a regression of the expected currency return on the interest rate differential. The one-month expected currency return is constructed by regressing monthly the currency return on lagged currency return, lagged log interest rate differential, and lagged log real exchange rates. The expectations beyond one month are obtained by taking the power of a VAR model of the regression variables. The model sample is constructed from averaging across 1,000 paths of 20,000 months as a proxy for the population equivalent.

Our model can match the deviations from UIP and the predictability reversal because of stochastic volatility of the country-specific factors (ν_t). The stochastic volatility creates an additional source of risk, which makes the relationship between the interest rate differential and the currency premium dependent of the volatility. When the volatility is low, risk premia are low and precautionary savings are weak, leading to a negative relationship between the interest rate differential and the currency risk premium. However, when the volatility is high, risk premia are high and precautionary savings are strong, leading to a positive relationship between the interest rate differential and the risk premium. In the short-run, the effect of the high volatility states dominates and the coefficient in the Fama regression is positive. As the volatility persistence is fairly low, the effect of the low volatility states dominate in the medium run and the coefficient becomes negative. In the long-run, the effect of the volatility dies out and there are no deviations from the UIP.

Table 6: Fama regression

	Data (nominal)		Data (real)		Model	
Interest rate differential	1.615***	1.778***	2.648***	2.922***	3.787	7.823
	(0.366)	(0.364)	(0.599)	(0.597)		
Real exchange rate	–	−0.017***	–	−0.017***		−0.030
	–	(0.006)	–	(0.006)		
R^2	0.018	0.026	0.018	0.026		
# obs.	4,565	4,565	4,565	4,565		

This table shows estimation results for panel regressions of the log currency return ($rx_{j,t+1}$) on the log interest rate differential (the foreign minus the US interest rate, $r_{j,t} - r_t$) and the log real exchange rate ($q_{j,t}$): $rx_{j,t+1} = \alpha_j + \beta\,(r_{j,t} - r_t) + \gamma q_{j,t} + \varepsilon_{j,t+1}$. The data covers G10 currencies and is sampled monthly during the period March 1976 to May 2020. The regression includes currency fixed effects; standard errors are clustered by month (*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$). The model columns report population moments, constructed from averaging across 1,000 simulated paths of 20,000 months.

6 Conclusion

This paper explores joint dynamics of cross-country wealth transfers and asset prices in times of stress and tranquility. We show that in times of stress the USD appreciates and therefore the US stock market does not fall as much as other stock markets. We propose a model in which US investors have a home bias in their portfolios, which dampens the shock to US wealth due to the USD appreciation. This effect is large enough to overturn the loss due to falling NFA and the wealth share of the US in the world economy increases, i.e., the US receives a wealth transfer from the ROW. In addition, the model can reproduce patterns for the currency risk premium that were challenging for earlier general equilibrium models.

We see at least three avenues for future research. First, our calibration focuses on G10 countries. However, emerging markets are important holders of US debt, and the bulk of wealth transfers to the US in times of stress comes from emerging markets. As our motivating empirical facts suggest, there are differences in the exposure to global shocks between the G10 countries and the emerging markets. It would be interesting to take these differences into account in a calibration of the model. This could help in better understanding the dynamics of wealth transfers between emerging and developed markets.

Second, in contrast to many papers in the international macro-finance literature, we consider an asymmetric calibration in which the US differs from all other countries in the world economy. Yet, as we have a tractable model, one could easily allow for a three region calibrations – the US, developed markets, and emerging markets. This could allow for a more nuanced exploration of the NFA dynamics and the currency risk premia around the world.

Third, we assume that markets are complete. This lets us solve the model in closed form, which gives transparency and tractability. However, there are important frictions that we do not model. One possible extension would be to incorporate realistic frictions affecting cross-border asset positions.

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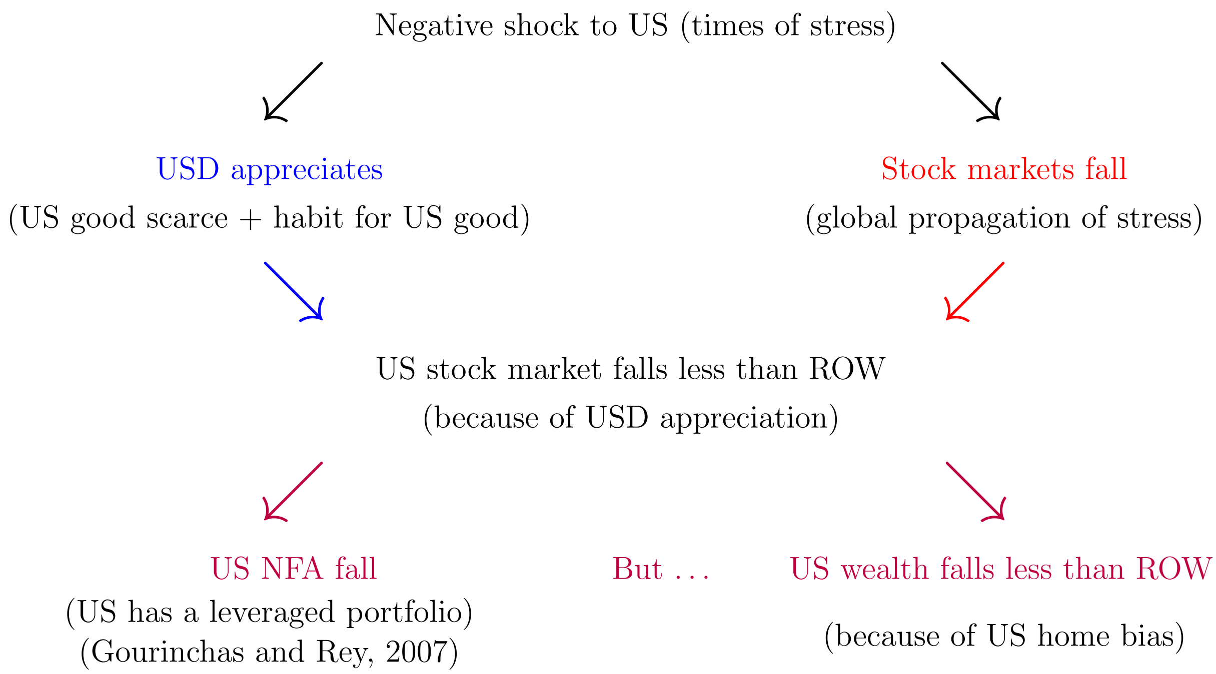
A Illustration of the economic mechanism

We illustrate below the main economic mechanism of our model.



B Data appendix

We retrieve monthly spot and one-month forward exchange rates from Barclays Bank International and Reuters (via Datastream) for the period January 1976 to May 2020. We consider the G10 currencies: the Australian dollar (AUD), Canadian dollar (CAD), euro (EUR), Japanese yen (JPY), Norwegian krona (NOK), New Zealand dollar (NZD), Swedish krona (SEK), Swiss franc (CHF), pound sterling (GBP), and US dollar (USD). We let the USD be the domestic currency and express all exchange rates in USD per unit of the foreign currency. For the CAD, EUR (spliced with the German mark before 1999), JPY, NOK, SEK, CHF, and GBP, the sample begins in January 1976; for the AUD and NZD, data availability makes the sample start in January 1985. The G10 data are further described in Chernov, Dahlquist, and Lochstoer (2022). We also collect data for emerging currencies: Colombia (COP), Indonesia (IDR), Republic of Korea (KRW), Malaysia (MYR), Mexico (MXN), Philippines (PHP), Singapore (SGD), Sri Lanka (LKR), Taiwan (TWD), and Turkey (TRY), and include them as they become available.

Log real USD are computed as $q_t = s_t + p_t^* - p_t$, where p_t^* and p_t are log consumer price indices obtained from the Organization for Economic Co-operation and Development (OECD) for the period January 1976 to May 2020. The statistical agencies in Australia and New Zealand release price indices on a quarterly basis. We therefore forward fill the price indices for the AUD and NZD in the months until the next quarter. This creates stale prices but avoids introducing future information into the economist's information set.

Our stock return data come from Global Financial Data (GFD). We compute excess returns in USD by subtracting the US 3-month T-Bill rate (also obtained from GFD) from total equity dollar returns (in logs).

Realized volatility in a given country i and month t is computed using daily data as

$$\text{Vol}_{i,t} = \sqrt{\frac{\pi}{2}} \sum_{k=1}^{K_{i,t}} \frac{|x_{i,k}|}{\sqrt{K_{i,t}}},$$

where $K_{i,t}$ is the number of observations in a given country/month. The daily excess return $x_{i,k}$ is approximated by index price changes. We use S&P 500 data for the US, obtained from GFD; for non-US countries, we use MSCI indices when available, and GFD indices otherwise.

To parallel the model, our NFA data include only equity and debt positions (and thus does not include FDI). Our main US series come from the Board of Governors of the Federal Reserve System, downloaded from FRED. We use Equity assets (ticker ROWCEAQ027S), Debt assets (DODFFSWCMI), Equity liabilities: (ROWCESQ027S), and Debt liabilities: (DODFF-SWCMI). These data are quarterly and are available since 1945. Net Exports (NX) are obtained from the OECD over the period Q1 1965 to Q4 2018. The GDP data also come from the OECD.

We compute implied one-month interest rate differentials using the covered interest rate parity (CIP): $i_t^* - i_t = s_t - f_t$, where s_t and f_t denote the log spot and forward exchange rates, respectively. Log excess returns for a US investor going long in a foreign currency are computed as $rx_{t+1} = s_{t+1} - f_t$.

We estimate real interest rate differentials as the fitted value of a projection of the realized real interest rate differential on the current nominal interest rate differential and the inflation differential over the previous month, in the spirit of Schorfheide, Song, and Yaron (2018).

C Proofs

Proof of Proposition 1. As we assume complete markets, we can solve the equilibrium by solving the corresponding central planner problem, Equation (17) in the paper, repeated here for convenience:

$$U\left(Y_t; H_t, a\right) = \max_{C_t^j} \sum_{j=1}^{N} a^j \sum_{i=1}^{N} \log\left(C_{i,t}^j - H_{i,t}^j\right) \quad s.t. \quad \sum_{j=1}^{N} C_{i,t}^j = Y_{i,t} \quad i = 1, \ldots, N, \tag{65}$$

where $Y_t = (Y_{1,t}, \ldots, Y_{N,t})$, $H_t = (H_{1,t}, \ldots, H_{N,t})$ and $a = \left(a^1, \ldots, a^N\right)$ for all t. The FOCs of the central planner problem in Equation (65)

$$\frac{a^j}{C_{i,t}^j - H_{i,t}^j} = \eta_{i,t}, \qquad i, j = 1, \ldots, N, \tag{66}$$

where $\eta_t = (\eta_{1,t}, \ldots, \eta_{N,t})$ are the Lagrange multipliers on the resource constraint in (65). Hence, we can relate the consumption of investor j and k through $\frac{a^j}{C_{i,t}^j - H_{i,t}^j} = \frac{a^k}{C_{i,t}^k - H_{i,t}^k}$. Imposing the market clearing in each consumption good and using $\sum_{j=1}^{N} \phi_i^j = 1$ from Assumption 1 and the fact that $\sum_{n=1}^{N} a^n = 1$, we have that

$$\left(C_{i,t}^j - H_{i,t}^j\right) = a^j \left(Y_{i,t} - H_{i,t}\right) \tag{67}$$

Next, we have that

$$\begin{aligned} f_{i,t}^j &= \frac{C_{i,t}^j}{Y_{i,t}} = a^j \left(1 - \frac{H_{i,t}}{Y_{i,t}}\right) + \phi_i^j \frac{H_{i,t}}{Y_{i,t}} \\ &= a^j + \left(\phi_i^j - a^j\right) \frac{H_{i,t}}{Y_{i,t}} \\ &= a^j + \left(\phi_i^j - a^j\right) \left(1 - s_{i,t}\right) \\ &= \phi_i^j + \left(a^j - \phi_i^j\right) s_{i,t} \end{aligned} \tag{68}$$

□

Proof of Proposition 2. We have from Proposition 1 that $f_{i,t}^j = \phi_i^j + \left(a^j - \phi_i^j\right) s_{i,t}$. Consider first the case of $\phi_i^j > a^j$. It immediately follows that $\frac{\partial f_{i,t}^j}{\partial s_i} < 0$ and therefore the minimum is attained for $s_i = s_i^{max}$. For the maximum we have that if $s_i \to 0$ then $f_i^j \to \phi_i^j$. The case of $\phi_i^j < a^j$ follows from a similar argument. □

Proof of Proposition 3. The individual optimization problem of the representative agent in country j is

$$U^j\left(C_t^j\right) = \max_{C_t^j} E\left[\int_0^\infty e^{-\rho t}\sum_{i=0}^{N} log\left(C_{i,t}^j - H_{i,t}^j\right) dt\right] \tag{69}$$

$s.t$

$$E\left[\int_0^\infty \sum_{i=0}^{N} M_{i,t} C_{i,t}^j dt\right] \le W_0^j. \tag{70}$$

The FOC of the above problem are

$$\frac{e^{-\rho t}}{C_i^j - H_{i,t}^j} = \kappa^j M_{i,t}, \tag{71}$$

where κ^j is the Lagrange multiplier associated with optimization problem in Equation (69).

Using the optimal consumption in Proposition 1 and noting that, as standard in the mapping between the planner's problem and competitive equilibrium, $\kappa^j = \frac{1}{a^j}$, we have

$$M_{i,t} = e^{-\rho t}\frac{1}{Y_{i,t} - H_{i,t}}. \tag{72}$$

The stochastic discount factor in country j is the discount factor that prices a basket with h_i^j units of good $i = 1, \ldots, N$, i.e.,

$$M_t^j = \sum_{i=1}^{N} h_i^j M_{i,t}. \tag{73}$$

□

Proof of Proposition 4. This follows from noting that the price of good i in terms of units of the consumption basket of country 1 is $p_{i,t} = \frac{M_{i,t}}{M_t^1}$. Using the expressions for $M_{i,t}$ and M_t^1 from

(72) and (73) and Assumption 3, we have

$$\begin{aligned} p_{i,t} &= \frac{M_{i,t}}{M_t^1} \\ &= \frac{M_{i,t}}{\sum_{l=1}^N h_l^1 M_{l,t}} \\ &= \frac{e^{-\rho t}\frac{1}{Y_{i,t}-H_{i,t}}}{\sum_{l=1}^N h_l^1 e^{-\rho t}\frac{1}{Y_{l,t}-H_{l,t}}} \\ &= \left(\frac{G_{i,t}}{Y_{i,t}}\right)\frac{1}{\sum_{l=1}^N h_l^1\left(\frac{G_{l,t}}{Y_{l,t}}\right)}, \end{aligned} \tag{74}$$

and it follows that the real exchange rate of country j relative to country 1 is

$$Q_t^j = \frac{\sum_{l=1}^N h_l^j\left(\frac{G_{l,t}}{Y_{l,t}}\right)}{\sum_{l=1}^N h_l^1\left(\frac{G_{l,t}}{Y_{l,t}}\right)}. \tag{75}$$

□

Proof of Proposition 5. The net exports of country 1 is the

$$NX_t = p_{1,t}\left(Y_{1,t} - C_{1,t}^1\right) - \sum_{i=2}^N p_{i,t}C_{i,t}^1. \tag{76}$$

Using the optimal consumption shares from Proposition 1, we have

$$\begin{aligned} NX_t &= p_{1,t}\left(Y_{1,t} - f_{1,t}^1 Y_{1,t}\right) - \sum_{i=2}^N p_{i,t} f_{i,t}^1 Y_{i,t} \\ &= p_{1,t}Y_{1,t}\left(1 - f_{1,t}^1\right) - \sum_{i=2}^N p_{1,t}Y_{1,t}\frac{p_{i,t}}{p_{1,t}}\frac{Y_{i,t}}{Y_{1,t}} f_{i,t}^1 \\ &= p_{1,t}Y_{1,t}\left(1 - f_{1,t}^1 - \sum_{i=2}^N \frac{G_{i,t}}{G_{1,t}} f_{i,t}^1\right) \\ &= p_{1,t}Y_{1,t}\left(1 - \sum_{i=1}^N \frac{G_{i,t}}{G_{1,t}} f_{i,t}^1\right) \\ &= p_{1,t}Y_{1,t}\left(1 - \sum_{i=1}^N \frac{G_{i,t}}{G_{1,t}}\left(\phi_i^1 + \left(a^1 - \phi_i^1\right)s_{i,t}\right)\right) \end{aligned} \tag{77}$$

□

Proof of Proposition 6. We have that the stock price of country i is

$$\begin{aligned} S_{i,t} &= p_{i,t} E_t \left(\int_t^{\infty} \frac{M_{i,u}}{M_{i,t}} Y_{i,u} du \right) && (78) \\ &= p_{i,t} Y_{i,t} PD_{i,t}, && (79) \end{aligned}$$

where

$$PD_{i,t} = E_t \left(\int_t^{\infty} \frac{M_{i,u}}{M_{i,t}} \frac{Y_{i,u}}{Y_{i,t}} du \right) \tag{80}$$

We can write the price-dividend ratio, $PD_{i,t}$, as

$$\begin{aligned} PD_{i,t} &= E_t \left(\int_t^{\infty} \frac{M_{i,u}}{M_{i,t}} \frac{Y_{i,u}}{Y_{i,t}} du \right) \\ &= E_t \left(\int_t^{\infty} \frac{\mathcal{R}_{i,u}}{\mathcal{R}_{i,t}} du \right) \\ &= \frac{1}{\mathcal{R}_{i,t}} F_{i,t} && (81) \end{aligned}$$

where the inverse surplus consumption ratio $\mathcal{R}_{i,t}$ is as defined in (6) and $F_{i,t} = E_t \left(\int_t^{\infty} \mathcal{R}_{i,u} du \right)$. We therefore have

$$\begin{aligned} F_{i,t} &= E_t \left(\int_t^{\infty} e^{-\rho(u-t)} \mathcal{R}_{i,u} du \right) \\ &= \int_t^{\infty} e^{-\rho(u-t)} E_t \left(\mathcal{R}_{i,u} \right) du \\ &= \int_t^{\infty} e^{-\rho(u-t)} E_t \left(G_{i,u} \right) E_t \left(G_u \right) du, && (82) \end{aligned}$$

where the last equality follows from the independence of $G_{i,t}$ and G_t. Assumption 3 implies that

$$E_t \left(G_{i,u} \right) = G_{i,t} e^{-\kappa_i (u-t)} + \bar{G}_i \left(1 - e^{-\kappa_i (u-t)} \right), \tag{83}$$

and

$$E_t \left(G_u \right) = G_t e^{-\kappa (u-t)} + \bar{G}_i \left(1 - e^{-\kappa (u-t)} \right). \tag{84}$$

Inserting Equations (83) and (84) into Equation (82) and integrating yields the result. □

Proof of Proposition 7. The wealth of investor j is

$$\begin{aligned} W_t^j &= E_t\left(\int_t^\infty \sum_{i=1}^N \frac{M_u^1}{M_t^1} p_{i,t} C_{i,t}^j du\right) \\ &= \sum_{i=1}^N p_{i,t} C_{i,t}^j \int_t^\infty E_t\left(\frac{M_{i,u}}{M_{i,t}} \frac{C_{i,u}^j}{C_{i,t}^j}\right) du \\ &= \sum_{i=1}^N p_{i,t} C_{i,t}^j \int_t^\infty E_t\left(e^{-\rho(u-t)} \frac{\mathcal{R}_{i,u}}{\mathcal{R}_{i,t}} \left(\frac{Y_{i,t}}{Y_{i,u}}\right)\left(\frac{Y_{i,u}}{Y_{i,t}}\right)\left(\frac{f_{i,u}^j}{f_{i,t}^j}\right)\right) du \\ &= \sum_{i=1}^N p_{i,t} Y_{i,t} \int_t^\infty E_t\left(e^{-\rho(u-t)} \frac{\mathcal{R}_{i,u}}{\mathcal{R}_{i,t}} \left(\phi_i^j + \left(a^j - \phi_i^j\right) \frac{1}{\mathcal{R}_{i,u}}\right)\right) du \\ &= \sum_{i=1}^N p_{i,t} Y_{i,t} \frac{F_{i,t}}{\mathcal{R}_{i,t}} \phi_i^j + \sum_{i=1}^N p_{i,t} Y_{i,t} \frac{1}{\mathcal{R}_{i,t}} \left(\frac{a^j - \phi_i^j}{\rho}\right) \\ &= \sum_{i=1}^N \phi_i^j S_{i,t} + \frac{b^j}{e^{\rho t} M_t^1}, \end{aligned} \tag{85}$$

where the inverse surplus consumption ratio $\mathcal{R}_{i,t}$ is as defined in (6) and the consumption share $f_{i,t}^j$ is from Proposition 1. □

Proof of Proposition 8. From Proposition 7 we have that

$$W_t^j = \sum_{i=1}^N \phi_i^j S_{i,t} + \frac{b^j}{e^{\rho t} M_t^1}. \tag{86}$$

If each country is initially endowed with the claim to their own output we have that $W_t^j = S_{j,t}$. Using this together with the fact that $b^j = \frac{1}{\rho}\left(N a^j - \sum_{i=1}^N \phi_i^j\right)$ and $\mathcal{R}_{i,0} = \bar{\mathcal{R}}_i$ for $i = 1, \ldots, N$, and then solving for a^j yields the result. □

Proof of Proposition 9. We can write ED_t^j as

$$ED_t^j = \frac{b^j}{e^{\rho t} M_t^1} = b^j \left(\sum_{i=1}^N h_i^j \frac{\mathcal{R}_{i,t}}{Y_{i,t}}\right)^{-1}. \tag{87}$$

Note that for $a^j > \frac{1}{N}$ we have that $b^j > 0$. Moreover, we have

$$\frac{\partial}{\partial \mathcal{R}_{i,t}} \frac{1}{\sum_{i=1}^{N} h_i^j \frac{\mathcal{R}_{i,t}}{Y_{i,t}}} = -\frac{1}{\left(\sum_{i=1}^{N} h_i^j \frac{\mathcal{R}_{i,t}}{Y_{i,t}}\right)^2} h_i^j \frac{1}{Y_{i,t}} < 0, \tag{88}$$

hence $\frac{\partial ED_t^j}{\partial \mathcal{R}_{i,t}} < 0$ if $b^j > 0$. □

Proof of Proposition 10. The stochastic discount factor of country j follows follows

$$\frac{dM_t^j}{M_t^j} = -r_t^j dt - \left(\theta_t^j\right)' dw_t. \tag{89}$$

Applying Ito's lemma to $M_t^j = e^{-\rho t} \sum_{i=1}^{N} h_i^j \frac{1}{Y_{i,t} - H_{i,t}}$ and equating the drift and diffusion coefficients with the drift and diffusion coefficient from Equation (89) yields the result. □

Proof of Proposition 11. The real exchange rate can be written as the ratio of SDF

$$Q_t^j = \frac{M_t^j}{M_t^1} \tag{90}$$

Applying Ito's lemma to Equation (90) yields the result. □

Proof of Proposition 12. The instantaneous return on stock i is

$$dR_{i,t} = \frac{dS_{i,t} + Y_{i,t} dt}{S_{i,t}} = \mu_{R_i,t} dt + \sigma_{R_i,t}^{'} dw_t \tag{91}$$

By no-arbitrage $\mu_{R_i,t} = r_t + \sigma_{R_i,t}^{'} \theta_t$. Applying Ito's lemma to the stock price in Proposition 6 to calculate the diffusion coefficients $\sigma_{R_i,t}$ yields the result. □

Proof of Proposition 13. Applying Ito's lemma to the wealth in Proposition 7 and matching the drift and diffusion coefficients with the drift and diffusion coefficients in the dynamic budget constraint in Equation (16), we derive the result in the proposition. □

D Additional Figures

In this appendix, we present additional figures that supplement our discussion in the main text.

Figure 7 shows the GDP and consumption shares of the US around the GFC, for the 2004-2014 period. Both shares increase during the GFC.

Figure 8 shows that the net exports are decreasing with the risk appetite level factor, $1/G$. In contrast to the US surplus consumption ratio, the level factor does not impact the exchange rate. Hence, the mechanism is quite different from that in Figure 4. First, exports are increasing due to the consumption home bias as in Figure 4. However, the imports increase is due to risk sharing and not to relative prices. This is because the US has a lower utility curvature for foreign goods. In good times when the surplus consumption ratio is high, the US consumes a larger fraction of foreign goods so that imports increase. In other words, the decreasing net exports are due to changes in the quantities of imported goods and not exchange rate changes.

E Alternative estimates of the US wealth share

In this appendix, we present alternative estimates of the US wealth share, which is based on the wealth data retrieved from the World Inequality Database (WID).[17] These national accounts data were initially assembled by Piketty and Zucman (2014) and have since been continually refined and augmented by the dedicated team of WID researchers. The wealth estimates in the WID database are constructed using a "bottom-up" approach, which adds up estimates of wealth following the U.N. System of National Accounts (SNA). Official national balance sheets use census-like methods to measure financial assets, liabilities, and real estate, collecting data at a specific point in time. Non-financial wealth is primarily assessed by aggregating past investment flows. The approach leaves out some important constituents of wealth such as

[17] See https://wid.world/data/.

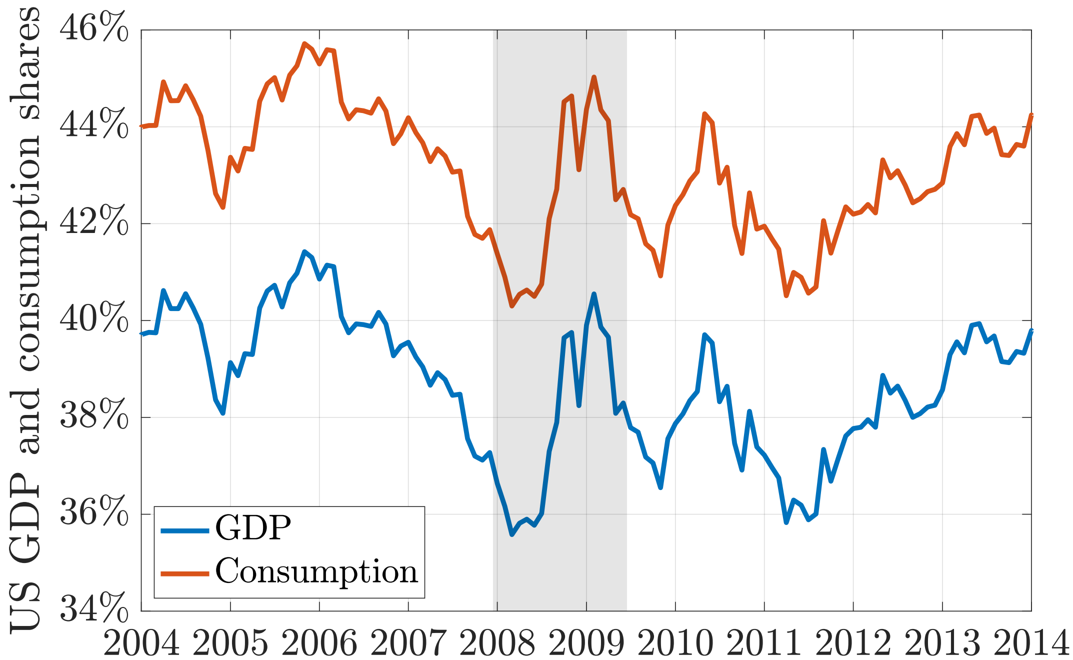


Figure 7: US GDP and consumption shares during the Great Financial Crisis
This figure shows US GDP as a fraction of the combined GDP of the G10 economies and US consumption as a fraction of the combined consumptions of the G10 economies.

human wealth.

We start with describing our dataset in detail.

E.1 GDP data

The GDP data are quarterly and retrieved from the OECD. The data are real and in local currency (OECD code: VOBARSA). We convert them to real USD using spot exchange rates.

E.2 Yearly private wealth

We next calculate real national wealth in USD using the real GDP series. We first obtain net market-value national wealth (code: nweal) and GDP (code: gdpro) data from the World Inequality Database (WID). These series are expressed in nominal terms. We calculate wealth-GDP ratios for all countries for which the data are available. We then multiply the wealth-GDP ratios with the real USD GDP series from the OECD. This gives us yearly real USD series.

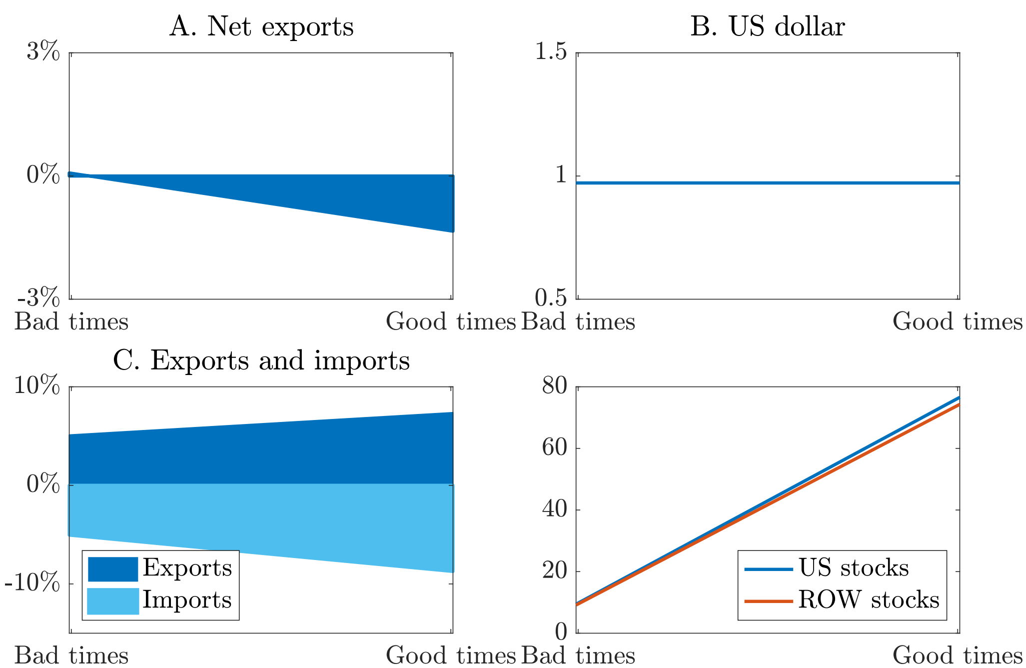


Figure 8: Exchange rates, stock markets, and net exports as a function of $1/G$. The figure plots the net exports (Panel A), the real exchange rate (USD against ROW currencies, Panel B), exports and imports (Panel C), and US stock prices (Panel D) as functions of the risk appetite level factor $(1/G)$. For all figures we keep the US habit level at its long-run mean $G_{US} = \bar{G}_{US}$. The parameters are the same as the parameters used in the calibration section (see Tables 1 and 3).

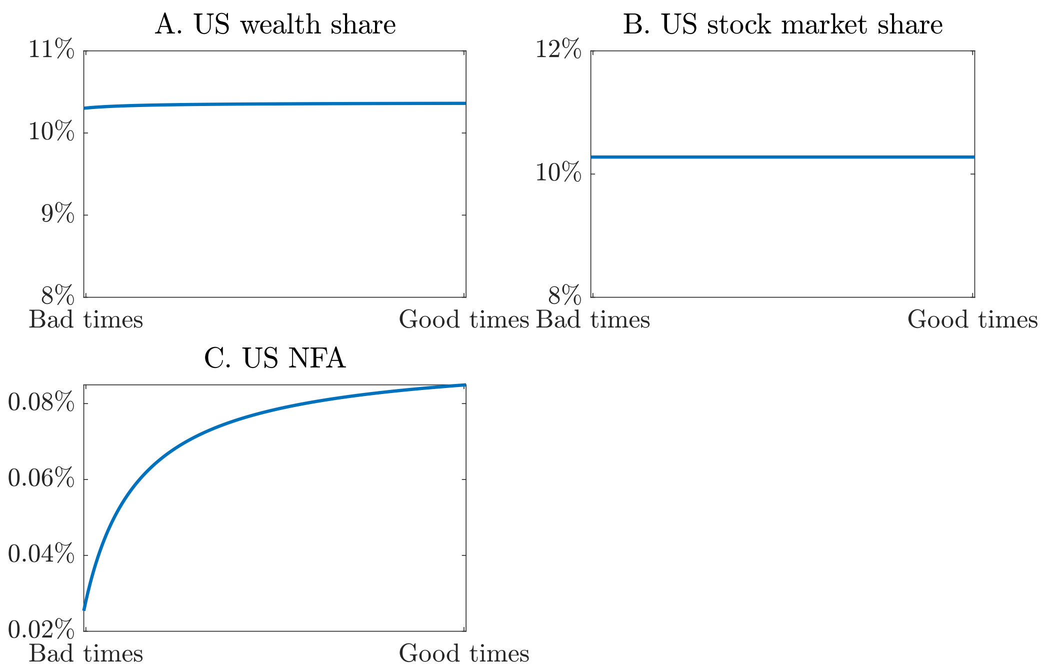


Figure 9: US wealth share, US stock market share, and net foreign assets as a function of $1/G$

The figure plots the US wealth scaled by world wealth (Panel A), the US stock market share in world stock market (Panel B), and the US NFA scaled by world wealth (Panel C) as functions of the risk appetite level factor ($1/G$). For all figures we keep the US habit level at its long-run mean $G_{US} = \bar{G}_{US}$. The parameters are the same as the parameters used in the calibration section (see Tables 1 and 3).

E.3 Monthly private wealth

To construct monthly series, we calculate returns on wealth following (Jordà, Knoll, Kuvshinov, Schularick, and Taylor 2019; henceforth JKKST) using Global Financial Data (GFD). These returns are calculated as a weighted average of equities, bonds, bills, and housing returns, where the weights are assumed to stay constant within a year. We first construct monthly real USD returns on equities, bonds, and bills from GFD. We do not have monthly housing returns and thus assume that returns in local currency are constant within a year.

To calculate weights, we use that WID include national non-financial assets series (code: pwnfa), housing (code: pwhou), and net wealth (code: pweal). The share of housing assets is obtained by dividing the value of housing assets by the total national wealth. We calculate the share of financial assets by subtracting the value of non-financial assets from the total national wealth and dividing the result by the total national wealth. The sum of share of housing and financial assets typically amount to 80–90% of wealth. We simply ignore the residual part and thus assume that wealth consists of housing and financial assets. Following JKKST, we assume that financial assets consist of equity, bonds, and bills. We obtain equity market capitalization series from Kuvshinov and Zimmermann (2022). We are thus able to calculate the shares of housing, equities, bonds, and bills. Again, following JKKST we allocate this last portion 50–50 between bonds and bills. We use the returns and weights data to calculate returns on wealth for each country over each month from January to November. These returns are then applied to the yearly wealth series, which are allocated to December.

E.4 Monthly human wealth

In one of our estimates, we include human weath in the construction of the countries' wealth shares. To construct the human wealth series, we first obtain compensation of employees (code: comhn) and GDP (code: gdpro) data from WID. We then calculate the yearly labor share series as comhn/gdpro. We multiply this labor share series with quarterly nominal local currency GDP data from the OECD. This gives us a nominal local currency labor compensation series,

L. We therefore assume that the labor-GDP ratio is equal to a consol. We assume that the unobserved consol yield y is equal to the 10-year yield plus one percent. (This assumption ensures that the rate is always positive.) Thus, the nominal wealth series is equal to L/y. We then multiply this series by real USD GDP to get the wealth in real USD, H.

E.5 Total wealth

Finally, we combine this series with the monthly private wealth series constructed earlier. Total wealth is defined as the sum of private wealth and human wealth and is available monthly.

E.6 The estimates of the US wealth share

We now reconstruct the wealth shares using data from the World Inequality Database (WID) from Piketty and Zucman (QJE 2014). The countries in our sample are Australia (AUS), Canada (CAN), Germany (DEU), France (FRA), United Kingdom (GBR), Italy (ITA), Japan (JPN), Sweden (SWE), and United States (USA). We convert these data to USD. To make it comparable to our main series we construct a monthly series. To do so, we follow JKKST to calculate returns on wealth within a year. Next, to construct the real wealth share, we start with the wealth-GDP ratios from the WID and convert them to real USD using real local currency GDP from the OECD quarterly data and converting back to USD using spot exchange rates. We then apply the monthly return on wealth series to obtain real USD monthly series. We find that over the period from 2007 to the end of the recession in mid-2009, the US wealth share is flat of slightly decreasing. Up until the end of 2008, the US wealth share was flat and even slightly increasing, primarily thanks to the USD appreciation. We remark that Japan's wealth share increased markedly in that period. The main driver of this is the JPY, which appreciated against the USD over the GFC.

Figure 10 shows the share of US wealth over the combined wealth of G10 markets over the 2002–2014 period.

Finally, we note that the biggest distinction between our wealth calculation attempts to

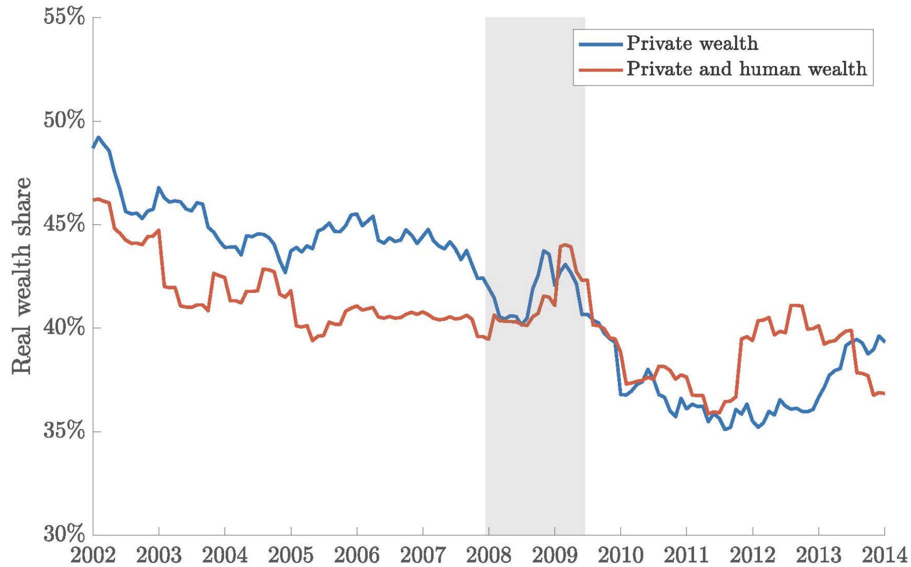


Figure 10: US wealth during the Global Financial Crisis
This figure shows the US wealth share when wealth is private wealth only (the sum of non-financial assets and financial assets minus financial liabilities of the household and non-profit sectors) and when wealth includes human wealth. All series are expressed in real USD and are converted to monthly frequency using available portfolio weights and asset returns as in JKKST. Human wealth is measured as the present value of the labor share of GDP, discounted using a long-term bond rate.

estimate and Piketty and Zucman (2014) is human capital. It is a major part of total wealth, but it is missing from the Piketty and Zucman data. In the literature, it is common to think of human capital as a bond (Lustig et al., 2013 **[Anna's note:** Merton?]). Specifically, human capital of the US is close to a USD-denominated bond. Fluctuations in the USD would to a large extent drive fluctuations in the value of a bond. If we enhance WID wealth with our estimate of human capital, we get a well-pronounced rise in the US wealth share up until the end of the recession in June 2009. We plot our estimate of US share of total wealth, which includes human wealth, in Figure 10.

F A simple model

In this appendix, we present a simple model, which generates our key economic mechanism. Specifically, in this model the USD appreciates in times of stress and the US receives a wealth transfer from the ROW during such episodes.

The model is a discrete-time version of the model in Pavlova and Rigobon (2007). Time indexed by t. There are two countries, Home (the US) and Foreign (the ROW). Each country produces its own perishable good. The output of the Home good at time t is denoted by Y_t and that of the Foreign good Y_t^*. Both Y and Y^* can be arbitrary positive stochastic processes.

A home stock market is a claim to the home output Y_t and the foreign a claim to Y_t^*. Assume that the financial markets include a sufficient number of Arrow-Debreu claims so that markets are complete.

Each country is populated by a representative investor with preferences over the Home and Foreign goods given by

$$E\sum_{t=0}^{T}\beta^t\gamma_t\Big[\phi\log C_{H,t}+(1-\phi)\log C_{F,t}\Big],\quad \text{(Home)}$$
$$E\sum_{t=0}^{T}\beta^t\Big[(1-\phi)\log C_{H,t}^*+\phi\log C_{F,t}^*\Big],\quad \text{(Foreign)}$$

where the quantities with an asterisk correspond to the Foreign consumption, $\phi > 0.5$ so as to create a home bias in consumption, $\beta < 1$ is a time discount factor, and $\gamma_t > 0$ denotes a Home demand shifter process. We make a mild technical assumption that γ_t is a martingale, that is, $E_t[\gamma_s] = \gamma_t$, $t \leq s$.[18]

Without the demand shifter γ_t, this is a Cole and Obstfeld (1991) economy, in which the wealth distribution is constant. Shocks to γ_t give rise wealth transfers. We interpret a shock to γ_t as a increase (or decrease) in Home demand for both goods, albeit with a higher increase in the demand for the Home good because of the home bias parameter $\phi > 0.5$.

We assume that the Home demand shifter γ_t is perfectly negatively correlated with the Home output Y_t. This assumption parallels our Assumption 3 in the main text and therefore our demand shifter can be interpreted as a reduced form for the fluctuations in the inverse

[18]The martingale assumption is not necessary, but simplify the calculation of the equilibrium. As we discuss below, in the main model the equivalent of γ_t is the inverse surplus consumption ratio which follows a mean reverting process. We could make a similar assumption here at the cost of more complicated expressions.

surplus consumption ratio in the main text.[19] We continue to interpret states in which the US (Home's) output is low as bad times. Our assumption on the demand shifter γ therefore implies that the US demand increases in bad times, or more precisely, following negative US output shocks.

Let p_t and p_t^* denote the prices of the Home and Foreign goods, respectively. Mirroring the main text, we define the prices of the consumption baskets in Home and Foreign as $P_t = h_H p_t + h_F p_t^*$ and $P_t^* = h_H^* p_t + h_F^* p_t^*$, where the weights reflect a home bias, i.e., $h_H > 0.5$, $h_F^* > 0.5$, $h_F < 0.5$, and $h_H^* < 0.5$. The real exchange rate is then $Q_t = P_t^*/P_t$. Unless specifically stated otherwise, all prices deported below are in terms of the Home consumption basket, the numeraire.

As in the main text, to characterize equilibrium allocation by solving the central planner's problem, i.e.,

$$\max_{C_t, C_t^*} aE \sum_{t=0}^{T} \beta^t \gamma_t \Big[\phi \log C_{H,t} + (1-\phi) \log C_{F,t}\Big] + (1-a)E \sum_{t=0}^{T} \beta^t \Big[(1-\phi) \log C_{H,t}^* + \phi \log C_{F,t}^*\Big],$$

$$s.t \quad C_{H,t} + C_{H,t}^* = Y_t, \quad C_{F,t} + C_{F,t}^* = Y_t^*,$$

where a is Home's Pareto weight. The consumption allocation that solves this problem is

$$C_{H,t} = \frac{a\gamma_t \phi}{a\gamma_t \phi + (1-a)(1-\phi)} Y_t, \qquad C_{F,t} = \frac{a\gamma_t(1-\phi)}{a\gamma_t(1-\phi) + (1-a)\phi} Y_t^*, \tag{92}$$

$$C_{H,t}^* = \frac{(1-a)(1-\phi)}{a\gamma_t \phi + (1-a)(1-\phi)} Y_t, \qquad C_{F,t}^* = \frac{a\phi}{a\gamma_t(1-\phi) + (1-a)\phi} Y_t^*. \tag{93}$$

We then solve for the exchange rate and stock prices prevailing in the decentralized equilibrium. Let W and W^* denote Home and Foreign wealth, respectively, defined as in the main text. The US NFA then continues to satisfy the relationship $NFA_t = W_t - S_t$. As we show below, the signs of the responses of the NFA to the underlying shocks depend on whether Home is a

[19] The structure of the demand shifter in this appendix differs slightly from that of the inverse surplus consumption ratio in the main text. Here, a demand shock shifts Home's demand for both goods, whereas in the main text the "demand" (habit) shock affects demand only for the Home-produced good.

net debtor ($NFA_t < 0$) or a net creditor ($NFA_t > 0$) country.

Define the terms of trade (ToT) as $ToT_t \equiv p_t^*/p_t$. Proposition 14 reports the key equilibrium quantities of interest.

Proposition 14. *(i) The equilibrium real exchange rate, stock prices, and the wealth distribution are given by*

$$ToT_t = \frac{a\gamma_t(1-\phi) + (1-a)\phi}{a\gamma_t\phi + (1-a)(1-\phi)} \frac{Y_t}{Y_t^*}, \tag{94}$$

$$Q_t = \frac{h_H^* + h_F^* ToT_t}{h_H + h_F ToT_t}, \tag{95}$$

$$S_t = \frac{1-\beta^{T-t+1}}{1-\beta} \frac{1}{h_H + h_F ToT_t} Y_t, \quad S_t^* = \frac{1-\beta^{T-t+1}}{1-\beta} \frac{ToT_t}{h_H + h_F ToT_t} Y_t^*, \tag{96}$$

$$NFA_t = \frac{a\gamma_t(1-\phi) - (1-a)(1-\phi)}{a\gamma_t\phi + (1-a)(1-\phi)} S_t, \tag{97}$$

$$\frac{W_t}{W_t + W_t^*} = \frac{a\gamma_t}{a\gamma_t + 1 - a}, \tag{98}$$

(ii) with the following comparative statics:

	Positive shock to:		
	Home demand, $\Delta\gamma_t$	Home output, ΔY_t	Foreign output, ΔY_t^*
Effects on ToT_t	$-$	$+$	$-$
Effects on Q_t	$-$	$+$	$-$
Effects on S_t	$+$	$+$	$+$
Effects on S_t^*	$-$	$+$	$+$
Effects on NFA_t	$+^\dagger$	$+^\dagger$	$+^\dagger$
Effects on $\frac{W_t}{W_t+W_t^*}$	$+$	0	0

†These results hold under the condition that Home is a net creditor at time t, i.e., $NFA_t > 0$. This condition can be expressed in terms of the primitives of the model as $a(1+\gamma_t) - 1 > 0$. For the last two columns, this condition is necessary and sufficient, while for the first column, it is just sufficient.

A proof of Proposition 14 is at the end of this appendix. Figure 11 shows that the simple model we consider here can deliver the same qualitative insights as our model in the main text. We again refer to bad times as times when the Home output is low. As we have assumed earlier in this appendix, adverse output shocks coincide with positive demand shocks at Home, and we take this correlation into account in Figure fig:simplemodel. Just like in the main text, we observe from the figure that in bad times, the real exchange rate of the US appreciates, the NFA drops, the US stock market performs better than its ROW counterpart and the US wealth share rises. The presence of demand shocks is critical for the results. That is, demand shocks in this simple model act similarly to an increase in the inverse surplus consumption ratio in the main text, delivering both the exchange rate appreciation and a rise in the wealth share in bad times.

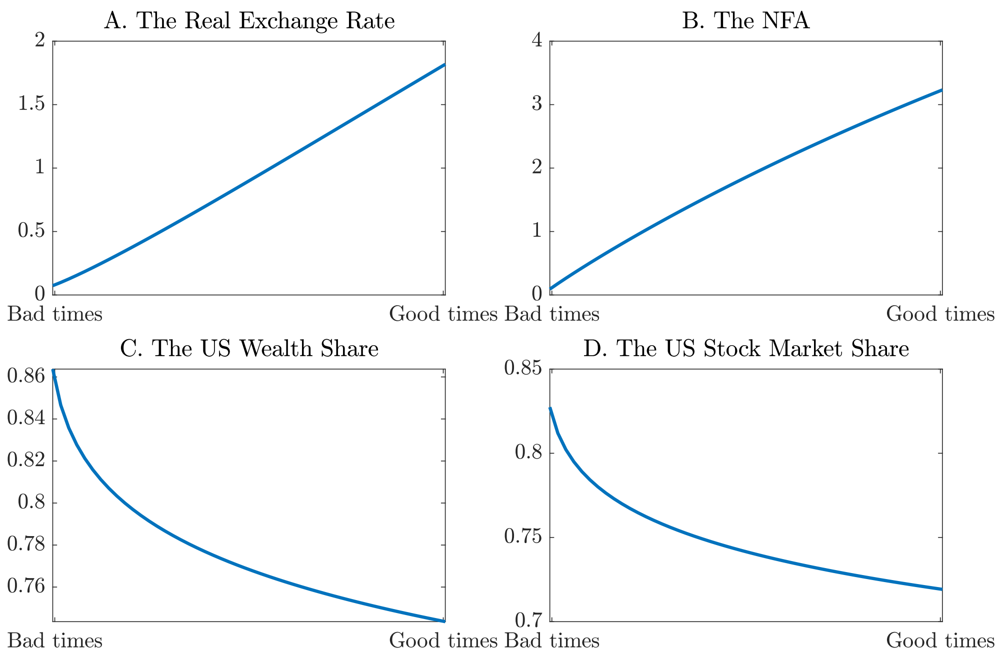


Figure 11: The real exchange rate, net foreign assets, US wealth share, and US stock market share

The figure plots the exchange rate (Panel A), the net foreign assets (Panel B), the US wealth share (Panel C), and the US stock market share (Panel D) as functions of the US output. Bad times corresponds to low levels of US output. We incorporate the perfect negative correlation between US output and demand shocks by assuming that $\gamma_t = \bar{\gamma} Y_t^{-\delta}$, where $\delta = 0.2$. We assume that the distribution of Y_t is such that $Y_t^{-\delta}$ is a martingale. This way of connecting the US output and demand shocks is only one of many ways. All plots are based on the US output ranging between 0.1 and 5 with $Y^* = 1$. The parameter values are $\phi = 0.95$, $a = 0.8$, $h_H = h_F^* = 0.95$, $h_F = h_H^* = 0.05$ and $\beta = 0.95$.)

As Proposition 14 implies, the NFA of Home may rise or fall in bad times, depending on whether Home is a net creditor or a net debtor country. Note from (97), that Home is a net creditor at time t if and only if $a(1 + \gamma_t) - 1 > 0$. The net creditor condition offers a more nuanced understanding of how NFA responds to underlying shocks. We leave this avenue for future work, since the purpose of this exercise is merely to present a very simple model that delivers our key mechanism for a wide range of parameter values.

We close with highlighting an important limitation of the simple model: While its qualitative insights and economic mechanisms are in line with our main model, its ability to match the asset pricing and exchange rate moments is inferior to that of the main model. It is possible to improve the simple model's quantitative performance by introducing additional elements, such as, for example, a mean-reverting component to the demand shifts. We leave this to future research.

Proof of Proposition 14. From Pavlova and Rigobon (2007), we have the following expression for the terms of trade:

$$ToT_t = \frac{a\gamma_t(1-\phi) + (1-a)\phi}{a\gamma_t\phi + (1-a)(1-\phi)} \frac{Y_t}{Y_t^*}.$$

The goods prices are therefore p_t and $p_t^* = \frac{a\gamma_t(1-\phi)+(1-a)\phi}{a\gamma_t\phi+(1-a)(1-\phi)} p_t$. Substituting p and p^* into the Home and Foreign price indices, we have

$$P_t = \left(h_H + h_F \frac{a\gamma_t(1-\phi) + (1-a)\phi}{a\gamma_t\phi + (1-a)(1-\phi)} \frac{Y_t}{Y_t^*} \right) p_t,$$
$$P_t^* = \left(h_H^* + h_F^* \frac{a\gamma_t(1-\phi) + (1-a)\phi}{a\gamma_t\phi + (1-a)(1-\phi)} \frac{Y_t}{Y_t^*} \right) p_t.$$

The real exchange rate is then

$$Q_t = P_t^*/P_t = \frac{h_H^* + h_F^* ToT_t}{h_H + h_F ToT_t}, \tag{99}$$

which we have expressed in terms on the ToT_t for brevity.

From Pavlova and Rigobon (2007), the good-specific stochastic discount factors—the analogues of $M_{i,t}$ in the main text—are given by

$$M_t = \beta^t \frac{a\gamma_t\phi + (1-a)(1-\phi)}{Y_t} \quad \text{and} \quad M_t^* = \beta^t \frac{a\gamma_t(1-\phi) + (1-a)\phi}{Y_t^*} \tag{100}$$

for the Home and Foreign goods, respectively. In terms of the numeraire basket of Home, Home's stochastic discount factor is

$$M_t^H = h_H M_t + h_F M_t^* = h_H \beta^t \frac{a\gamma_t \phi + (1-a)(1-\phi)}{Y_t} + h_F \beta^t \frac{a\gamma_t(1-\phi) + (1-a)\phi}{Y_t^*}. \tag{101}$$

As in the main model, Home's (country 1's in the main model) basket serves as the numeraire and therefore the stochastic discount factor in (101) should be used to discount any cash flow, expressed in units of the numeraire. From equation (28), good prices expressed in terms of the numeraire, p_t and p_t^*, are

$$p_t = \frac{M_t}{M_t^H} = \beta^t \frac{a\gamma_t \phi + (1-a)(1-\phi)}{\left(h_H \beta^t \frac{a\gamma_t\phi + (1-a)(1-\phi)}{Y_t} + h_F \beta^t \frac{a\gamma_t(1-\phi)+(1-a)\phi}{Y_t^*}\right) Y_t} = \frac{1}{h_H + h_F ToT_t}, \tag{102}$$

$$p_t^* = \frac{M_t^*}{M_t^H} = \beta^t \frac{a\gamma_t(1-\phi) + (1-a)\phi}{\left(h_H \beta^t \frac{a\gamma_t\phi + (1-a)(1-\phi)}{Y_t} + h_F \beta^t \frac{a\gamma_t(1-\phi)+(1-a)\phi}{Y_t^*}\right) Y_t^*} = \frac{ToT_t}{h_H + h_F ToT_t}. \tag{103}$$

The price of the Home stock is the present value of its cash flows. Using the discrete-time analog of equation (78), we have the following expression of the Home stock price (in terms of the numeraire):

$$\begin{aligned}
S_t &= E_t \sum_{u=t}^{T} \frac{M_u^H}{M_t^H} p_u Y_u \\
&= \frac{1}{M_t^H} E_t \sum_{u=t}^{T} M_u^H \frac{M_u}{M_u^H} Y_u \\
&= \frac{1}{M_t^H} E_t \sum_{u=t}^{T} \beta^u \left[a\gamma_u \phi + (1-a)(1-\phi)\right] \\
&= \frac{1}{M_t^H} \left[a\gamma_t \phi + (1-a)(1-\phi)\right] \sum_{u=t}^{T} \beta^u \\
&= \frac{1-\beta^{T-t+1}}{1-\beta} \frac{1}{h_H + h_F ToT_t} Y_t,
\end{aligned}$$

where we substituted in M_t and M_t^H from (100) and (101), respectively, and used the property that γ_u is a martingale in the fourth equality. The derivation of S_t^* in (96) is analogous.

We compute Home's wealth, W_t, as the present value of Home's consumption expenditure from time t onwards, that is,

$$\begin{aligned} W_t &= E_t \sum_{u=t}^{T} \frac{M_u^H}{M_t^H}(p_u C_{H,u} + p_u^* C_{F,u}) \\ &= \frac{1}{M_t^H} E_t \sum_{u=t}^{T} \beta^u a \gamma_u \\ &= \frac{1-\beta^{T-t+1}}{1-\beta}\,\beta^t \frac{a\gamma_t}{M_t^H}, \end{aligned}$$

where we have first substituted in the consumption sharing rules from (92)–(93) and then used the property that γ_t is a martingale. Analogously, one can show that Foreign's wealth is given by

$$W_t^* = \frac{1-\beta^{T-t+1}}{1-\beta}\,\beta^t \frac{1-a}{M_t^H},$$

which leads to the wealth share expression (98).

To compute Home's NFA, we first note that our expressions for W_t and S_t imply that

$$W_t = \frac{a\gamma_t}{a\gamma_t\phi + (1-a)(1-\phi)} S_t.$$

Using the expression $NFA_t = W_t - S_t$ for the NFA, derived in the main text, and simplifying, we arrive at (97).

Finally, to prove the comparative statics results, we utilize the closed-form expressions (94)–(98). We take partial derivatives of all quantities of interest with respect to γ_t, Y_t, and Y_t^* and sign the resulting expressions. This is somewhat tedious but straightforward. In determining the signs, we make use of the assumptions we have made on the parameters in the preferences and in the numeraire basket, most notably the assumptions that give rise to a consumption home bias. □