Measuring Unobserved Expected Inflation

By

Rafi Melnick*

The Interdisciplinary Center Herzliya

January 2016

Abstract

The aim of this study is to develop an eclectic but robust model that allows for a better measure of expected inflation and allows testing for all sorts of biases. Improving the measure of expected inflation is of critical importance for conducting monetary policy. In many circumstances, indicators of expected inflation move in opposite directions, and this divergence may be critical for the setting of the interest rate. The model is estimated for a special set of Israeli data via the Kalman filter methodology. We test for systematic biases, for a better normalization of the model, for liquidity problems and for inflation risk—all possibly present in current measures of expected inflation.

JEL: E31, E44, E52, E58.
Keywords: Expected Inflation, Forward-looking Indicators, Measuring Unobservables.

*This paper was motivated by the responsibilities I had as a member of the Monetary Committee of the Bank of Israel. I would like to thank Jacob Boudoukh, Alex Cukierman, Stanley Fischer, Alex Ilek, Roy Stein and Mark Watson for helpful discussions; two anonymous referees for their comments and suggestions; and Roy Stein and Aviel Shpitalnik for help with the data.

1. Introduction

Assessing the rate of expected inflation is of critical importance for the task of designing and undertaking monetary policy. Choosing the required monetary policy with a forward-looking horizon in an uncertain economic future is a constant challenge for the monetary authorities. The purpose of this paper is to offer an econometric approach to obtain an operational measure of expected inflation.

The implementation of a proper forward-looking inflation targeting regime[1], with or without a Taylor type interest rate rule, requires a quantitative measure of expected inflation. This is of critical importance because the adoption of an inflation targeting strategy for the conduct of monetary policy[2] has become a preferred strategy for an increasing number of countries.[3] Therefore, the approach developed here may have broad applications.

Expected inflation is not directly observed; therefore, practically speaking, the execution of monetary policy requires an estimate. We will refer here to the different estimates of expected inflation as indicators: inflation forecasts, expert surveys, expected inflation based on financial markets and others.

Monetary policy decisions based on these indicators could be problematic because they may be subject to different sorts of errors such as:

- Biases in the case of surveys and in the case of forecasts. Cargill and Meyer (1985) find a systematic 10 percent bias in the forecasts based on the Livingston

[1] Inflation targeting is the current approach for monetary policy in Israel.
[2] In a forward-looking inflation targeting regime, the rate of interest is set according to the difference between expected inflation and the inflation target.
[3] In Appendix 1, we present a list of 29 countries, given in Roger (2009), that have adopted IT and the approximate date of adoption.

Index. Laster et al. (1999) discuss the possible rational bias in macroeconomic forecasts. Stock and Watson (2007) explore why the U.S. inflation has become more difficult to forecast, and Frenkel et al. (2013) discuss strategic behavior of professional forecasters.

- Risk and liquidity problems in the case of indicators based on financial markets. Kandel et al. (1996) estimate an inflation risk premium in nominal interest rates. Pflueger and Viceira (2011) find a high liquidity premium, a large average real interest rate risk premium and a smaller inflation risk premium.
- Model dependence and possibly misspecification.
- Measurement errors and pure noise.

In many circumstances, the indicators move in opposite directions. This divergence may be critical for the setting of the interest rate.[4] In real world situations, policy makers need to identify, by alternative methods, the correct level and change of inflationary expectations to avoid monetary policy mistakes.

Our approach requires a number of indicators that encompass expected inflation as a common factor. Our approach utilizes a cross-section of indicators of one-year-ahead expected inflation that move together over time and permit the estimation of a dynamic common factor. This is similar to the approach implemented in Gottlieb et.al (1985), who estimate expected inflation in a multiple indicators setup using wages, interest rate and the velocity of money as indicators of expected inflation in a high inflation period in Israel. Reis and Watson (2010), who specify a common factor model on different

[4] In most cases, the Central Bank sets a short-term interest rate.

components of consumer goods prices in the US to decompose their change to three components, two of relative price changes and one of general changes in all inflation rates.[5] Mertens (2012) estimates trend inflation based on survey expectations, the term structure of interest rates and realized inflation. His model allows an assessment of whether inflation expectations are anchored.

Improving the measurement of expected inflation is of great importance in diminishing monetary policy mistakes. In our study, we develop an eclectic but robust model that provides a better measure of expected inflation and allows for testing all sorts of biases. We implement our model using monthly indicators of expected inflation in Israel from 2000 to 2011, a period of stable and low inflation defined in Cukierman and Melnick (2015) as price stability, and we examine our model in an out-of-sample period from 2012 to 2014.

In the next section, we present the Israeli indicators and their stochastic properties. The basic model is presented in section 3. A simple version of the model is estimated in section 4. Testing for systematic biases and for a better normalization, liquidity problems and inflation risk effects leads to the estimation of the complete model and the derivation of expected inflation. The results are presented and discussed in section 5. In section 6, we measure expected inflation out of the estimation sample of our model to check for robustness. Conclusions are offered in the last section.

3. The Data

[5] They label this component as “pure” inflation. Within the aggregate sources of variation, they estimate that pure inflation accounts for 15 to 20 percent of the variability in PCE inflation.

Our study focuses on a period of low and stable inflation in Israel, from January 2000 until December 2011. During this period, expected inflation fluctuated around 2 percent, the center of the inflation target range.[6] Our data consist of monthly observations of three indicators of expected inflation at a yearly horizon. We therefore estimate expected inflation at this horizon.[7] The indicators are as follows:

π_1 = the breakeven inflation between one-year maturity indexed and unindexed bonds issued by the Israeli government (hereinafter, breakeven inflation, denoted DPE).

π_2 = the mean of the one-year-ahead forecasts of inflation regularly provided to the Bank of Israel by professional forecasters from the financial sector in Israel[8] (hereinafter, inflation forecast, denoted DPF).

π_3 = the mean breakeven inflation for nominal and real interest rates set by private banks in Israel as a benchmark for over-the-counter transactions[9] (hereinafter, banks' breakeven benchmark, denoted DPB)

In Figure 1 and Table 1, we present the indicators clearly displaying the end result of a successful inflation stabilization process that started with a stabilization program in 1985 (see Bruno and Piterman (1988)) and gradually progressed in a step-like function

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[6] Only in 2003, with a delay recognizing the decline of the rate of inflation after a prolonged disinflation process that started in 1985, the inflation target range was set at 1 to 3 percent. In the years 2000, 2001 and 2002, the upper and lower limits of the target were 4 to 3, 3.5 to 2.5 and 3 to 2 percent respectively.

[7] With daily data, the time horizon of DPE and DPB could differ from that of DPF since the last includes the forecast of the current CPI to be published with a delay. In our study we use monthly averages so this problem weakens. In any case, this problem does present a systematic bias since the forecast of the current CPI could be higher or lower than the one included in the market value of the breakeven measures.

[8] The number of forecasters has varied over time.

[9] The data to compute this indicator has been supplied regularly to the Bank of Israel by the commercial banks since January 2008. The indicator is computed at the Statistical division of the Bank of Israel. For the period before 2008, we updated the series using interest rates of commercial banks in Israel on indexed and unindexed credit and deposits with a maturity of up to one year.

described in Liviatan and Melnick (1999), reaching price stability in the year 2000[10]. From Figure 1, we can clearly see that the indicators generally move closely together, but they also diverge in non-trivial ways at several dates. Most of the time, they are within the long-range inflation target range (1 to 3 percent). An important decline in all of the indicators is observed at the end of 2008, immediately after the collapse of Lehman Brothers-a variation of the data that enriches our sample. In Table 1, we see that the mean of the three indicators is near 2 percent with a relatively small standard deviation of between 0.68 and 0.84 percent.

Figure 1. Monthly Indicators of Expected Inflation
(Percent, January 2000 - December 2011)

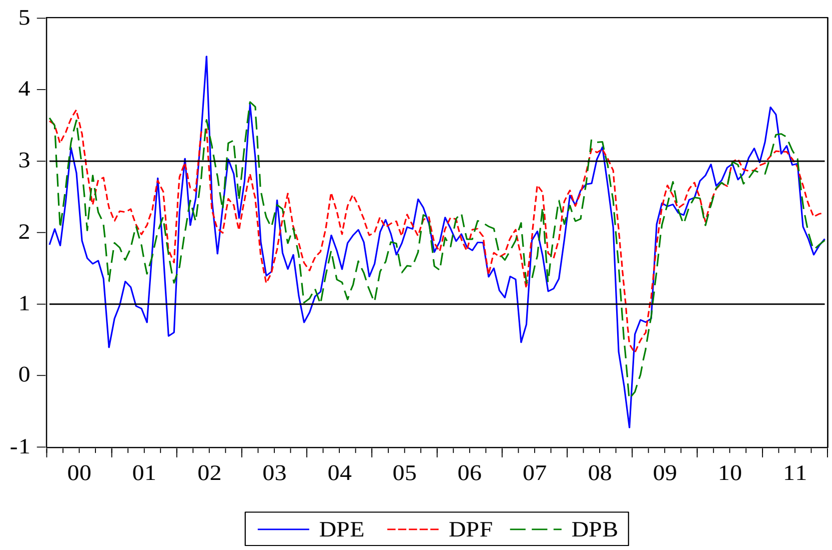

Table 1. Descriptive Statistics
(Percent, 144 observations, January 2000 – December 2011)

	DPE	DPF	DPB
Mean	1.98	2.31	2.11
Median	1.93	2.31	2.08
Maximum	4.46	3.72	3.83
Minimum	-0.73	0.32	-0.32

[10] There are no data for all the indicators before 2000.

Std. Dev.	0.84	0.62	0.77
	Correlation (t statistics)		
	DPE	**DPF**	**DPB**
DPE	1.00 --		
DPF	0.73 (12.9)	1.00 --	
DPB	0.77 (14.3)	0.78 (14.8)	1.00 --

DPE is the breakeven inflation, DPF is the inflation forecast, and DPB is the Banks' breakeven benchmark.

The stochastic properties of the indicators are critical for the proper econometric specification of the model developed in the next section. In Table 2, we test for unit roots in the data. The results show that when the test does not include a constant, the presence of a unit root cannot be rejected for the levels. (Here, the levels are the rates of change.)

However, when the test includes a constant, the unit root hypothesis is rejected in all of the indicators. For the first differences the presence of a unit root is strongly rejected. When testing for cointegration, both the Johansen test and the Engle and Granger test reject the hypothesis of non-cointegration as defined in Engle and Granger (1987).

Table 2. Augmented Dickey-Fuller[11]
Unit Root Tests

Null Hypothesis	Levels	Levels	First Differences

[11] A methodological problem appears when testing for unit roots when the series do not have a trend. When a constant is included, non-rejection of unit roots means the series has a drift, contradicting the no-trend appearance of the data. When testing without a constant, rejection of the unit root means that the series converges to zero, contradicting the obvious non-zero mean of the data.

		With no constant t-Statistic (p value*)	With constant t-Statistic (p value*)	t-Statistic (p value*)
DPE has a unit root		-1.19	-3.68	-12.02
		(0.21)	(0.01)	(0.00)
DPF has unit root		-1.42	-3.85	-11.45
		(0.14)	(0.00)	(0.00)
DPB has unit root		-1.05	-4.26	-10.30
		(0.26)	(0.00)	(0.00)
Test critical values:	1%	-2.58	-3.47	-3.47
	5%	-1.94	-2.88	-2.88
	10%	-1.62	-2.58	-2.57

***MacKinnon (1996) one-sided p-values**

4. The Model

The time series properties of our data could support two types of econometric specifications. In the first type (hereinafter, the NS model), expected inflation is non-stationary, and it is the source of non-stationarity of the indicators (the results with no constant in Table 2). In the second type (hereinafter, the SM model), expected inflation is stationary. In both types, the common stochastic factor is expected inflation, π_t^e. The specification of our model is similar to the single-index dynamic model developed by Stock and Watson (1991) to estimate the coincident indicator for the US economy. In our study, the common unobserved component is expected inflation.[12] Our indicators are directly influenced by expected inflation.[13]

The specification of the model is:

(1) $\pi_t = AX_t + \gamma\pi_t^e + \varepsilon_t$

(2) $\beta(L)\pi_t^e = \eta_t$

[12] In Stock and Watson (1991) the common trend is the unobserved state of the economy. Along the same lines, Melnick and Golan (1993) estimated a coincident indicator for the Israeli economy.
[13] A similar multiple indicators model was applied by Gottlieb et al. (1985) to study inflationary expectations in Israel during the high inflation era.

(3) $\rho(L)\varepsilon_t = u_t$

Where:

π_t is a k × 1 vector of k different indicators of expected inflation in period t.

A is a k × q coefficient matrix.

X_t is a q × k matrix of exogenous or predetermined variables in period t.

γ is a k × 1 vector of parameters.

π_t^e is the unobserved scalar of expected inflation in period t.

ε_t is a k × 1 vector of idiosyncratic stochastic shocks in period t.

B(L) is a scalar lag polynomial and ρ(L) is a lag polynomial matrix.

u_t is a k × 1 vector of Gaussian stochastic disturbances.

Each indicator could be influenced by exogenous or predetermined variables (X). This channel allows testing for omitted variables in the simple specification where the indicators are a function of expected inflation only. This is the methodology channel we use to test for biases in the relation between π_t^e and π_{it}. The idiosyncratic stochastic shocks are uncorrelated at all leads and lags and they are uncorrelated with all of the leads and lags of expected inflation. It is well known that under these assumptions, the model is identified if k>=3.[14] The basic structure of our model is that the correlation between the different indicators is explained by the common factor π_t^e. The stochastic

[14] The exclusion of some variables in X from the different indicators equations can add identification restrictions and testable hypotheses.

properties of the indicators in Stock and Watson (1991) have stochastic trends and they are not cointegrated, dictating a difference model specification.

In our case, we have two options: for the case that the unit root is not rejected and series are not stationary (the NS case), the data dictates a model specified in levels[15], imposing a unit root in the π_t^e process. For the case that the unit root is rejected (the SM case), the series are stationary, and the π_t^e process is modeled and estimated as stationary. In the next section, we first address the NS model and then the SM model.

5. Estimation of a simple model

The model is estimated by maximum likelihood via the Kalman Filter. In the simple model, there are no exogenous (X) variables except for a constant term. The casting of the model in state space formulation is explained in Appendix 2. From equation (1), it is clear that multiplying γ and dividing π_t^e by an arbitrary constant yields identical models; therefore, a normalization is needed to fix the units of measurement of the parameters and the unobserved common factor. This is done by setting $\gamma_1 = 1$. This identification normalization is inconsequential.[16] To determine the length of the polynomial lags $\beta(L)$ and $\rho(L)$, we start with a six-lag specification and omit the statistically insignificant lags. In both types of models, this procedure leads to a first order lag model in the ε process and a second order in the π_t^e process.[17]

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[15] Actually, the levels here are the first differences. i.e. expected inflation is the first difference of the expected price level.

[16] Stock and Watson (1991) normalize the $\mathrm{var}(\eta_t) = 1$. In our case, it is more intuitive to fix the units of measurement of the unobserved expected inflation to the units of measurement of the breakeven inflation.

[17] The lowest value for the Akaike (AIC) information criteria and others, is achieved for a four lags dynamic specification of expected inflation. However, with four lags, the estimated expected inflation

Table 3. Estimated single-index expected inflation, simple NS model
January 2000 – December 2011
(Std. Errors in parenthesis)

	Indicator		
Parameter	**DPE**	**DPF**	**DPB**
$\boldsymbol{a_i}$	-0.03	0.70	0.55
	(1.08)	(0.83)	(0.83)
$\boldsymbol{\gamma_i}$	1.00	0.76	0.76
	(normalization)	(0.05)	(0.08)
$\boldsymbol{\rho_i}$	0.47	0.88	0.68
	(0.12)	(0.05)	(0.08)
$\boldsymbol{\sigma_{ui}}$	0.09	0.02	0.09
	(0.01)	(0.01)	(0.01)

$$\boldsymbol{\pi_t^e = \underset{(0.10)}{1.30}\pi_{t-1}^e - \underset{(0.10)}{0.30}\pi_{t-2}^e + \eta_t \qquad \sigma_\eta = \underset{(0.02)}{0.15}}$$

Log Likelihood = - 141.5

First, we estimate a simple model in which X includes only a vector of ones (a constant). The simple model ignores all other variables (inflation risk, liquidity risk, forecast biases and others) to be included and tested in the next section.[18] The NS model is presented in Table 3, and the SM model is presented in Table 3a.[19] By simple observation of the models, it is clear that the estimated parameters are similar. In the SM model, the constant in the expected inflation process is not significantly different than zero, casting doubt on the stationarity of the process. However, the sum of the

mean is larger than the mean of all the indicators. Its standard deviation is larger than the standard deviation of the two lags specification. The two lags specification produces expected inflation with a mean within the means of the indicators (see table 5). Therefore, our preferred specification is a two lags dynamic structure for expected inflation.

[18] A problem we encounter in the estimation is the flat nature of the likelihood function. We addressed the problem by exploring different initial conditions and assuring the model converges to the same parameter values.

[19] The Eviews program and output for both models is available upon request.

coefficients is statistically smaller than one, indicating a convergence to zero that is clearly contradictory to the data presented in Figure 1.

Table 3a. Estimated single-index expected inflation, simple SM model
January 2000 – December 2011
(Std. Errors in parenthesis)

	Indicator		
Parameter	**DPE**	**DPF**	**DPB**
$\boldsymbol{a_i}$	-0.21	0.59	0.45
	(1.73)	(1.30)	(1.29)
$\boldsymbol{\gamma_i}$	1.00	0.75	0.74
	(normalization)	(0.05)	(0.09)
$\boldsymbol{\rho_i}$	0.51	0.87	0.68
	(0.12)	(0.05)	(0.07)
$\boldsymbol{\sigma_{ui}}$	0.09	0.02	0.09
	(0.01)	(0.01)	(0.02)

$$\boldsymbol{\pi_t^e = \underset{(0.40)}{0.46} + \underset{(0.10)}{1.20}\pi_{t-1}^e - \underset{(0.11)}{0.41}\pi_{t-2}^e + \eta_t \qquad \sigma_\eta = \underset{(0.02)}{0.13}}$$

Log Likelihood = - 130.5

From the simple model, we obtain some preliminary conclusions:

1. The constants (the α_i parameters) are not significantly different than zero. This is an indication that the different variables are not systematically biased indicators of expected inflation. Therefore, we could be led to reject, for example, the following hypotheses: that the breakeven inflation includes a non-zero mean inflation risk premium or a non-zero mean liquidity premium; that the inflation forecast contains a non-zero mean forecast error; or that the banks' breakeven

benchmark is systematically biased, potentially as the breakeven inflation, or is systematically manipulated by the banks.

2. The idiosyncratic shocks reveal strong persistence captured statistically by significant serial correlation (the ρ_i parameters). It is possible that this serial correlation is a result of an inflation/liquidity risk type premium that is time dependent with zero mean (no constant), but its presence could bias the estimation of expected inflation. This is the main issue addressed in the next section.
3. Although adding two additional lags for the π_t^e process marginally improves the different information criteria (Akaike, AIC, Schwartz, SC, and Hannan-Quinn, HQ), the preferred specification for π_t^e is a second order dynamic process because the estimated mean of π_t^e is within the mean of the indicators and it has lower standard deviation.[20] In the NS model, we impose the unit root restriction in the π_t^e process, no constants and the sum of the beta parameters equals 1. In the SM model, the π_t^e is specified as stationary with constant and unconstrained beta parameters.
4. From a methodological perspective, although the presence or the absence of a unit root in the data dictates different models, it seems that in practice, in our case, the choice of specifications does not matter a great deal.

6. Testing and completing the model

The hypotheses in this section are tested by a likelihood ratio Wald test that is $\chi^2_{(n)}$ distributed. The first hypothesis we test, H1, is a restriction of the parameter such that

[20] This outcome may be a result of the relative small sample size.

the average of the expected value of the three indicators equals the mean of expected inflation. Because in the simple model the constants are not significantly different from zero, this restriction implies that expected inflation is unbiased with respect to the indicators.

(4) $E(\pi_1 + \pi_2 + \pi_3)/3 = \pi^e$

Conditioning on $\alpha_i = 0$ for all i, H1 is not rejected for the NS model with $\chi^2_{(1)} = 0.15$, and with $\chi^2_{(1)} = 0.16$ for the SM model. This result leads naturally to a new normalization of the γ coefficient for both models.

(5) $(\gamma_1 + \gamma_2 + \gamma_3)/3 = 1$

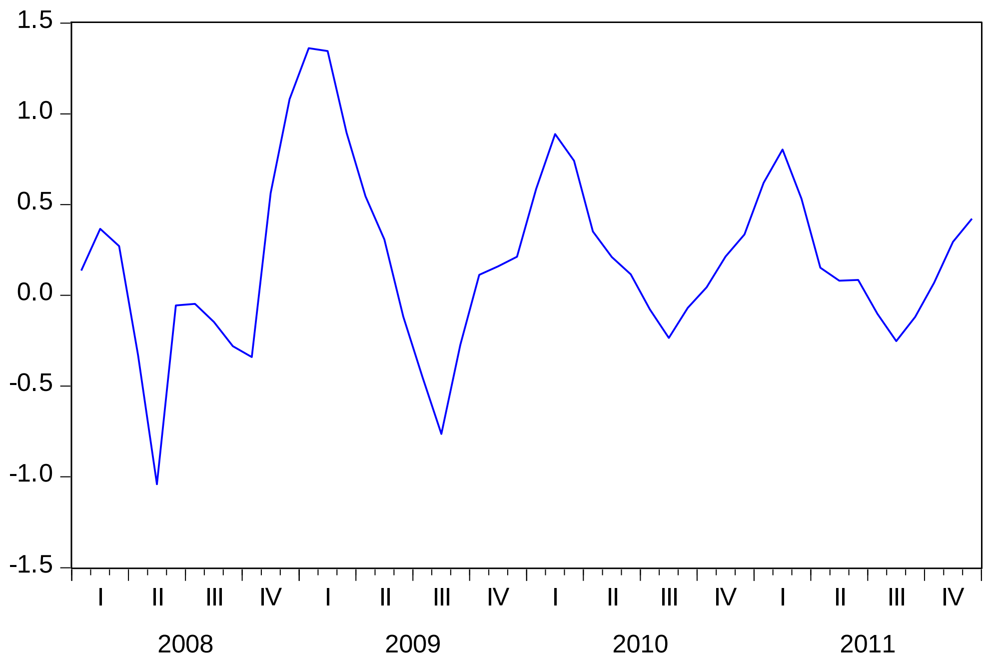
The non-rejection of H1 together with the zero constant illustrates that, on average, the three variables in our model are unbiased indicators of expected inflation. We also tested a stronger restriction hypothesis that $\gamma_1 = \gamma_2 = \gamma_3 = 1$, and it was rejected.[21]

In the second hypothesis, H2, we address a possible bias in the indicators due to liquidity issues. This issue was first raised in Israel by Stein (2012) when he tested for possible deviation of the breakeven inflation and the forecast inflation due to seasonality in the Consumer Price Index (CPI). Although we are dealing with one-year-ahead indicators that should naturally be unaffected by short-term-monthly or quarterly-seasonality, the problem could appear, as shown in Stein (2012), due to discontinuity in the maturity of indexed bonds.

[21] The joint hypothesis, $\alpha i = 0$ and $\gamma_i = 1$ for all i, is also rejected.

Assume that we need a breakeven inflation for the period t+12 but we have only indexed bonds with maturity of t+12+k and t+12-j, and k and j are not zero. If within that horizon there is a strong seasonal CPI[22], the market has to interpolate between the t+12+k and t+12-j returns and evaluate the seasonal effect on the t+12 return. It turns out, as shown in Stein (2012), that there is a significant seasonal/liquidity factor in the breakeven inflation starting in 2008. The estimated seasonal/liquidity factor is presented in Figure 2.

Figure 2: The Sesonal/Lquidity Factor
2008 - 2011



Source: Research Department, Bank of Israel

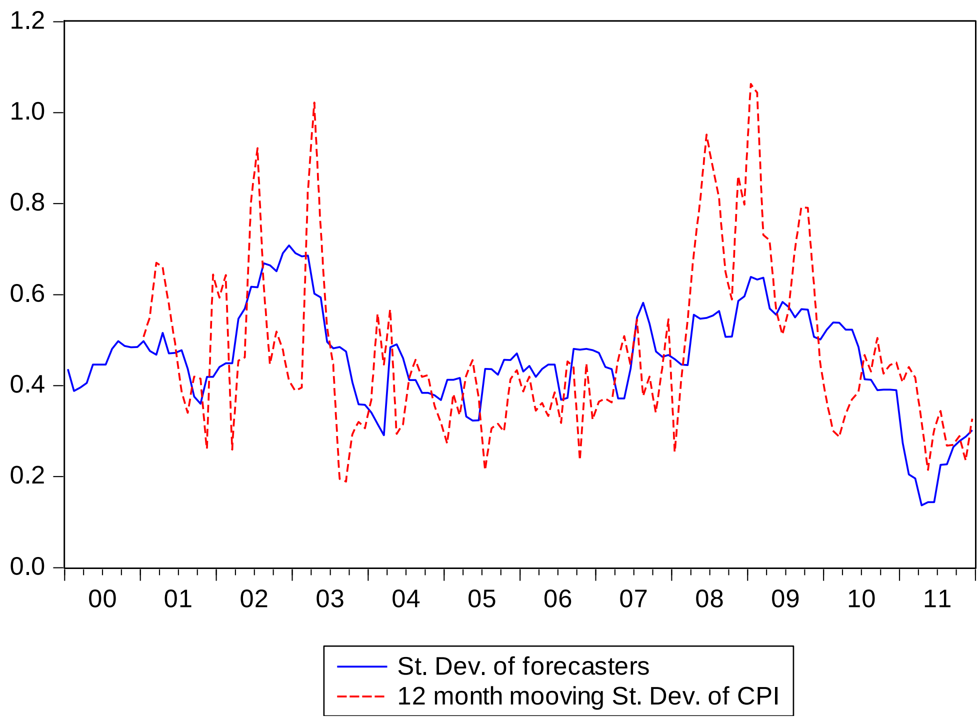
We use the estimated factor as an additional variable in our model. The omission of this factor is not rejected in the inflation forecast or in the banks' breakeven benchmark equations, but it is strongly rejected in both models for breakeven inflation. It seems that for indicators that are not direct market variables, the seasonal/liquidity bias is not significant. However, for the indicator that is a direct market variable, the breakeven

[22] In the Israeli CPI, there is a strong upward seasonal factor in the month of April.

inflation, the seasonal/liquidity bias is highly significant. Taking this bias into consideration helps us to extract a better measure of expected inflation.

In the third hypothesis, H3, we address a possible inflation risk bias. Following Cukierman (1984), we use two proxies for inflation risk to test for this bias. First, we use the standard deviation of the 12-months-ahead inflation forecasts reported regularly to the Bank of Israel by professional financial market forecasters.[23] Second, we employ the 12-month moving standard deviation of the monthly change of the CPI. These two measures of inflation uncertainty are presented in Figure 3.

Figure 3. Proxies of Inflation Risk
(2000 - 2011)



In our model, the first proxy performs better. It seems that although both proxies are highly correlated, the higher variation of the first and the fact that it is a forward-

[23] This variable can be computed from January 2001.

looking measure captures the inflation risk better. Again, omitting the inflation risk variable is not rejected in the inflation forecast and banks' breakeven benchmark equations, but it is rejected in both models for breakeven inflation.[24] As in the seasonal/liquidity factor, the inflation risk bias is present only in the breakeven inflation that is a direct market value.

The complete model is similar to the simple model for the inflation forecast and the banks' breakeven benchmark indicators. The breakeven equation is augmented to include the seasonal/liquidity and the inflation risk variables. The mean of the γ coefficients is restricted to equal 1.

Table 4. Estimated single-index expected inflation complete NS model
January 2000–December 2011
(Std. Errors in parentheses)

	Indicator		
Parameter	**DPE**	**DPF**	**DPB**
$\boldsymbol{a_i}$	-0.80	0.57	0.39
	(2.35)	(1.55)	(1.57)
$\boldsymbol{\gamma_i}$	1.29	0.85	0.86
	(0.06)	(0.04)	(restriction**)
a_1^*	0.72		
	(0.20)		
a_2^*	0.46		
	(0.25)		
$\boldsymbol{\rho_i}$	0.33	0.88	0.69
	(0.17)	(0.05)	(0.06)
$\boldsymbol{\sigma_{ui}}$	0.06	0.02	0.08
	(0.01)	(0.01)	(0.01)

$$\boldsymbol{\pi_t^e = \underset{(0.11)}{1.32}\pi_{t-1}^e - \underset{(0.11)}{0.28}\pi_{t-2}^e + \eta_t \qquad \sigma_\eta = \underset{(0.02)}{0.12}}$$

Log Likelihood = - 128.20

*a_1 is the seasonal/liquidity parameter; a_2 is the inflation risk parameter.
** The mean of the γ's is restricted to equal 1.

[24] The rejection is at the 10% level.

The estimated NS model is presented in Table 4, and the SM model is presented in Table 4a.[25] For the NS case, the simple model is rejected against the complete model with $\chi^2_{(3)}$ = 28.17, and for the SM case, the simple model is rejected against the complete model with $\chi^2_{(3)}$ = 30.5.

As in the simple model, the differences between the estimates of the NS and SM models are negligible. The SM model reaches a lower level for the likelihood function. On the margin, we therefore prefer the SM model.

Table 4a. Estimated single-index expected inflation complete model
January 2000–December 2011
(Std. Errors in parentheses)

	Indicator		
Parameter	**DPE**	**DPF**	**DPB**
$\boldsymbol{a_i}$	-0.90	0.54	0.36
	(3.53)	(2.27)	(2.32)
$\boldsymbol{\gamma_i}$	1.30	0.85	0.85
	(0.06)	(0.05)	(restriction**)
a_1^*	0.77		
	(0.20)		
a_2^*	0.44		
	(0.25)		
$\boldsymbol{\rho_i}$	0.33	0.87	0.70
	(0.19)	(0.05)	(0.07)
$\boldsymbol{\sigma_{ui}}$	0.06	0.02	0.09
	(0.02)	(0.01)	(0.01)

$$\boldsymbol{\pi_t^e = \underset{(0.60)}{0.44} + \underset{(0.10)}{1.22}\pi_{t-1}^e - \underset{(0.10)}{0.44}\pi_{t-2}^e + \eta_t \qquad \sigma_\eta = \underset{(0.01)}{0.10}}$$

Log Likelihood = - 115.92

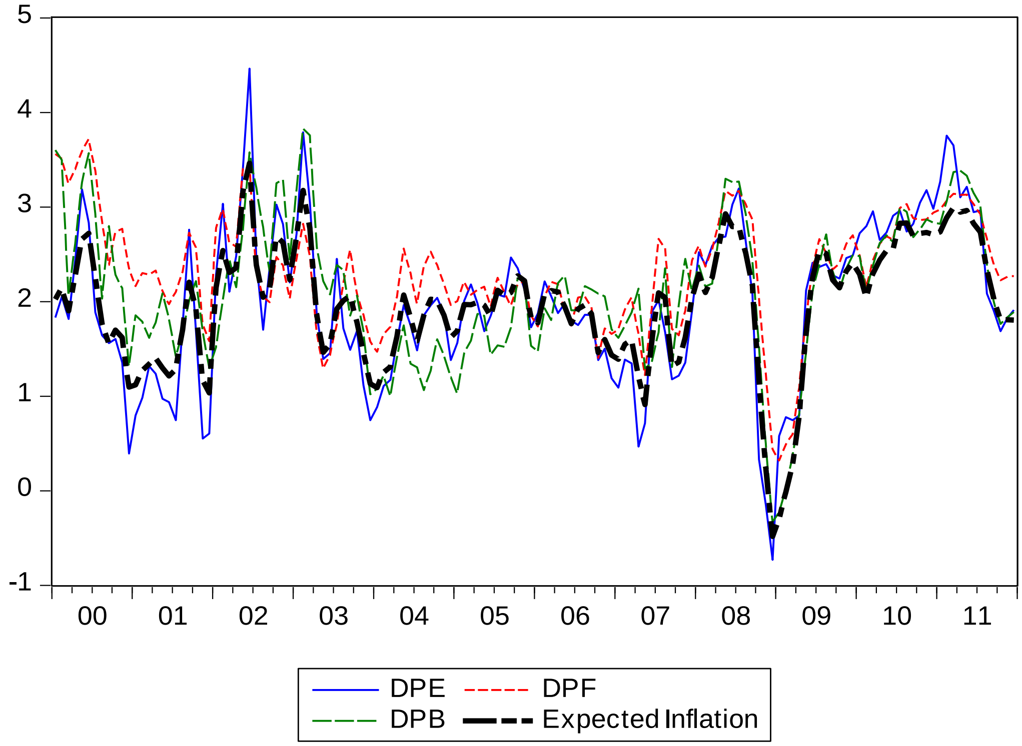
*a_1 is the seasonal/liquidity parameter, a_2 is the inflation risk parameter.
** The mean of the γ's is restricted to equal 1.

The expected inflation extracted by the complete model together with the three indicators is presented in Figure 3. Descriptive statistics of expected inflation and the

[25] The Eviews program and output are available upon request.

indicators are given in Table 5. The expected inflation series (for the SM model[26]) is given in Appendix 3.

Figure 4. Expected Inflation and the Indicators



The complete model confirms the preliminary conclusion of the simple model that the inflation forecast and the breakeven bank benchmark are unbiased indicators of expected inflation. It seems that these indicators, which are not market indicators, do not suffer from biases due to market imperfection, which are hard to take into account.

This is not the case for the breakeven inflation; we cannot reject the hypothesis of two types of biases. The first is the seasonal/liquidity bias. From our estimation, we confirm Stein's (2012) finding that the breakeven inflation includes a statistically significant seasonal/liquidity bias. This bias has a zero mean; therefore, it was not

[26] The expected inflation extracted from the NS model is almost identical.

detected by the simple model. It is serially correlated; therefore, it was captured by the ρ_1 parameter. The second is the inflation risk bias. The complete model allows estimating this bias, which has a statistically significant mean of 21 basis points (t=32). It seems that the parameters in the simple model are downward biased due to the omitted variables, which possibly explains the insignificant estimate of the α_1 (the constant) parameter of the breakeven equation in the simple model. The inflation risk bias is also serially correlated and captured by the ρ_1 parameter. The estimated ρ_1 in the completed model is smaller than the one estimated in the simple model. Practically speaking, our result indicates that expected inflation derived by the breakeven inflation is upward biased by 21 basis points on average.

Table 5. Descriptive Statistics of Expected Inflation
(Percent, 144 observations, January 2000 – December 2010)

	DPE	**DPF**	**DPB**	**Expected Inflation**
Mean	1.98	2.31	2.11	2.01
Median	1.94	2.31	2.08	2.06
Maximum	4.46	3.72	3.83	3.50
Minimum	-0.73	0.32	-0.32	-0.41
Std. Dev.	0.84	0.62	0.77	0.65

Correlation
(t statistics)

	DPE	**DPF**	**DPB**	**Expected Inflation**
DPE	1.00			
	--			
DPF	0.73	1.00		
	(12.88)	--		
DPB	0.77	0.78	1.00	
	(14.20)	(14.72)	--	
Expected Inflation	0.93	0.84	0.84	1.00
	(30.94)	(18.32)	(18.71)	--

The descriptive statistics and correlations of the indicators are repeated here for comparison with those of expected inflation. DPE is the breakeven inflation, DPF is the inflation forecast and DPB is the banks' breakeven benchmark.

7. Out-of-sample calculation of expected inflation

In Table 6, we present descriptive statistics of the out-of-sample forecast of expected inflation for the period from January 2012 to December 2014. The forecast shows that the parameters of the model capture the out-of-sample period reasonably well. An interesting result here is the decline in the correlation of DPE with the estimated expected inflation. This is explained by the severe seasonal/liquidity problem during this period, when the discontinuity in the one-year-ahead maturity of indexed bonds became an acute problem for this indicator. Fortunately, the seasonal variable included in the model addresses this issue correctly.

Table 6. Descriptive Statistics of Expected Inflation
(Percent, 36 0ut-of-sample observations, January 2012 – December 2014)

	DPE	DPF	DPB	Expected Inflation
Mean	1.79	1.80	1.54	1.80
Median	1.77	1.84	1.64	1.86
Maximum	2.84	2.56	2.50	2.53
Minimum	0.35	0.71	0.36	0.75
Std. Dev.	0.62	0.47	0.54	0.41

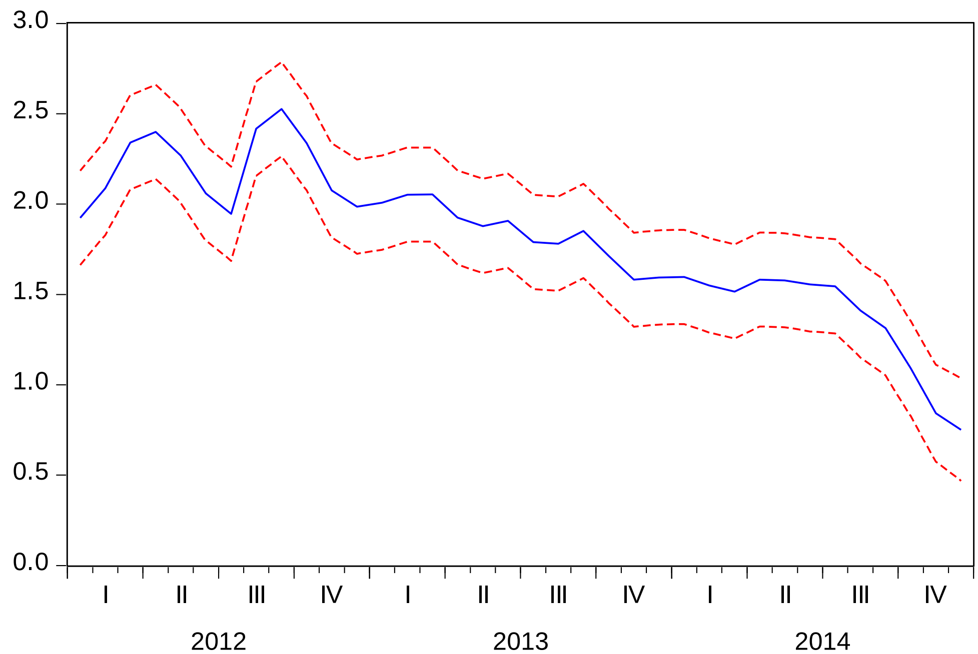
Correlation
(t statistics)

	DPE	DPF	DPB	Expected Inflation
DPE	1.00			
	--			
DPF	0.85	1.00		
	(9.55)	--		
DPB	0.88	0.97	1.00	
	(10.75)	(24.55)	--	
Expected Inflation	0.89	0.96	0.96	1.00
	(11.36)	(19.93)	(20.30)	--

DPE is the breakeven inflation, DPF is the inflation forecast and DPB is the banks' breakeven benchmark.

In Figure 5, we present the out-of-sample calculation of expected inflation and a 95 percent confidence interval, using the complete model that was estimated from January 2000 to December 2011. The forecast shows that the parameters of the model capture the out-of-sample period reasonably well. Expected inflation seems to be well anchored at approximately 2 percent up to the end of 2013, the center of the inflation targeting range set by the government. After that, a clear decline in expected inflation is estimated. This is consistent with the slowdown in economic activity, the decline in oil and commodity prices and the drop in world inflation, especially in Europe.[27] At the end of our forecast, expected inflation approaches the lower limit of inflation target range.

Figure 5. Expected Inflation - Out of Estimation Sample Forecast
January 2012 - December 2014



In Table 7, we compare the mean and standard deviation of actual and expected inflation and test the hypothesis that the mean of unexpected inflation is equal to zero.[28]

Table 7. Expected and Actual Inflation
(Annual rate, Percent)

[27] Most imports to Israel originate in Europe.
[28] Unexpected inflation is computed as the difference between 12-month-ahead expected inflation and the actual inflation for the same period.

	Estimation Sample Jan 00–Dec 11		test[1]	Forecast Sample Jan 12–Dec 14		test[1]
	Inflation	Expected Inflation		Inflation	Expected Inflation	
Mean	2.23	2.01	0.22 (0.20)	0.45	1.80	-1.35 (0.12)
Std. Dev.	2.14	0.65		1.01	0.41	
Observations	144	144		36	36	

[1]Test for the equality of the mean, standard error in parenthesis.

In the estimation sample, January 2000 to December 2011, we cannot reject the hypothesis that mean unexpected inflation is zero and that both mean inflation and mean expected inflation are different from 2 percent, the center of the inflation targeting range. However, it is clear that the fluctuations of inflation are much larger than those of expected inflation, with standard deviations of 2.14 and 0.65 percent respectively. In the forecast sample, January 2012 to December 2014, we reject the equal mean hypothesis. It seems that the sharp decline in world inflation due mainly to the decline in world oil and commodity prices had a large impact on inflation in Israel. It seems that this decline was interpreted, by the general public, as a temporary deviation of inflation from the target, therefore expected inflation did not decline much and it remained within the inflation target range. The decline in the standard deviation of inflation seems to be related to the decline in the path-through coefficient from changes in the exchange rate to prices in Israel; the decline in the standard deviation of expected inflation seems to be related to the process of improvement of anchoring of expected inflation in Israel; both of which are documented in Cukierman and Melnick (2015). At the end of our forecast sample, there are signs that anchoring of expected inflation is declining as expected inflation approaches the lower limit of the inflation target range.

8. Conclusion

Our study offers and implements an eclectic but robust time series model that provides a new way to measure expected inflation while obtaining better results and allowing testing for the presence of biases. This is done utilizing contemporaneous indicators of expected inflation and extracting the common dynamic factor via the Kalman filter methodology.

Testing the hypothesis that expected inflation is unbiased with respect to the indicators led to a new normalization of the γ coefficient, which helped to set the units of measurement of expected inflation to its natural units – percent – similar to the units of measurement of the indicators.

We find that the indicators that are not direct market indicators – the inflation forecast and the banks' breakeven benchmark – do not suffer from the seasonal/liquidity bias and the inflation risk bias. But the breakeven inflation derived from financial market contains a seasonal/liquidity bias and an inflation risk bias. The seasonal/liquidity bias has a zero mean, as expected, but the inflation risk bias contains a non-zero mean estimated at 21 basis points. Both biases are strongly serially correlated.

The out-of-sample calculation of expected inflation for the period January 2012 – December 2014 indicates that the parameters of the model capture expected inflation reasonably well out of the estimating sample. Expected inflation seems to be anchored within the inflation targeting range set by the government, especially after 2003,[29] and declining towards the end of our sample. At the end of 2014 the decline in expected inflation may indicate a deterioration of expected inflation anchoring.

[29] This supports the finding of improved expected inflation anchoring found in Cukierman and Melnick (2015)

Our study demonstrates that since the year 2000 expected inflation in Israel has been well anchored close to the center of the inflation target range, but it has also deviated temporarily from the target range and returned to it. We estimated two specifications, the NS (not stationary case) and the SM (the stationary case). It is well known that in the NS model, expected inflation does not have a mean reversal property since the series is not stationary. One possible explanation of the mean reversal type behavior of expected inflation in this period is the successful inflation targeting monetary policy implemented by the Bank of Israel. A word of caution is needed here since nothing lasts forever, and in a non-stationary environment, a shock to expected inflation can take actual inflation away from the target range. On the other hand, if we stay with the SM model, as recommended here, the mean reversal property of expected inflation can be interpreted as a strong level of public trust that deviations of inflation from the target range will be properly dealt with by the Bank of Israel.

References

Bruno, M. and S. Piterman, (1988), “Israel’s Stabilization: A Two Years’ Review” in M. Bruno, G. Di Tella, R. Dornbusch and S. Fischer, eds.’ *Inflation Stabilization: The Experience of Israel, Argentina, Brazil, Bolivia and Mexico.* MIT press.

Cargill, F. T. and R. A. Meyer, (1985), “Forecast Evaluation of Livingston Data on Inflation Rates and Inflation Uncertainty”, *Decision Sciences,* 16, 161-176.

Cukierman, A, (1984), *Inflation, Stagflation, Relative Prices, and Imperfect Information,* Cambridge University Press.

Cukierman, A., and R. Melnick (2015). "The Conquest of Israeli Inflation and Current Policy Dilemmas", in Offenbacher, A. (Ed.) Maintaining Price Stability: The Bank of Israel's Sixth Decade. Bank of Israel, Jerusalem (13-62).

Engle, R. F. and C.W.J. Granger, (1987), “Cointegration and Error Correction: Representation, Estimation and Testing”, *Econometrica,* 55 251-76.

Frenkel, M., J. C. Rülke and L. Zimmermann, (2013), “Do Private Forecasters Chase After IMF or OECD Forecasts?”, *Journal of Macroeconomics,* In Press.

Gottlieb, D., R. Melnick and S. Piterman, (1985), "Inflationary Expectations in Israel: A Multiple Indicators Approach", *Journal of Business and Economic Statistics,* 3, 112-117.

Hamilton D. J. (1994), *Time Series Analysis*, Princeton New Jersey: Princeton University Press

Kandel, S., A. R. Ofer and O. Sarig, (1996), “Real Interest Rates and Inflation: Ex-Ante Empirical Analysis”, *The Journal of Finance”,* 51, 205-225.

Laster, D., P. Bennett and I. S. Geoum, (1999), “Rational Bias in Macroeconomic Forecasts”, *The Quarterly Journal of Economics,* Vol. 113, No. 1, 293-318.

Liviatan, N. and R. Melnick, (1999), “Inflation and Disinflation by Steps in Israel”, in L. Leiderman ed. *Inflation and Disinflation in Israel,* Research Department, Bank of Israel.

Melnick R. and Y. Golan, (1993), "Measurement of Business Fluctuations in Israel", *Bank of Israel Economic Review*, 67, 1-20.

Mertens, E. (2011), “Measuring the Level and Uncertainty of Trend Inflation”, Finance and Economics Discussion Series 2011-42, Federal Reserve Board, Washington D.C.

Pflueger, C. E., and L. M. Viceira, (2011), “Return Predictability in the Treasury Market: Real Rates, Inflation, and Liquidity”, NBER Working Paper No. 16892

Reis, R. and M. Watson, (2010), “Relative Goods’ Prices, Pure Inflation and The Phillips Correlation”, *American Economic Journal: Macroeconomics,* 2, 128-157.

Roger, S. (2009), “Inflation Targeting at 20: Achievements and Challenges”, IMF Working Paper WP/09/236.

Stein, R. (2012), “The Effect of Seasonality in the CPI on Inflation Expectations”, Research Department, Bank of Israel. Discussion Paper 2012.06 (Hebrew).

Stock, J. H. and M. W. Watson (1991), “A Probability Model of the Coincident Economic Indicator” in Kajal Lahiri and Geoffrey H. Moore, eds., *Leading Economic Indicators: New Approaches and Forecasting Records.* Cambridge, England: Cambridge University Press.

Stock, J. H. and M. W. Watson (2007), "Why Has U.S. Inflation Become Harder to Forecast?", *Journal of Money, Credit and Banking,* Supplement to Vol. 39, N0. 1.

Appendix 1. Countries and approximate adoption dates of inflation targeting

Table A1.1 Approximate adoption dates of inflation targeting

Country	**Date**	**Country**	**Date**
New Zealand	1990 q1	Korea	2001 m1
Canada	1991 m2	Mexico	2001 m1
United Kingdom	1992 m10	Iceland	2001 m3
Sweden	1993 m1	Norway	2001 m3
Finland	1993 m2	Hungary	2001 m6
Australia	1993 m4	Peru	2002 m1
Spain	1995 m1	Philippines	2002 m1
Israel	1997 m6	Guatemala	2005 m1
Czech Republic	1997 m12	Slovakia	2005 m1
Poland	1998 m10	Indonesia	2005 m7
Brazil	1999 m6	Romania	2005 m8
Chile	1999 m9	Turkey	2006 m1
Colombia	1999 m9	Serbia	2006 m9
South Africa	2000 m2	Ghana	2007 m5
Thailand	2000 m5		

Source: Table 2.1: Roger (2009)

Appendix 2. Casting the model in State-Space formulation[30]

Using the notation given in Hamilton (1994) the state-space representation of a dynamic system is given by:

The State equation

(A.1) $\xi_{t+1} = F\xi_t + v_{t+1}$

The Observation equation

(A.2) $y_t = A'x_t + H'\xi_t + w_t$

In our model:

(A.3) $$\xi_t = \begin{bmatrix} \pi_t^e \\ \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}$$

(A.4) $$F = \begin{bmatrix} \beta & 0 & 0 & 0 \\ 0 & \rho_1 & 0 & 0 \\ 0 & 0 & \rho_2 & 0 \\ 0 & 0 & 0 & \rho_3 \end{bmatrix}$$

[30] We are closely following Stock and Watson (1991)

$$(A.5)\quad v_t = \begin{bmatrix} \eta_t \\ u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix}$$

$$(A.6)\quad y_t = \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \\ \pi_{3t} \end{bmatrix}$$

$$(A.7)\quad H = \begin{bmatrix} \gamma_1 & 1 & 0 & 0 \\ \gamma_2 & 0 & 1 & 0 \\ \gamma_3 & 0 & 0 & 1 \end{bmatrix}$$

A includes the constant and possibly other coefficients related to additional exogenous explanatory variables. x_t is a column of ones and possibly other additional exogenous explanatory variables correspondingly and $w_t = 0$. If the model contains richer dynamic processes in π_t^e or ϵ_{it} the model can be augmented correspondingly as is done in Stock and Watson (1991).

Appendix 3. Expected Inflation January 2000 – December 2011

Jan-00	2.041	Jan-03	2.726	Jan-06	1.827	Jan-09	-0.229
Feb-00	2.163	Feb-03	3.226	Feb-06	2.121	Feb-09	0.065
Mar-00	1.941	Mar-03	2.813	Mar-06	2.180	Mar-09	0.346
Apr-00	2.312	Apr-03	1.890	Apr-06	2.161	Apr-09	0.878
May-00	2.719	May-03	1.486	May-06	2.004	May-09	1.797
Jun-00	2.784	Jun-03	1.581	Jun-06	1.825	Jun-09	2.350
Jul-00	2.320	Jul-03	1.977	Jul-06	1.986	Jul-09	2.615
Aug-00	1.817	Aug-03	2.086	Aug-06	2.032	Aug-09	2.569
Sep-00	1.612	Sep-03	2.140	Sep-06	1.938	Sep-09	2.262
Oct-00	1.763	Oct-03	1.859	Oct-06	1.529	Oct-09	2.160
Nov-00	1.688	Nov-03	1.532	Nov-06	1.660	Nov-09	2.328
Dec-00	1.167	Dec-03	1.197	Dec-06	1.505	Dec-09	2.421
Jan-01	1.186	Jan-04	1.154	Jan-07	1.465	Jan-10	2.283
Feb-01	1.347	Feb-04	1.315	Feb-07	1.627	Feb-10	2.024
Mar-01	1.418	Mar-04	1.382	Mar-07	1.669	Mar-10	2.268
Apr-01	1.481	Apr-04	1.709	Apr-07	1.276	Apr-10	2.440
May-01	1.370	May-04	2.148	May-07	0.974	May-10	2.547
Jun-01	1.280	Jun-04	1.909	Jun-07	1.646	Jun-10	2.584
Jul-01	1.355	Jul-04	1.652	Jul-07	2.177	Jul-10	2.865
Aug-01	1.784	Aug-04	1.925	Aug-07	2.118	Aug-10	2.871
Sep-01	2.288	Sep-04	2.095	Sep-07	1.380	Sep-10	2.732
Oct-01	2.006	Oct-04	2.080	Oct-07	1.406	Oct-10	2.744
Nov-01	1.217	Nov-04	1.917	Nov-07	1.668	Nov-10	2.750
Dec-01	1.074	Dec-04	1.681	Dec-07	2.081	Dec-10	2.729
Jan-02	2.194	Jan-05	1.750	Jan-08	2.330	Jan-11	2.759
Feb-02	2.593	Feb-05	2.031	Feb-08	2.117	Feb-11	2.911
Mar-02	2.350	Mar-05	2.031	Mar-08	2.285	Mar-11	3.007
Apr-02	2.401	Apr-05	2.053	Apr-08	2.631	Apr-11	2.969
May-02	3.216	May-05	2.013	May-08	2.992	May-11	2.999
Jun-02	3.503	Jun-05	1.905	Jun-08	2.887	Jun-11	2.862
Jul-02	2.411	Jul-05	2.176	Jul-08	2.917	Jul-11	2.773
Aug-02	2.051	Aug-05	2.123	Aug-08	2.656	Aug-11	2.388
Sep-02	2.149	Sep-05	2.155	Sep-08	2.282	Sep-11	2.074
Oct-02	2.708	Oct-05	2.332	Oct-08	1.238	Oct-11	1.851
Nov-02	2.652	Nov-05	2.272	Nov-08	0.313	Nov-11	1.847
Dec-02	2.263	Dec-05	1.885	Dec-08	-0.408	Dec-11	1.853