The Voting Premium[*]

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Abstract

This paper develops a unified theory of blockholder governance and the voting premium. It explains how and why a voting premium emerges in the absence of takeovers and controlling shareholders. The model features a minority blockholder and dispersed shareholders, who trade shares in a competitive market, and those who own shares after trading vote on a proposal. A voting premium can emerge from the blockholder's desire to influence who exercises control, rather than from exercising control himself. The model shows that empirical measures of the voting premium generally do not reflect the value of voting rights, and that the voting premium can be negligible even when the allocation of voting rights is important. Moreover, the model can explain a negative voting premium, which has been documented in several studies. It arises because of free-riding by dispersed shareholders on the blockholder's trades, which increases the price impact of trading voting shares and makes them less liquid than non-voting shares. The model also has novel implications for the relationship between the voting premium and the severity of conflicts of interest between shareholders, the price of a separately traded vote, and competition for control among blockholders.

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1 Introduction

Voting is a central mechanism of corporate governance. It empowers shareholders of publicly traded companies to elect directors, approve major corporate transactions, and decide on governance, social, and environmental policies. Most corporations have blockholders who are large enough to influence voting outcomes (La Porta et al. (1999); McCahery, Sautner, and Starks (2016); Edmans and Holderness (2017); Dasgupta, Fos, and Sautner (2020)). Blockholders' desire to accumulate voting power and exert corporate influence can affect stock prices and give rise to a voting premium.

The asset pricing implications of control rights have been studied extensively. The theoretical literature followed Grossman and Hart (1988) and Harris and Raviv (1988), and attributed the voting premium almost exclusively to control contests and takeovers. This is puzzling in light of a large empirical literature in this area. First, the most common measure of the voting premium is arguably the dual-class premium, which appears to be largest in economies in which control contests are rare, and which does not disappear when regulation requires equal treatment of non-voting shares in takeovers.[1] Second, while all studies find the voting premium to be positive on *average*, many studies document negative voting premiums for some firms, which is difficult to explain in a model with bidding contests. Third, studies that construct non-voting shares synthetically to estimate the voting premium find that it is largest around shareholder meetings compared to other periods of the year (e.g., Kalay, Karakas, and Pant (2014)), which highlights the importance of voting on proposals for the existence of a voting premium. Last, a more recent empirical literature estimates the voting premium from fees in equity lending markets or price changes around record dates. These studies create a new puzzle, because they usually find negligible values for voting rights, which appears to be in conflict with the earlier literature.

Overall, these gaps and conflicting conclusions suggest that the theoretical underpinnings of the voting premium are still incomplete. To address these challenges, this paper develops a unified theory of blockholder governance and the voting premium. We study how and why a voting premium emerges in the absence of takeovers or controlling shareholders, which is

[1] See our extensive discussion of the empirical literature in Section 8 and in the Appendix, which shows large dual-class premiums for France, Israel, and Italy, among others, whereas the lowest dual-class premiums are found in the US.

arguably the empirically most relevant setting.

We analyze a model with a continuum of atomistic dispersed shareholders and one minority blockholder. The baseline model features one-share-one-vote. Shareholders first trade with each other in a competitive stock market. Those who own shares after trading then vote on a proposal at a shareholder meeting. Shareholders observe a public signal about the quality of the proposal before they cast a vote, and the proposal is approved if enough votes are cast in favor. Shareholders differ in their preferences for the proposal: While some shareholders are skeptical and need a lot of evidence to vote in favor, others are more disposed toward the proposal and generally support it. Such heterogeneity may arise from differences in investment horizons; tax status; ownership of other firms; attitudes toward risk, corporate governance philosophies, and social and political ideologies.[2]

In our framework, the voting outcomes, the composition of the shareholder base, and asset prices are all endogenous. In equilibrium, the proposal is approved if and only if the public signal about its quality exceeds a certain cutoff. Shareholders are heterogeneous, so the proposal is accepted too often from the point of view of some, and rejected too often from the perspective of others. We call the shareholder who fully agrees with the decision rule implied by the cutoff the "median voter;" the median voter's identity completely characterizes the expected voting outcome. Importantly, the median voter can be either a dispersed shareholder or a blockholder, and his identity is determined by the composition of the shareholder base after trading. Hereafter, the term "median voter" is used interchangeably with the expected voting outcome.

The blockholder and dispersed shareholders trade in anticipation of the expected voting outcome and its impact on their valuations; shareholders' valuations could differ because of heterogeneous preferences. Trading reallocates cash flow rights and voting rights across shareholders, since shares are bundles of both. Price-taking dispersed shareholders trade only for cash flow reasons, i.e., if the share price differs from their private valuations. By contrast, the blockholder can be pivotal for the voting outcome, so he may also purchase shares to influence

[2]See the following literature on each of these issues: Investor time horizons: Bushee (1998) and Gaspar, Massa, and Matos (2005); tax status: Desai and Jin (2011); conflicts of interest and common ownership: Cvijanovic, Dasgupta, and Zachariadis (2016) and He, Huang, and Zhao (2019); attitudes to corporate governance and social and political ideologies: Bolton et al. (2020) and Bubb and Catan (2021). Hayden and Bodie (2008) provide a comprehensive overview of different sources of shareholder heterogeneity.

the voting outcome, that is, to push the median voter in his preferred direction.

The equilibrium share price can be decomposed into two terms. The first term captures the market clearing price in the hypothetical scenario in which all shareholders anticipate exactly the same decision rule as the one that actually arises, but take it as exogenously given. This price would emerge if trading of shares did not reallocate voting rights across shareholders, e.g., if trade happened after the record date or if shares did not contain voting rights. The second term is the additional component in the stock price that arises exactly because the trading of shares reallocates the voting rights across shareholders, moves the median voter, and thereby changes the value of the shares for small shareholders. We call this term the *voting premium* and show that it reflects the blockholder's equilibrium net marginal payoff from buying one additional voting right.

Our theoretical definition of the voting premium has two appealing empirical counterparts. First, our definition of the voting premium captures the dual-class premium: In an extension to a dual-class setting, the price differential between voting and non-voting shares reflects the blockholder's net marginal payoff from buying an additional voting right. Second, the voting premium can be considered as the difference between the pre-record date and the post-record date share price. The expected voting outcome is the same at both moments in time, but after the record date, trading no longer reallocates voting rights for decisions taken at the upcoming meeting. However, our definition of the voting premium is more general and extends to single-class firms and to dates other than the record date.

We show that a positive voting premium can arise in equilibrium even though there are no takeovers in our model and the blockholder does not obtain a controlling stake. Intuitively, as the blockholder buys more shares, he moves the median voter, who becomes more similar to the blockholder. If the blockholder accumulates enough shares, he even becomes the median voter himself. However, since voting rights are not traded separately from cash flow rights, this accumulation of voting power requires dispersed shareholders who like the expected voting outcome to sell more of their shares. Their heterogeneous valuations create an upward-sloping supply function to the blockholder, which results in price impact.

The voting premium equals the blockholder's benefit of purchasing one additional voting right, net of the price impact of this additional purchase on his entire trade. If his price impact is significant, the blockholder optimally limits his accumulation of shares and, hence, his voting power; then the voting premium is positive. Conversely, if price impact is moderate, the blockholder will buy sufficiently many shares to become the median voter. Then, any further purchases would leave the voting outcome unchanged and the voting premium is zero. Therefore, our model can explain empirical studies that document a negligible voting premium (e.g., Christoffersen et al. (2007) and Fos and Holderness (2020)).

The case of a zero voting premium illustrates the general principle that the voting premium does not reflect the economic value of voting rights, because it captures only the blockholder's *marginal* value from an additional vote, net of his price impact and evaluated at his optimal ownership level. However, the blockholder's total value of voting rights reflects his average willingess to buy votes, which includes all the infra-marginal trades from his initial endowment to his equilibrium ownership. In this respect, the voting premium underestimates the importance of voting rights. This observation is important for interpreting empirical findings, since some proxies for the voting premium measure the marginal value of a vote (dual-class share premium; price drop on record days), whereas others are more related to the average value of voting rights (block premium).

For the same reasons, the voting premium is not a good measure of voting power. Voting power is related to the blockholder's likelihood to be pivotal and swing the voting outcome. Since an increase in the blockholder's voting power decreases his distance from the median voter, and thus his valuation of a marginal vote, the magnitude of the voting premium is generally unrelated to the blockholder's voting power. The relationship between voting power and the voting premium can even be negative when the blockholder becomes the median voter himself: Then his voting power is large and the voting premium is zero. This discussion underscores that the voting premium does not emerge from exercising control, but rather from influencing who exercises control.

Our model can also rationalize a negative voting premium, which has been documented in several studies. A negative voting premium implies that the blockholder limits his purchases of voting shares in order to commit to a reduced influence on the voting outcome. Such a strategy may appear puzzling because the benefit of a marginal vote to the blockholder is always positive, since the value of his endowment always increases if he moves the median voter toward himself. However, if the blockholder has a small endowment, his main focus is on his trading profits, which can become negative if his price impact is sufficiently large. Consequently, the blockholder buys fewer shares compared to a scenario in which shares do not have voting rights, and the voting premium becomes negative. To see how this can happen, consider a scenario in which the blockholder favors acceptance of a certain proposal, e.g., adoption of an environmentally friendly production technology, but dispersed shareholders are on average more environmentalist and value this voting outcome even more than the blockholder himself. This increases the price at which they supply shares: they free ride on the blockholder's trades. Then the blockholder's price impact can be so large that further purchases would increase the stock price even more than his own valuation, giving rise to a negative voting premium.

The discussion above reflects the more general insight that the voting rights embedded in the shares can either amplify or attenuate the price impact of trades. If the blockholder's trades move the median voter in the direction preferred by dispersed shareholders, they increase the price at which they supply their shares to the blockholder. Then his price impact is amplified compared to a scenario in which shares do not have voting rights. However, if the blockholder is in conflict with dispersed shareholders, then his trades push the median voter away from their desired point and thus reduce the price at which they are willing to sell, attenuating price impact. Overall, this argument implies that liquidity, if measured by price impact, is endogenous in our setting and generally differs between voting and non-voting shares. Thus, we can also view the negative voting premium through the lens of endogenous liquidity, and our explanation for the negative voting premium is broadly consistent with the empirical literature. This literature sometimes attributes the negative voting premium to the lower liquidity of superior voting shares. However, the differential liquidity of voting and non-voting shares in our model arises endogenously from the impact of the blockholder's trades on dispersed shareholders' valuations.

The literature often associates the voting premium with conflicts of interest and sometimes uses it as a measure for private benefits of control. Hence, we investigate how the voting premium depends on the divergence between the preferences of the blockholder and dispersed shareholders. This relationship is nuanced. In some cases, a higher voting premium is associated with a higher payoff for the blockholder and lower payoffs for small shareholders, in line with the argument that the voting premium indicates a conflict of interest. However, in other cases, the voting premium is higher when both, the blockholder's and the small shareholders' payoffs are higher. This happens if the preferences of small shareholders are skewed, or with a supermajority requirement; in both cases the average small shareholder differs from the median voter. Hence, the voting premium is not necessarily positively associated with the severity of conflicts between different shareholders, and, therefore, is probably also not a good measure for them.

We extend the model in a number of ways to explore additional questions. First, we consider a setting in which voting rights are traded separately, e.g., through share lending, and show that the price of a separately traded vote can be zero even if the voting premium for a share in which voting and cash flow rights are bundled is strictly positive. Second, we introduce multiple blockholders to analyze how the competition among them and their heterogeneous preferences affect the voting premium. Last, we consider a setting in which decisions are made not by voting, but by managers who consider the preferences of the entire shareholder base. We show that the blockholder's trades can then give rise to an "influence premium" in the share price, which is different from the voting premium and can even be larger.

After concluding our theoretical analysis, in Section 8 we use our insights to shed some light on the large number of empirical studies on the voting premium. We distinguish five different methodologies that have been used to measure the voting premium and argue that the large differences of estimates across methodologies are unsurprising. We also discuss findings on the time-series variation and cross-sectional variation of the voting premium in light of our model.

Overall, our paper makes three contributions. First, it examines the trading between small and large shareholders and the ownership structure of the firm in a context in which blockholders affect voting outcomes without majority control. Second, it contributes to our understanding of asset prices by showing how and when a voting premium emerges when blockholders can acquire voting control only through securities in which cash flow rights are bundled with voting rights. Third, it provides guidance to the empirical literature by showing how different proxies for the voting premium are related and why they may be different from each other.

2 Discussion of the literature

We contribute a new theory of the value of voting rights. The primary approach in the literature, pioneered by Grossman and Hart (1988) and Harris and Raviv (1988), considers settings with control contests of firms with dual-class shares. In this approach, rival bidders and incumbent managers differ in their ability to generate cash flows that are shared by all shareholders, and in their valuation of private benefits from controlling the firm. Bidders compete for control and pay a premium to the holders of the voting shares.[3] Studies in this literature have explored a range of alternative settings, including different types of admissible bids; variation in the ability to extract private benefits; settings without a free-rider problem; and frictions from asymmetric information.[4] Moreover, some studies have considered deviations from the one-share one-vote principle through trading in derivatives rather than in non-voting shares (e.g., Blair, Golbe, and Gerard (1989); Kalay and Pant (2010); Burkart and Lee (2010); Dekel and Wolinsky (2012)). Independently of the details, a wide range of settings give rise to a voting premium in a bidding contest. Our theory contributes to this literature by showing how a voting premium emerges without takeovers and control contests. The critical point of departure is the way in which the blockholder's (respectively, bidder's) higher willingness to pay translates into a higher stock price. In control contests, this happens because competition among bidders or the free-rider problem force bidders to pay a higher price (see Bergström and Rydqvist (1992)) and Zingales (1995) for similar observations). However, neither mechanism is present in our model, which does not feature majority control. Instead, in our setting, the blockholder's trades affect the reservation prices of small shareholders, which generates an upward-sloping supply curve to the blockholder.[5] This may give rise to a positive voting premium even if the blockholder does not acquire majority control, and a negative voting premium may result if this supply curve becomes sufficiently steep.

A complementary literature analyzes a market in which votes trade separately from shares.[6] While these papers differ significantly regarding their chosen settings and normative conclusions, they all conclude that the value of separately traded votes is negligible, either because

[3] Burkart and Lee (2008) survey theoretical work on the role of the security-voting structure and the control premium in the context of takeovers.

[4] Types of admissible bids: Vinaimont and Sercu (2003); Dekel and Wolinsky (2012); variation in whether one party has private benefits: none: Bergström and Rydqvist (1992); one party is the main case in Grossman and Hart (1988); both parties: Vinaimont and Sercu (2003); Burkart, Gromb, and Panunzi (1998); there is no free-rider problem in Bergström and Rydqvist (1992); asymmetric information: Burkart and Lee (2010). Some empirical contributions also include further modeling efforts to motivate specific empirical analyses, e.g., Zingales (1995); Rydqvist (1996).

[5] An upward-sloping supply curve is also present in takeover models with majority control of Stulz (1988) and Burkart, Gromb, and Panunzi (1998). The latter also introduce the term "upward-sloping supply function."

[6] It is largely motivated by concerns about the incentives created by decoupling votes from cash flow rights ("empty voting"), triggered by a sequence of papers by Hu and Black, e.g., Hu and Black (2007); Hu and Black (2015).

dispersed shareholders value votes in proportion to their probability of being pivotal (Neeman and Orosel (2006); Brav and Mathews (2011); Speit and Voss (2020)) or because uninformed shareholders would like their votes to be picked up and cast by informed shareholders (Esö, Hansen, and White (2014)).[7] As our analysis emphasizes, the price of a vote traded separately can be very different from the price of a vote that is traded in conjunction with cash flow rights. In particular, in our extension to a separate market for votes, the voting premium for shares that combine cash flow and voting rights can be strictly positive even though the price of separately traded votes is zero.

The only approach that has derived a significant voting premium without control contests is Rydqvist (1987), who builds on Milnor and Shapley (1978) and introduces the notion of an oceanic Shapley value to the analysis of dual-class shares. The critical step here is that the ocean of atomistic shareholders can *collectively* become pivotal and thus value their voting power.[8] However, this leaves open how these atomistic shareholders resolve their collective action problem. In our setting, each dispersed shareholder maximizes only his individual payoff.

Our paper also contributes to the literature on the equilibrium ownership structure of firms and the analysis of blockholders. A large strand of this literature is on direct intervention by blockholders ("voice").[9] Another strand of this literature analyzes how trading by blockholders affects governance through its impact on stock prices and managers' incentives ("exit").[10] By contrast, in our setting, the blockholder exercises influence by affecting the identity of the median voter. This is empirically important because many blockholders, notably financial institutions, rely on voting to influence firms' policies.[11] Dhillon and Rossetto (2015), Bar-Isaac and Shapiro (2020), and Meirowitz and Pi (2021) also consider blockholder models with

[7]These papers focus on vote trading in corporations. A related literature in political science examines how vote trading allows agents with a higher intensity of preferences to buy votes from those who care about the decision less. See, e.g., Casella, Llorente-Saguer, and Palfrey (2012) and the literature surveyed in that paper.

[8]Rydqvist (1987) develops an empirical measure of relative voting power that is widely used (e.g., Zingales (1994); Chung and Kim (1999); Nenova (2003)).

[9]See Admati, Pfleiderer, and Zechner (1994), Bolton and von Thadden (1998), Kahn and Winton (1998), and Maug (1998) for earlier contributions to this literature. See the surveys of Edmans (2014), Edmans and Holderness (2017), and Dasgupta, Fos, and Sautner (2020) for more recent work and further details.

[10]See Admati and Pfleiderer (2009), Edmans (2009), Edmans and Manso (2011), as well as the surveys cited in the previous footnote.

[11]Thus, our paper contributes to the broader literature on corporate voting (e.g., Maug and Rydqvist, 2009; Levit and Malenko, 2011; Van Wesep, 2014; Malenko and Malenko, 2019; and Cvijanovic, Groen-Xu, and Zachariadis, 2020).

voting, but differently from our paper, they do not study the voting premium and focus on the effects of blockholders on, respectively, the risk taking of the firm and information aggregation.

More broadly, our paper is related to an earlier literature on the existence of equilibrium and the objectives of the firm in a context with incomplete markets and shareholders with heterogeneous preferences.[12] In particular, Drèze (1985) and DeMarzo (1993) develop models with the board of directors as a group of controlling blockholders. To this literature, we contribute by analyzing the voting premium and a richer characterization of the interplay between small shareholders and blockholders. This also distinguishes our paper from Levit, Malenko, and Maug (2020), who analyze trading and voting by atomistic shareholders – a setting in which the voting premium does not arise.

3 Model

Consider a publicly traded firm, which is initially owned by a continuum of measure one of dispersed shareholders and one large blockholder. The blockholder is endowed with $\alpha \in [0, 1)$ shares, and each dispersed shareholder is endowed with $e = 1 - \alpha$ shares, so the total number of outstanding shares is 1. In the baseline setting, each share has one vote. There is a proposal on which shareholders vote. The proposal could relate to director elections, M&As, executive compensation, corporate governance, or social and environmental policies. The proposal can either be approved ($d = 1$) or rejected ($d = 0$).

Preferences. Shareholders' preferences over the proposal depend on two components, which reflect a common value and private values. The common value component depends on an unknown state $\theta \in \{-1, 1\}$: if $\theta = -1$ ($\theta = 1$), accepting the proposal is value-decreasing (increasing). In other words, the common value is maximized if the policy matches the state ($d = 1$ if $\theta = 1$), as common in the strategic voting literature, e.g., Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996).

Shareholders also have private values from the proposal, which reflect the heterogeneity in their preferences. For simplicity, we refer to these private values as biases and denote them by b. A shareholder with bias $b > 0$ ($b < 0$) receives additional (dis)utility if the proposal is

[12] See Gevers (1974), Drèze (1985), DeMarzo (1993), and Kelsey and Milne (1996).

accepted. The distribution of biases b among the initial dispersed shareholders is given by a publicly known differentiable cdf G, which has full support with positive density g on $\left[-\bar{b}, \bar{b}\right]$, where $\bar{b} \in (0,1)$. Differences in shareholders' preferences can stem from time horizons, private benefits, social or political views, common ownership, risk aversion, or tax considerations, and we expand on some of these sources at the end of this section. As noted in the introduction, the evidence for preference heterogeneity is pervasive.

The value of a share from the perspective of a dispersed shareholder with bias b is

$$v\left(d, \theta, b\right) = v_0 + \left(\theta + b\right) d, \tag{1}$$

where $v_0 \geq 0$ ensures that shareholder value is always non-negative. Notice that because of heterogeneous preferences, shareholders apply different hurdle rates for accepting the proposal: a shareholder with bias b would like the proposal to be accepted if and only if his expectation of $\theta + b$ is positive. We will refer to shareholders with a higher b as being "more activist".

The blockholder has the same preference structure as dispersed shareholders, except that his bias is β. Thus, the value of a share from the perspective of a blockholder is $v\left(d, \theta, \beta\right)$.

Timeline. All shareholders are initially uninformed about the state θ and have the same prior about its distribution, which we specify below. Shareholders first trade and then vote on the proposal. This timing allows us to focus on how trading affects the composition of the voter base, which is crucial for the analysis of the voting premium. At the trading stage, each dispersed shareholder can buy any number of shares x, where $x < 0$ corresponds to the shareholder selling shares. A dispersed shareholder's utility from buying x shares is

$$u\left(d, \theta, b, x; \gamma, e\right) = \left(e + x\right) v\left(d, \theta, b\right) - \frac{\gamma}{2} x^2, \tag{2}$$

where $\gamma > 0$ captures trading frictions, e.g., illiquidity, transaction costs, or wealth constraints, which limit shareholders' ability to build large positions in the firm.[13] Since the mass of investors is bounded and $\gamma > 0$, our model effectively features limits to arbitrage.

Similarly, the blockholder can buy any number of shares y, and his utility from buying y

[13]Since each dispersed investor has a zero mass, his trade x and endowment e are infinitesimal quantities. With a slight abuse of notation, e also denotes the total endowment held by dispersed investors.

shares is $u(d, \theta, \beta, y; \eta, \alpha)$, where $u(\cdot)$ is given by (2) and $\eta \geq 0$ captures the blockholder's trading costs. All results continue to hold for $\eta = 0$. We assume that neither dispersed shareholders nor the blockholder find it in their best interest to short sell.[14] Moreover, and for simplicity, we assume that the blockholder submits his order y first, and dispersed shareholders observe y and submit their orders next. Effectively, dispersed shareholders trade shares by submitting limit orders.[15]

We denote the market clearing share price by p. After the market clears, but before voting takes place, all shareholders observe a public signal about the state θ, which may stem from disclosures by management, proxy advisors, or analysts.[16] Let $q = \mathbb{E}[\theta|\text{public signal}]$ be shareholders' posterior expectation of the state following the signal. For simplicity, we assume that the public signal is q itself, and that q is distributed according to a differentiable cdf F with mean zero and full support with positive density f on $[-\Delta, \Delta]$, where $\Delta \in (\bar{b}, 1)$. Thus, the ex-ante expectation of θ is zero. In what follows, we always refer to $H(q^*) \equiv \Pr[q > q^*]$ rather than to the cdf. The symmetry of the support of q around zero is not necessary for any of the main results.

After observing the public signal q, each shareholder votes the shares he owns after the trading stage. Hence, we assume that the record date, which determines who is eligible to participate in the vote, is after the trading stage. This timeline applies well to important votes, such as the votes on M&As, proxy fights, and high-profile shareholder proposals, which are typically known well ahead of the record date. The proposal is accepted if at least fraction $\tau \in (0, 1)$ of all shares are cast in favor; otherwise, the proposal is rejected. We assume that the blockholder's initial stake and ability to buy shares are not large enough to grant him the power to accept the proposal unilaterally, as well as to veto the proposal solely with his own votes, i.e., $\alpha + y < \min\{\tau, 1 - \tau\}$ in any equilibrium.[17]

[14] Lemma 3 in the Online Appendix shows that if $\alpha > 0$ and γ is sufficiently large then there are no short sales in equilibrium. If $\alpha = 0$, which we analyze as a special case and separately from Proposition 2, the no-short-selling constraint can bind for the blockholder, but it does not change our main results.

[15] As equation (35) shows, if γ is sufficiently large, then the market clearing price is monotonic in y, and thus, whether the limit order is conditioned on the price itself or the blockholder's trade is immaterial for our analysis.

[16] In practice, proxy advisors' recommendations (and management's response) are on average released about one month after the record date. See, for example, Fig. 1 in Li, Maug, and Schwartz-Ziv (2022).

[17] Lemma 3 in the Online Appendix shows that if $\alpha < \min\{\tau, 1 - \tau\}$ and γ is sufficiently large, then $\alpha + y < \min\{\tau, 1 - \tau\}$ in any equilibrium.

We analyze subgame perfect Nash equilibria in undominated strategies of the voting game. The restriction to undominated strategies is common in voting games, which usually impose the equivalent restriction that dispersed shareholders vote as-if-pivotal.[18] This implies that an investor with bias b, whether he is a dispersed shareholder or the blockholder, votes in favor of the proposal if and only if

$$b + q > 0. \tag{3}$$

Applications of the model. The model introduced above can map into several applications, which we develop in Section B of the Online Appendix. In the first application, we consider investors with heterogeneous time horizons who vote between a short-termist and a long-termist investment strategy, such as in a proxy contest organized by a short-termist activist. Shareholders' valuations of the firm under each strategy depend on the likelihood that the strategy succeeds (common value), but also on their time horizons (private values): long-termist shareholders put a higher weight on long-term cash flows relative to short-term cash flows. In this application, shareholders' trading and voting strategies, as well as the voting premium, are given by the same expressions (up to a constant) as in the baseline model. In the second application, we microfound shareholders' valuations in eq. (1) via a model with heterogeneous beliefs, where shareholders' biases b and β capture differences in beliefs (or "sentiment") regarding the value of the proposal, rather than differences in preferences. The third application captures a setting with private benefits of control. The blockholder can dilute the assets of the firm if the proposal is approved, which leads him to favor the proposal relative to dispersed shareholders ($\beta > b$). Finally, the fourth application captures costly monitoring: if the proposal is approved, the blockholder can incur a private cost to monitor the manager and increase firm value. Since dispersed shareholders benefit from monitoring but do not incur its cost, they favor the proposal more than the blockholder ($b > \beta$).

[18] See, e.g., Baron and Ferejohn (1989) and Austen-Smith and Banks (1996). This restriction helps rule out trivial equilibria, in which shareholders are indifferent between voting for and against because they are never pivotal.

4 Equilibrium

We begin by showing that for any trading outcome, proposal approval at the voting stage takes the form of a cutoff decision rule:

Lemma 1. *In any equilibrium, there exists q^* such that the proposal is approved by shareholders if and only if $q > q^*$.*

Intuitively, this is because all shareholders value the proposal more if it is more likely to be value-increasing, i.e., if $\theta = 1$ is more likely.

We proceed in several steps. First, for any possible blockholder's trade y, Sections 4.1 and 4.2 characterize the trading of dispersed shareholders and the voting stage as a function of y. In Section 4.3, we solve for the optimal trading strategy of the blockholder, y^*, and for the equilibrium share price.

4.1 Trading of dispersed shareholders

Motivated by Lemma 1, suppose that dispersed shareholders expect the proposal to be accepted if and only if $q > q_e^*$ for some cutoff q_e^* (we later derive the equilibrium cutoff such that shareholders' expectations are rational). Let $v(b, q_e^*)$ denote the valuation of a shareholder with bias b prior to the realization of q, as a function of the cutoff q_e^*. Then

$$v(b, q_e^*) = \mathbb{E}\left[v\left(\mathbf{1}_{q>q_e^*}, \theta, b\right)\right], \tag{4}$$

where the indicator function $\mathbf{1}_{q>q_e^*}$ equals one if $q > q_e^*$ and zero otherwise, and $v(d, \theta, b)$ is defined by (1). Then (4) can be rewritten as

$$v(b, q_e^*) = v_0 + (b + \mathbb{E}[\theta | q > q_e^*]) H(q_e^*), \tag{5}$$

which increases in b. Dispersed shareholders are price takers, so for any expected share price p, each dispersed shareholder solves

$$\max_x \left\{(e + x) v(b, q_e^*) - xp - \frac{\gamma}{2}x^2\right\} \tag{6}$$

and optimally chooses

$$x\left(b, q_e^*, p\right) = \frac{v\left(b, q_e^*\right) - p}{\gamma}. \tag{7}$$

Thus, shareholder b buys shares if his valuation exceeds the market price, $v\left(b, q_e^*\right) > p$, sells shares if $v\left(b, q_e^*\right) < p$, and does not trade otherwise. Given the blockholder's order y, the market clears if and only if

$$\int_{-\bar{b}}^{\bar{b}} x\left(b, q_e^*, p\right) g\left(b\right) db + y = 0, \tag{8}$$

which gives the market clearing price

$$p^*\left(y, q_e^*\right) = \gamma y + v\left(\mathbb{E}\left[b\right], q_e^*\right). \tag{9}$$

The equilibrium share price increases in y, and the price impact of the blockholder's trade is larger if γ is larger. Thus, we can interpret γ as measuring the illiquidity of the market, i.e., the inverse of γ reflects market depth. In addition, the share price (9) increases in the valuation of the average dispersed shareholder. Intuitively, if dispersed shareholders' valuation (conditional on q_e^*) is higher, they are willing to supply shares to the blockholder only at a higher price.

From (5), (7), and (9), dispersed shareholders' demand as a function of the blockholder's trade can be written as

$$x\left(b, y, q_e^*\right) = \frac{1}{\gamma}\left(b - \mathbb{E}\left[b\right]\right) H\left(q_e^*\right) - y. \tag{10}$$

The post-trade ownership structure. Next, we characterize the post-trade ownership structure. After the trading stage, the blockholder owns $\alpha + y$ shares, a dispersed shareholder with bias b owns $1 - \alpha + x\left(b, y, q_e^*\right)$ shares, and all dispersed shareholders collectively own $1-\alpha-y > 0$ shares. Thus, the proportion of shares owned post-trade by dispersed shareholders with bias b, conditional on the expected decision rule q_e^* and blockholder's trade y, is given by

$$r\left(b; y, q_e^*\right) \equiv g\left(b\right) \frac{1 - \alpha + x\left(b, y, q_e^*\right)}{1 - \alpha - y}. \tag{11}$$

Note that $r(b;y,q_e^*)$ is a density function, i.e., $\int_{-\bar{b}}^{\bar{b}} r(b;y,q_e^*)\,db = 1$. Thus, the post-trade dispersed shareholder base is characterized by the cdf $R(b;y,q_e^*)$ given by

$$\begin{aligned} R(b';y,q_e^*) &= \int_{-\bar{b}}^{b'} r(b;y,q_e^*)\,db \\ &= G(b')\left(1 - \frac{\mathbb{E}[b] - \mathbb{E}[b|b<b']}{\gamma}\frac{H(q_e^*)}{1-\alpha-y}\right), \end{aligned} \tag{12}$$

where the second equality follows from (10) and (11). The cdf R characterizes the post-trade dispersed shareholder base, whereas G characterizes the pre-trade dispersed shareholder base. Note that $R(b) < G(b)$ for any b, i.e., R dominates G in the sense of first-order stochastic dominance. Hence, trading shifts the shareholder base in such a way that more activist shareholders own a larger proportion of the firm after trading. Moreover, $R(b';y,q_e^*)$ increases in q_e^*; hence, a more activist decision rule (lower q_e^*) makes the post-trade shareholder base more activist. Intuitively, shareholders' heterogeneous attitudes towards the proposal create gains from trade, so the shareholder base moves in the direction of the expected outcome.

4.2 Voting

The composition of the post-trade shareholder base determines the voting outcome. To derive the conditions under which the proposal is approved, we first analyze the votes of dispersed shareholders. Denote by $s(q;y,q_e^*)$ the number of votes cast by dispersed shareholders in favor of the proposal if signal q is realized, the blockholder traded y shares, and the expected decision rule is q_e^*. Then,

$$s(q;y,q_e^*) \equiv (1-\alpha-y)(1-R(-q;y,q_e^*)), \tag{13}$$

which is the number of shares held by dispersed shareholders, $1-\alpha-y$, multiplied by the proportion of dispersed shareholders for whom $b > -q$.

The blockholder is pivotal for the outcome only if at least $\tau-(\alpha+y)$ but no more than τ dispersed shareholders vote to support the proposal, i.e., if and only if

$$\tau-(\alpha+y) < s(q;y,q_e^*) < \tau. \tag{14}$$

Otherwise, if $s(q;y,q_e^*) < \tau-(\alpha+y)$ ($s(q;y,q_e^*) > \tau$), the proposal fails (succeeds) independently of the vote of the blockholder. From (13), the support of dispersed shareholders is increasing in the signal q. Define the bounds $\underline{q} \equiv s^{-1}(\tau - \alpha - y; y, q_e^*)$ and $\overline{q} \equiv s^{-1}(\tau; y, q_e^*)$, so that the blockholder is pivotal whenever the signal is in the intermediate range $q \in [\underline{q}, \overline{q}]$. Whenever $q < \underline{q}$ ($q > \overline{q}$), dispersed shareholders' support for the proposal is so low (high) that the proposal fails (succeeds) even if the blockholder supports (rejects) it.

We describe the voting outcome by characterizing the identity of the *median voter*, who is defined as the shareholder whose individual vote always coincides with the collective decision on the proposal. In other words, whenever the median voter votes in favor (against), the proposal is accepted (rejected). Let $b_{MV}(\beta, y, q_e^*)$ denote the bias of the median voter if the expected decision rule is q_e^*, the blockholder traded y shares, and his bias is β. There are three possible cases, which define $b_{MV}(\beta, y, q_e^*)$:

(i) If $\beta > -\underline{q}$ then, the blockholder is very activist and supports the proposal whenever he is pivotal. The proposal is accepted if and only if $s(q; y, q_e^*) + \alpha + y \geq \tau$, i.e., whenever $q \geq \underline{q}$. Hence, the proposal passes if and only if the dispersed shareholder with bias $b = -\underline{q}$ votes in favor. This shareholder is then the median voter, i.e., $b_{MV}(\beta, y, q_e^*) = -\underline{q}$.

(ii) If $\beta < -\overline{q}$ then, the blockholder has a large bias against the proposal and votes against whenever he is pivotal. The proposal is accepted if and only if $s(q; y, q_e^*) \geq \tau$, i.e., whenever $q \geq \overline{q}$. Hence, the proposal passes if and only if the dispersed shareholder with bias $b = -\overline{q}$ votes in favor. This shareholder is then the median voter, i.e., $b_{MV}(\beta, y, q_e^*) = -\overline{q}$.

(iii) If $-\overline{q} < \beta < -\underline{q}$ then, the blockholder is pivotal if $q \in [\underline{q}, \overline{q}]$ and votes in favor if and only if $q \geq -\beta$. Hence, the proposal is accepted if and only if the blockholder votes in favor, so the blockholder is the median voter, $b_{MV}(\beta, y, q_e^*) = \beta$.

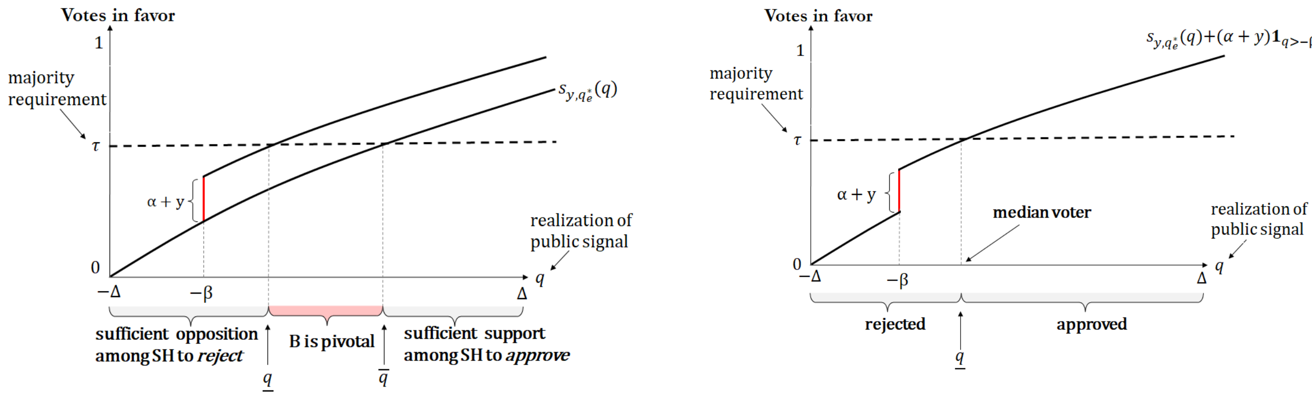

Figure 1 - The pivotal voter and the median voter

We conclude that if shareholders anticipate decision rule q_e^* when trading, then the decision rule at the voting stage is characterized by the three cases above. The first case is illustrated in Figure 1, which plots the number of votes in favor of the proposal as a function of the signal q. The left panel indicates the range in which the blockholder is pivotal; the right panel indicates the approval range and the location of the median voter.

The distinction between the pivotal voter and the median voter is an important implication of this argument. A voter is pivotal if his vote can sway the decision on the proposal. Only the blockholder can be pivotal in our setting, since all other shareholders are atomistic. In contrast, the median voter is the shareholder whose vote always coincides with the decision on the proposal. In the first two cases above, the blockholder is extreme and the median voter is a dispersed shareholder closer to the center of the distribution of votes. The distinction between the median voter and the pivotal voter will play a key role in our analysis of the voting premium (see Section 5).

In equilibrium, shareholders' expectations q_e^* must be consistent with the actual decision rule. Hence, an equilibrium can be found as a fixed point of q_e^* such that $-b_{MV}(\beta, y, q_e^*) = q_e^*$, where $b_{MV}(\beta, y, q_e^*)$ is defined by the three cases above. Using this logic, the equilibrium at the voting stage is characterized as follows.

Proposition 1 (Voting stage). *If the blockholder trades y shares, then the proposal is ap-*

proved if and only if $q > q^*(y)$, where $q^*(y)$ solves

$$-b_{MV}(\beta, y, q^*) = q^*. \tag{15}$$

There exists $\overline{\gamma} < \infty$ such that if $\gamma > \overline{\gamma}$, the solution of (15) is unique. In this case, there exists $\overline{y}$ such that if $y > \overline{y}$, the median voter is the blockholder ($-q^*(y) = \beta$), whereas if $y < \overline{y}$, the median voter is a dispersed shareholder with bias $-q^*(y) \neq \beta$, and $|\beta + q^*(y)|$ decreases in y.

In general, there can be multiple solutions to (15), and hence multiple equilibria at the voting stage. This is because for small γ, the shifts in the shareholder base are sensitive to the expected decision rule q_e^*, which can give rise to self-fulfilling expectations.[19] However, if γ is large enough, then dispersed shareholders trade less aggressively, the distribution of the post-trade shareholder base is less sensitive to q_e^*, and the equilibrium is unique. From this point on, we focus on parameterizations for which the equilibrium at the voting stage is unique.

Importantly, Proposition 1 shows that the blockholder can change the identity of the median voter, $-q^*(y)$, and thus the vote outcome, with his trades y. By buying more shares, the blockholder exerts more influence on the voting outcome, which pushes the bias of the median voter closer to β, as captured by the result that $|\beta + q^*(y)|$ decreases. This can be seen in the left panel of Figure 2, which shows that a larger y pushes $-q^*(y)$ to the left, closer to $-\beta$. Once the blockholder buys enough shares ($y > \overline{y}$), the vote outcome exactly coincides with the blockholder's own voting rule, so the blockholder becomes the median voter (see the right panel in Figure 2). The accumulation of shares beyond $\bar{y}$ increases the probability of the blockholder being pivotal, but does not change the expected vote outcome, that is, the identity of the median voter.

There are two complementary reasons why the accumulation of shares by the blockholder moves the median voter closer to him. First, more shares give the blockholder more voting power. Second, as the blockholder buys more, the composition of the dispersed shareholder base changes in favor of those who are more aligned with the blockholder. For example, if the blockholder is very activist, then the dispersed shareholders become more activist after

[19] In particular, the cdf of the post-trade shareholder base, given by (12), increases in q_e^*, and hence a more activist *expected* decision rule (lower q_e^*) makes the post-trade shareholder base more activist. A more activist shareholder base, in turn, is more likely to approve the proposal for any given signal, leading to a lower *realized* cutoff for approving the proposal, confirming the ex-ante expectations.

trading. (Recall that $R\left(b';y,q_e^*\right)$ increases in q_e^* in (12).)

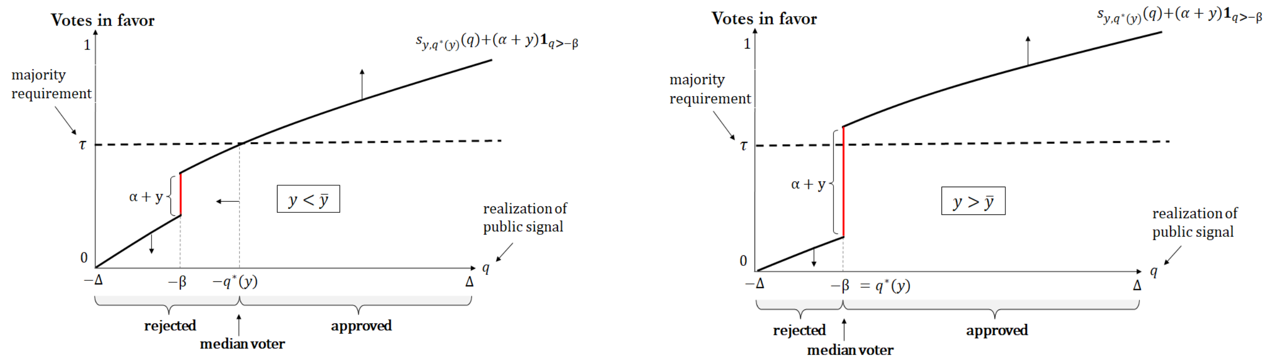

Figure 2 - The effect of the blockholder's trade y on the equilibrium median voter $q^*(y)$

4.3 Blockholder trading

Given the blockholder's trade y, all shareholders correctly anticipate that the decision rule at the voting stage will be $q^*\left(y\right)$, as given by (15), and that the market clearing price will be

$$p^*\left(y\right) = \gamma y + v\left(\mathbb{E}\left[b\right], q^*\left(y\right)\right) \tag{16}$$

from (9). In equilibrium, the blockholder chooses y to maximize

$$\Pi\left(y\right) \equiv \left(\alpha + y\right) v\left(\beta, q^*\left(y\right)\right) - yp^*\left(y\right) - \frac{\eta}{2}y^2. \tag{17}$$

The marginal effect of buying additional shares on the blockholder's expected payoff is

$$\frac{d\Pi\left(y\right)}{dy} = \underbrace{\frac{\partial\Pi\left(y\right)}{\partial y}}_{MPC(y)} + \underbrace{\frac{\partial\Pi\left(y\right)}{\partial\left(-q^*\left(y\right)\right)}\frac{\partial\left(-q^*\left(y\right)\right)}{\partial y}}_{MPV(y)} = MPC\left(y\right) + MPV\left(y\right). \tag{18}$$

The term $MPC\left(y\right)$ is the *marginal payoff from buying cash flow rights.* It can be thought of as the blockholder's marginal payoff from trading in a hypothetical scenario in which the decision rule is set exogenously at the level $q^*\left(y\right)$ and is not affected by the blockholder's trades. This term equals

$$MPC\left(y\right) = \left(\beta - \mathbb{E}\left[b\right]\right) H\left(q^*\left(y\right)\right) - \left(2\gamma + \eta\right) y. \tag{19}$$

Intuitively, if $\beta > \mathbb{E}[b]$ ($\beta < \mathbb{E}[b]$), the blockholder values shares more (less) than the average dispersed shareholder, which creates gains from trade. The term $MPV(y)$ is the *marginal payoff from buying voting rights*. It captures the blockholder's additional incentives to trade in order to change the decision rule, i.e., to shift the median voter $-q^*(y)$. In Section 5 below, we show that $MPV(y)$ is closely connected to the voting premium.

The next proposition characterizes the equilibrium of the game, including the blockholder's optimal trading strategy. As before, we focus on the case when the equilibrium is unique and, accordingly, assume that γ is large enough.

Proposition 2 (Equilibrium). *Suppose the blockholder has an endowment $\alpha > 0$. There exists $\overline{\gamma} < \infty$ such that if $\gamma > \overline{\gamma}$, the equilibrium exists and is unique. In this equilibrium:*

(i) The blockholder trades y^* shares, where

$$y^* = \frac{1}{2\gamma + \eta}\left(\beta - \mathbb{E}[b]\right) H(q^*(y^*)) + \frac{1}{2\gamma+\eta} MPV(y^*), \tag{20}$$

and a dispersed shareholder with bias b trades $x^*(b)$ shares, where

$$x^*(b) = \frac{1}{\gamma}\left(b - b^*\right) H(q^*(y^*)) - \frac{1}{2\gamma+\eta} MPV(y^*) \tag{21}$$

and

$$b^* = \frac{\gamma}{2\gamma+\eta}\beta + \left(1 - \frac{\gamma}{2\gamma+\eta}\right)\mathbb{E}[b]. \tag{22}$$

(ii) The share price is

$$p^* = v\left(b^*, q^*(y^*)\right) + \frac{\gamma}{2\gamma+\eta} MPV(y^*). \tag{23}$$

(iii) The bias of the median voter is $-q^*(y^*)$, where $q^*(\cdot)$ is defined in Proposition 1.

The blockholder's optimal trade y^* consists of two terms, which are related to decomposition (18). The first term reflects trading for cash flow reasons: the blockholder has an incentive to buy shares if and only if his valuation is higher than the average dispersed shareholder's valuation, $\beta > \mathbb{E}[b]$. The second term reflects the additional trading of voting shares because of the embedded voting rights and is proportional to $MPV(y^*)$. The expression for dispersed shareholders' trades $x^*(b)$ follows directly from (10). Intuitively, trading shifts the dispersed shareholder base towards the expected outcome (see the discussion after eq. (12) above), and their combined supply of shares equals the blockholder's demand. Finally, the equilibrium stock price consists of two terms, and we focus on this decomposition and its properties in the next section.

5 The voting premium

This section presents our main results. We proceed in several steps. We define the voting premium in Section 5.1 and clarify its determinants in Section 5.2. In Section 5.3 we characterize the equilibrium properties of the voting premium. Finally, in Section 5.4 we discuss the novel implications about the voting premium that emerge from our analysis.

5.1 Defining the voting premium

We start by defining the voting premium and relating it to MPV. Consider again the hypothetical scenario in which the voting rule is set exogenously at $q^*(y^*)$. Such a scenario may reflect cases in which trading does not reallocate voting rights among investors, so the median voter is unaffected.[20] Since in this hypothetical scenario the voting rule is exogenous, we have $\frac{\partial(-q^*(y^*))}{\partial y} = 0$, $MPV(y^*) = 0$, and the blockholder's first-order condition (18) reduces to $MPC(y^*) = 0$. A corollary of Proposition 2 shows that the share price in this scenario reflects the valuation of the shareholder with bias b^*, where b^* is defined in (22).[21]

Corollary 1 *If the voting rule is set exogenously at $q^*(y^*)$, the equilibrium share price is given by*

$$p_{CF}(q^*(y^*)) = v(b^*, q^*(y^*)). \tag{24}$$

Next, we define the *voting premium* as

$$VP(y^*) \equiv p^* - p_{CF}(q^*(y^*)), \tag{25}$$

[20] This hypothetical scenario also captures cases in which the company's board and management have decision rights over the proposal and implement a decision rule $q^*(y^*)$ that is exogenous to the composition of the shareholder base.

[21] According to (21), when $MPV(y^*) = 0$, the shareholder with bias b^* can be thought of as the marginal trader, since he is indifferent between buying and selling shares at the equilibrium share price.

i.e., the difference between the share price (23) that arises when the voting rule is determined endogenously by the post-trade shareholder base, and the share price in the hypothetical scenario when the voting rule is set exogenously at the same level $q^*(y^*)$. Proposition 2 and Corollary 1 imply that the voting premium is proportional to the blockholder's marginal payoff from buying voting rights:

$$VP(y^*) = \frac{\gamma}{2\gamma+\eta} MPV(y^*). \tag{26}$$

Hence, the voting premium reflects the additional component of the stock price that arises from the blockholder's incentive to influence the voting outcome.

There are two empirical counterparts of our definition of the voting premium. First, we can think of it as the difference between the stock prices right before and after the record date (Fos and Holderness (2020)). These prices reflect the same expected voting rule, but shares traded right before the record date have voting rights for the upcoming shareholder meeting, whereas shares traded right after the record date do not.[22] The second empirical counterpart is the *dual-class premium*, which we formalize in an extension to a dual-class share structure in Section 6. However, our definition of the voting premium is broader than these two empirical measures and suggests a way to isolate the voting premium as a component of the stock price also in single-class firms and on dates other than the record date.

5.2 Determinants of the voting premium

To understand the determinants of the voting premium, we rewrite (25) as

$$VP(y) = \underbrace{\frac{\partial(-q^*(y))}{\partial y}}_{\text{ability to move median voter}} \times \underbrace{\left[\underbrace{(\alpha+y)\frac{\partial v(\beta, q^*(y))}{\partial(-q^*)}}_{\text{marginal benefit of a vote}} - y\underbrace{\frac{\partial p^*(y)}{\partial(-q^*)}}_{\text{price impact of a vote}}\right]}_{\text{incentives to move median voter } = \frac{\partial \Pi(y)}{\partial(-q^*(y))}} \times \frac{\gamma}{2\gamma+\eta}. \tag{27}$$

Thus, the voting premium can be decomposed into the blockholder's *ability* to influence the identity of the median voter, and his *incentives* to do so.

The ability of the blockholder to influence the median voter depends on how his trades

[22] Our model is static in nature, so our analogy to trades around the record date abstracts from dynamic aspects of trade that could potentially affect the share price.

y affect $-q^*(y)$. According to Proposition 1, there exists $\overline{y}$ such that if $y > \overline{y}$, then the blockholder is the median voter and the accumulation of additional shares does not change it. Therefore, if $y > \overline{y}$, then $\frac{\partial(-q^*(y))}{\partial y} = 0$ and the blockholder cannot change the voting outcome even if he had incentives to do so. According to (27), the voting premium is then zero. Intuitively, since the blockholder's trades have no impact on the voting outcome, he is not willing to pay a premium for the additional voting rights he is accumulating.

Proposition 1 also shows that if $y < \overline{y}$, then $\frac{\partial(-q^*(y))}{\partial y} \neq 0$ and the blockholder's trades change the identity of the median voter. In this case, the voting premium also depends on the incentives of the blockholder to move the median voter. Based on (27), these incentives consist of two components. The first component captures how a marginal change in the median voter affects the blockholder's valuation of his post-trade stake in the firm, $\alpha + y$, and we refer to it as the *marginal benefit of a vote*. From (5) and (27),

$$\textit{Marginal benefit of a vote} = (\alpha + y)\frac{\partial v(\beta, q^*(y))}{\partial(-q^*)} = (\alpha + y)(\beta + q^*(y)) f(q^*(y)). \qquad (28)$$

By buying additional shares, the blockholder moves the median voter $-q^*(y)$ closer to his own bias β (see Proposition 1), which increases the blockholder's valuation of his stake. Thus, the blockholder always values a marginal vote for its impact on his stake, that is, the marginal benefit of a vote is always positive.

The second component captures the blockholder's additional incentives to move the median voter due to the associated *price impact*, which in turn affects the blockholder's gains from trade. Based on (16), the effect of a marginal change in the median voter $-q^*$ on the stock price is

$$\textit{Price impact of a vote} = \frac{\partial p^*(y)}{\partial(-q^*)} = (\mathbb{E}[b] + q^*(y)) f(q^*(y)). \qquad (29)$$

The sign and magnitude of the price impact of a vote depend on $\mathbb{E}[b]$, the bias of the average dispersed shareholder. Intuitively, the market clearing price reflects the reservation price at which dispersed shareholders are willing to supply their shares, so the price impact of a vote depends on whether the resulting change in the median voter benefits or hurts dispersed shareholders. The sign of this effect is generally ambiguous and we discuss it in detail in Section 5.4.4.

5.3 Properties of the voting premium

The discussion in Section 5.2 highlights that a key determinant of the voting premium is the location of the median voter. The next result characterizes the equilibrium median voter and the voting premium as functions of the blockholder's bias β.

Proposition 3 *Suppose the conditions of Proposition 2 are satisfied, and let*

$$\overline{\beta} \equiv G^{-1}\left(\frac{1-\tau}{1-\alpha}\right) \tag{30}$$

$$\underline{\beta} \equiv G^{-1}\left(\frac{1-\tau-\alpha}{1-\alpha}\right). \tag{31}$$

(i) If $\beta \in (\underline{\beta}, \overline{\beta})$, then the median voter is the blockholder and the voting premium is zero.

(ii) If $\beta > \overline{\beta}$ ($\beta < \underline{\beta}$), then the median voter is a dispersed shareholder with a smaller (larger) bias toward the proposal than the blockholder, $-q^*(y^*) < \beta$ ($-q^*(y^*) > \beta$). The voting premium is strictly positive and increases (decreases) in β.

Figure 3 illustrates Proposition 3. The bold black curve plots the bias of the median voter and the blue curve plots the equilibrium voting premium, both as functions of the blockholder's bias. There are two distinct scenarios. First, suppose that the preferences of the blockholder are moderate, $\beta \in (\underline{\beta}, \overline{\beta})$. In this region, the blockholder becomes the median voter since he needs to buy only few shares to do so, and the black curve coincides with the 45-degree line. Hence, the blockholder's ability to move the median voter and the voting premium are both zero. This observation further highlights that it is the median voter, and not the pivotal voter, that affects the voting premium (see Section 4.2). In fact, a larger blockholder (larger α) is not only more likely to be pivotal. He is also more likely to be the median voter, with an associated voting premium of zero, since the length of the interval $(\underline{\beta}, \overline{\beta})$ increases in α.

Second, if the blockholder's preferences are more extreme, $\beta > \overline{\beta}$ (or $\beta < \underline{\beta}$), then the blockholder does not become the median voter, as it would require buying too many additional shares. Then the median voter has a smaller (larger) bias toward the proposal than the blockholder, and the black curve is below (above) the 45-degree line. Now the blockholder does have the ability to move the median voter, and he benefits more from doing so the further he is away from the median voter. Accordingly, the voting premium is positive and increases as β becomes more extreme. Importantly, even though the blockholder's net marginal benefit of a vote is positive, he refrains from buying more voting shares, because he also considers the cost from his own price impact.

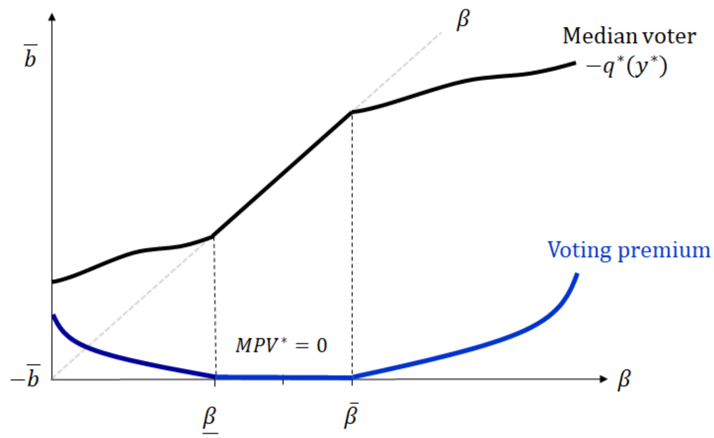

Figure 3 - Equilibrium median voter and voting premium.

5.4 Implications

In this section we discuss the key implications of our analysis of the voting premium.

5.4.1 Voting premium vs. total value of voting rights

Our analysis highlights that the voting premium is likely to *underestimate* the overall value of voting rights. A zero voting premium does not imply that the blockholder does not value voting rights, or that he would not benefit from further influencing the voting outcome. Indeed, the incentives to move the median voter, captured by $\frac{\partial \Pi(y)}{\partial(-q^*)}$, will generally differ from zero, even if the blockholder is already the median voter.[23] Instead, a zero voting premium only implies that the blockholder cannot influence the position of the median voter through additional trades.

Furthermore, the blockholder's overall benefits from accumulating voting rights can be positive even if the marginal benefits are zero, because these marginal benefits are evaluated at the blockholder's equilibrium trade y^*. By contrast, the overall benefits from owning voting rights also come from the blockholder's infra-marginal trades. These infra-marginal trades

[23] To see this, note that if $-q^*(y) = \beta$, then $\frac{\partial \Pi(y)}{\partial(-q^*)} = y(\beta - \mathbb{E}[b]) f(-\beta)$, which generally differs from zero. Intuitively, by changing the median voter further, the blockholder affects his gains from trade with small shareholders since the effect of the median voter on their valuations differs from the effect of the median voter on his own valuation (see Section 5.4.4 for a more in-depth discussion).

affect the voting outcome ($q^*(y^*) \neq q^*(0)$), even if the equilibrium voting premium is zero. Hence, empirical measures of the voting premium that measure the value of a marginal vote, such as the dual class premium or the ex-record date price drop, are likely to underestimate the overall value of voting rights.

5.4.2 Exercising control vs. influencing who exercises control

The voting premium does not emerge from exercising control, but from influencing who exercises control. At the margin, the blockholder does not pay a voting premium for being in control of the voting outcome. Instead, he pays the voting premium to influence the identity of the dispersed shareholder who becomes the median voter. In fact, the voting premium turns to zero when the blockholder becomes the median voter, in which case the voting outcome is aligned with the blockholder' bias.

As such, the voting premium is generally unrelated, or can even be negatively related, to measures of voting power. If the blockholder's voting power is large, he is the median voter himself, so the voting premium is zero. In contrast, if the blockholder's voting power is small, his marginal payoff from moving the median voter is strictly positive. He has both ability and, under the conditions of Proposition 2, also the incentives to do so, so the voting premium is positive as well.

5.4.3 Conflicts of interest and the voting premium

Divergence between the blockholder and small shareholders. The literature often relates the voting premium to conflicts between majority blockholders and minority shareholders. The idea is that a large voting premium may be associated with a lower payoff of dispersed shareholders, since the blockholder exploits his voting power to advance his own agenda at the expense of others. In this section, we show that this intuition is not always correct.

We start by defining the aggregate equilibrium payoff of dispersed shareholders as

$$W^* \equiv \int_{-\bar{b}}^{\bar{b}} u^*(b)\, g(b)\, db, \tag{32}$$

where $u^*(b)$ is the expected payoff of a dispersed shareholder with bias b:

$$u^*(b) = (e + x^*(b))\, v(b, q^*(y^*)) - x^*(b)\, p^* - \frac{\gamma}{2} x^*(b)^2, \tag{33}$$

and $x^*(b)$, $q^*(y^*)$, and p^* are defined in Proposition 2. The next result shows how W^* is related to the voting premium.

Proposition 4. *Suppose the conditions of Proposition 2 are satisfied.*

(i) If $\overline{\beta} \le \mathbb{E}[b] < \beta$, then both W^* and the voting premium strictly increase with β.

(ii) Suppose $\mathbb{E}[b] \le \beta < \underline{\beta}$.

- *(a) If $\alpha > \underline{\alpha}$, where $\underline{\alpha}$ is defined in Lemma 4 in the Online Appendix, then both W^* and the voting premium strictly decrease with β.*
- *(b) If α is sufficiently small, then W^* strictly increases in β, whereas the voting premium strictly decreases with β.*

The striking result in Proposition 4 is that a change in β may be associated with both an increase in the voting premium and an increase in the payoff of small shareholders (parts (i) and (ii.a)). In other words, a larger voting premium does not necessarily indicate a greater conflict between the blockholder and small shareholders. To understand the intuition, note that W^* can be written as

$$W^* = ev(\mathbb{E}[b], q^*) + \frac{1}{2\gamma} \mathbb{E}\left[(v(b, q^*) - p^*)^2\right]. \tag{34}$$

The first term in (34) is dispersed shareholders' aggregate payoff from their endowment, whereas the second term reflects the aggregate gains from trade of dispersed shareholders, both among themselves and with the blockholder. To facilitate the explanation, we focus on the first term and abstract from trading profits.

Part (i) describes the case in which $\overline{\beta} < \beta$. If γ is large, as under the conditions of the proposition, then the median voter is approximately $\overline{\beta}$. As the blockholder becomes more activist, he moves away from the median voter, which gives him stronger incentives to change the median voter and increases the voting premium (see Proposition 3 and the discussion of Figure 3). At the same time, $\overline{\beta} < \mathbb{E}[b]$ implies that the median voter is less activist than the average dispersed shareholder. Then, as the blockholder becomes more activist, his accumulation of voting shares moves the median voter closer to the average preferences of dispersed shareholders, increasing their payoff as well. As a result, the aggregate payoff of small shareholders and the voting premium move in the same direction.

There are two scenarios that can lead to such a situation. First, consider a situation with a simple majority requirement in which the post-trade distribution of dispersed shareholders' preferences is right-skewed. This means that more activist small shareholders feel much more strongly about the advantages of the proposal than their less activist peers. The median voter does not reflect this asymmetric intensity of preferences, but it is relevant for the payoff W^*, which is based on the average and not the median payoff. Second, suppose that the post-trade distribution of preferences is symmetric, but the proposal is subject to a supermajority requirement ($\tau > 0.5$). The supermajority requirement introduces a conservative bias into the voting process and thereby reduces the bias of the median voter (note that $\overline{\beta}$ and $\underline{\beta}$ both decrease in τ from (30) and (31).

In both scenarios, the voting rule is too conservative from the perspective of an average small shareholder. Then a more activist blockholder becomes a countervailing force against this conservative bias and increases the aggregate payoff of small shareholders. Notably, this happens even though the distance between the blockholder and the average small shareholder increases with β. Hence, what ultimately matters for small shareholders is not whether the blockholder is closer to them, but whether he moves the median voter closer to them.[24] At the same time, the increased distance between a more activist blockholder and the median voter increases the voting premium in both scenarios (Proposition 3).

Part (ii.a) describes a symmetric case, in which $\beta < \underline{\beta}$. The intuition in this case is the mirror image of the intuition for part (i) and is omitted for brevity. Finally, part (ii.b) gives sufficient conditions for the voting premium and the payoff of small shareholders to move in opposite directions. In this case, a larger voting premium is associated with a larger conflict between the blockholder and the small shareholders of the firm, consistent with the commonly expressed intuition.[25]

[24] The additional requirement of $\mathbb{E}[b] < \beta$ in part (i) of Proposition 4 is a sufficient condition that guarantees that the gains from trade of small shareholders (which contribute to W^*) also increase in β.

[25] Unlike part (i) of Proposition 4, parts (ii.a) and (ii.b) require additional conditions on α. In particular,

Divergence among dispersed shareholders. A positive voting premium emerges in our setting only if the blockholder is able to move the median voter. The next result shows that this can happen only if there is some heterogeneity of preferences among dispersed shareholders.

Corollary 2 *Suppose that all dispersed shareholders have the same bias b, which differs from that of the blockholder, $b \neq \beta$. Then the voting premium is zero.*

The situation described in Corollary 2 could arise in a setting in which dispersed shareholders have the same valuation of cash flows and the blockholder can reduce these cash flows by diluting the assets of the firm if the proposal is accepted. Then all dispersed shareholders have the same b and $b < \beta$, since the blockholder benefits more from the proposal. This example is discussed in more detail in Section B of the Online Appendix, where we also discuss an example for the opposite case with $b > \beta$.

The key point is that in our setting, the blockholder does not acquire majority control. If small shareholders are homogeneous, the median voter is always a small shareholder and the blockholder does not have the ability to change his bias. Hence, heterogeneity among dispersed shareholders is critical for the existence of a positive voting premium in our setting.

5.4.4 Liquidity of voting vs. non-voting shares

Price impact is often used as a measure of liquidity. Liquidity measured in this way is endogenous in our setting and arises because voting rights are bundled with cash flow rights in voting shares. As a consequence, the liquidity of voting shares generally differs from that of non-voting shares. To see this, note from (16) that the total price impact of the blockholder's trade is

$$\frac{dp^*}{dy} = \gamma + \frac{\partial p^*}{\partial(-q^*)}\frac{\partial(-q^*(y))}{\partial y} = \gamma + \underbrace{(\mathbb{E}[b] + q^*)\, f(q^*)}_{\textit{price impact of a vote}}\frac{\partial(-q^*(y))}{\partial y}. \tag{35}$$

The first term, γ, reflects dispersed shareholders' trading costs and would be present even absent voting considerations, e.g., for non-voting shares. The second term reflects the indirect effect through the influence of the blockholder's trades on the median voter, $-q^*(y)$, and is pro-

requiring $\alpha > \underline{\alpha}$ in (ii.a) (and a sufficiently small α in (ii.b)) guarantees that the bias of the median voter increases (decreases) with β. For more details on the role of α when $\beta < \underline{\beta}$, see the discussion of Lemma 4 in the Online Appendix.

portional to the price impact of a vote in equation (29). The sign of the indirect effect depends on whether the resulting change in the median voter benefits or hurts dispersed shareholders.

If the blockholder's and dispersed shareholders' interests are aligned (e.g., $-q^*(y^*) < \min\{\beta, \mathbb{E}[b]\}$), then the blockholder's trades move the median voter in the direction preferred by both (see Section 5.4.3 for a discussion of two scenarios in which this can occur). This benefits dispersed shareholders and increases the price at which they are willing to supply additional shares; accordingly, the price impact of a vote is positive. Essentially, dispersed shareholders *free-ride* on the blockholder's trades. As a result, the supply function is steeper, the price impact of the blockholder's trades is amplified, and the liquidity of a voting share is smaller compared to a scenario without voting considerations.

By contrast, if the blockholder's and dispersed shareholders' interests are in conflict (e.g., $\mathbb{E}[b] < -q^*(y^*) < \beta$), then the blockholder's trades move the median voter away from dispersed shareholders. This hurts dispersed shareholders and reduces the reservation price at which they are willing to supply additional shares, so the price impact of a vote (29) is negative. Accordingly, the supply function is flatter, the price impact of the blockholder's trades is attenuated, and the liquidity of a voting share is now greater than that of a non-voting share.

5.4.5 Price impact and the sign of the voting premium

Based on (27), the incentive part of the voting premium is a combination of the marginal benefit of a vote (28) and the price impact of a vote (29). Thus, although the marginal benefit of a vote is always positive, the blockholder only benefits from buying additional shares if his price impact is not too strong. In particular, the voting premium is positive only if moving the median voter increases the value of the blockholder's stake by more than it increases the costs of his trades. This argument has important implications for the sign of the voting premium.

Conflict and a positive voting premium. If the blockholder's trades move the median voter in the direction that hurts dispersed shareholders, the voting premium is positive. This is because in this case, a more activist median voter not only increases the value of the blockholder's stake, but also reduces the price he has to pay to dispersed shareholders for a marginal share. Interestingly, this also implies that the voting rights embedded in the shares have opposite effects on the *price* of the shares and the *price impact* of trades: the embedded voting rights increase the price of the shares but decrease the price impact of trades.

Free-riding and a negative voting premium. If the blockholder's trades move the median voter in the direction that benefits dispersed shareholders, dispersed shareholders free-ride on the blockholder's trades and increase the price at which they are willing to sell shares. As a result, the blockholder's combined incentives to move the median voter, and hence the voting premium, decrease. In general, this does not prevent a positive voting premium. For example, if the blockholder has a positive endowment α and the trading frictions γ are large (as under the assumptions of Proposition 2), then the voting premium is always non-negative. Intuitively, in this case, the marginal benefit of a vote is sufficiently large because the blockholder's ability to move the median voter in his preferred direction increases the value of his entire post-trade stake $\alpha + y$, whereas his price impact only applies to his trade y. However, if the blockholder has no endowment, then a negative voting premium may arise:

Proposition 5. *Suppose $\alpha = 0$. There exists $\overline{\gamma} < \infty$ such that if $\gamma > \overline{\gamma}$, then the equilibrium exists and is unique. In equilibrium, the voting premium is negative if and only if $\mathbb{E}[b] < \beta < G^{-1}(1-\tau)$. In this case, the blockholder buys shares ($y^* > 0$) and the share price exhibits a negative voting premium: $p^* < p_{CF}(q^*(y^*))$ in (25).*

Intuitively, if $\beta < G^{-1}(1-\tau)$, then the blockholder is less activist than the median voter, so the median voter becomes less activist as the blockholder buys shares, $\frac{\partial(-q^*(y))}{\partial y} < 0$. Moreover, (27)-(29) continue to hold for $\alpha = 0$ and imply that the voting premium $VP(y)$ simplifies to

$$\frac{\partial(-q^*(y))}{\partial y} y \left[\frac{\partial v(\beta, q^*)}{\partial(-q^*)} - \frac{\partial p^*}{\partial(-q^*)}\right] \frac{\gamma}{2\gamma + \eta} = \frac{\partial(-q^*(y))}{\partial y} y (\beta - \mathbb{E}[b]) f(q^*(y)) \frac{\gamma}{2\gamma + \eta}, \quad (36)$$

which is simply the marginal effect of moving the median voter on the blockholder's net trading profits and equals the relative sensitivity of the blockholder's and small shareholders' valuations to a change in the median voter. Thus, the blockholder's net marginal payoff from moving the median voter is negative if the average dispersed shareholder is even *less* activist than the blockholder ($\mathbb{E}[b] < \beta$): Then a less activist median voter increases the valuation of the average dispersed shareholder, and thereby the stock price, even more than the valuation of the blockholder.

The scenario in Proposition 5 obtains because the interests of the blockholder and those of dispersed shareholders are aligned, but dispersed shareholders are more extreme and benefit more from the resulting change in the voting outcome. For example, the blockholder may be reluctant to support a certain management proposal, but dispersed shareholders may be more strongly biased against this proposal than the blockholder himself. Then, even though the blockholder values the voting rights per se (the marginal benefit of a vote (28) is positive), his overall incentives to accumulate voting rights become negative. As a result, the blockholder buys fewer shares than if he could buy cash flow rights separately, i.e., if they were not bundled with voting rights. Thus, free-riding by dispersed shareholders results in a negative voting premium if the blockholder has no endowment and cares only about his trading profits.

The negative voting premium is directly related to the differential liquidity of voting and non-voting shares discussed in Section 5.4.4. If the price impact of trading voting shares is much stronger than the price impact of trading non-voting shares (the second term in (35) is large), then the blockholder's demand for voting shares can be smaller than his demand for non-voting shares, even though he values the voting rights per se. This results in a negative premium on the price of voting shares or, in some sense, an "illiquidity discount" due to the attached voting rights. Moreover, while Propositions 2 and 5 imply that for large γ, a negative voting premium arises only when the blockholder has no initial endowment ($\alpha = 0$), the existence of a negative voting premium is more general: the same intuition implies that if γ is not too large, a negative voting premium can also arise for small but strictly positive α.

6 Dual-class shares

In this section, we investigate how the MPV is related to measures of the voting premium estimated in the empirical literature (see Section 8). To do so, we extend our model to a setting with two classes of shares with different voting rights. Specifically, suppose that in addition to the traded voting shares, investors can also trade non-voting shares. This setting can capture companies with a dual-class share structure (e.g., Zingales (1995); Nenova (2003)).

We assume that the blockholder and each dispersed shareholder are endowed with $\hat{\alpha} \in [0, 1]$ and $\hat{e} \in [0, 1 - \hat{\alpha}]$ non-voting shares, respectively, so that the total number of outstanding non-voting shares lies in the interval $[0, 1]$. Notice that we allow for the supply of non-voting shares to be zero (i.e., $\hat{\alpha} = \hat{e} = 0$), which could capture the creation of non-voting securities in derivatives markets. Denote by $\hat{x}$ and $\hat{y}$ the trades of dispersed shareholders and the blockholder in the non-voting shares, respectively. The utility of dispersed shareholders is given by

$$\hat{u}\left(d, \theta, b, x, \hat{x}; \gamma, e, \hat{e}\right) = \left(e + x\right) v\left(d, \theta, b\right) - \frac{\gamma}{2} x^2 + \left(\hat{e} + \hat{x}\right) v\left(d, \theta, b\right) - \frac{\gamma}{2} \hat{x}^2, \tag{37}$$

which means that in this extension, γ is best interpreted as capturing trading costs, rather than as risk aversion. We assume that the trading costs are the same for these two securities, to make sure that the price differential between voting and non-voting shares does not stem from differences in the microstructure of these markets. Similarly, the blockholder's utility is given by $u\left(d, \theta, \beta, y, \hat{y}; \eta, \alpha, \hat{\alpha}\right)$. Notice that in principle, shorting of non-voting shares is feasible. However, we assume that the trading costs γ and η are large enough, so that $e + x + \hat{e} + \hat{x} > 0$ and $\alpha + y + \hat{\alpha} + \hat{y} > 0$, i.e., the net cash flow exposure of each investor is always non-negative.

Suppose first that the decision rule q^* is exogenous, similar to the hypothetical scenario considered in Section 5. With an exogenous decision rule q^*, the trading strategies of all investors in each market are given by the expressions in Proposition 2, assuming that $\frac{\partial(-q^*(y))}{\partial y} = 0$, and hence, $MPV = 0$. Indeed, if the decision rule is not affected by trading, then the existence of the market for non-voting shares does not affect trading in the market for voting shares, and vice versa. This is because investors have no budget constraints and the trading costs apply to each market separately. Moreover, although the endowments of non-voting shares could be different from the endowments of voting shares, the trading quantities are the same as in our model, since they are invariant to the levels of the endowment. This observation implies that with an exogenous cutoff q^*, the prices of voting and non-voting shares must be identical. Indeed, given $(y, \hat{y})$ and q^*, the difference in prices is

$$p\left(y, q^*\right) - p\left(\hat{y}, q^*\right) = \gamma y + v\left(\mathbb{E}\left[b\right], q^*\right) - \left(\gamma \hat{y} + v\left(\mathbb{E}\left[b\right], q^*\right)\right) = \gamma\left(y - \hat{y}\right),$$

and since $y = \hat{y}$, the two prices are the same.

Next, consider the model where q^* is determined endogenously by voting. By assumption, the net positions of dispersed shareholders and the blockholder are always non-negative. Therefore, a dispersed shareholder with bias b votes for the proposal if and only if $q + b > 0$, and the blockholder votes for the proposal if and only if $q + \beta > 0$. Note also that for a given q^* and

$(y, \hat{y})$, the trading strategies of dispersed shareholders in the voting and non-voting shares are the same as in the baseline model. Thus, the identity of the median voter as a function of the blockholder's trade, namely $q^*(y)$, is determined as in the baseline model (see Proposition 1). This implies that given y, the median voter is unaffected by $\hat{y}$, i.e., the trades that take place in the market for non-voting shares. However, the presence of non-voting shares changes the blockholder's trades of voting shares, because he internalizes the effect of the voting outcome on the value of his non-voting shares. The objective of the blockholder becomes:

$$\max_{y,\hat{y}} \Pi(y, \hat{y}) = (\alpha + y) v(\beta, q^*(y)) - yp^*(y) - \frac{\eta}{2}y^2 \tag{38}$$

$$+ (\hat{\alpha} + \hat{y}) v(\beta, q^*(y)) - \hat{y}\hat{p}^*(\hat{y}) - \frac{\eta}{2}\hat{y}^2. \tag{39}$$

We obtain the following result:

Proposition 6 (Dual-class shares) *If the blockholder and dispersed shareholders can trade in voting and non-voting shares, the voting premium is:*

$$p^*_{voting} - p^*_{non-voting} = \gamma(y^* - \hat{y}^*) = \frac{\gamma}{2\gamma + \eta} MPV(y^*, \hat{y}^*), \tag{40}$$

where

$$MPV(y, \hat{y}) = \frac{\partial(-q^*(y))}{\partial y} f(q^*(y)) \left[(\alpha + \hat{\alpha})(q^*(y) + \beta) + (y + \hat{y})(\beta - \mathbb{E}[b])\right]. \tag{41}$$

Proposition 6 shows that the dual-class voting premium is proportional to $MPV(y, \hat{y})$, which is the analog of (26) in the baseline model. Thus, the blockholder's marginal payoff from buying voting rights translates into an actual price difference between voting and non-voting shares.[26]

Note also that $MPV(y^*, \hat{y}^*)$ depends on $\hat{y}^*$, which means that the volume of trades in the market for non-voting shares affects the blockholder's incentives to buy voting shares, and hence the voting premium. Intuitively, the blockholder's position in non-voting shares gives him additional incentives to change the median voter for the same reasons as his position

[26] The price differential, and hence the positive voting premium, is possible since the number of investors who can demand shares in our model is bounded, namely, there are limits to arbitrage.

in voting shares and increases his marginal benefit of a vote, but it also changes dispersed shareholders' valuations and thus the price he has to pay for non-voting shares.

7 Extensions

In this section, we briefly discuss several other implications of the baseline model and its extensions. The complete analysis and explanation of these results is in the Online Appendix.

Exit and a positive voting premium. In the context of our baseline model, we show that although the blockholder has the power to gain influence over the voting outcome by buying additional shares, he may nevertheless choose to do the opposite: sell shares to dispersed shareholders and thereby give up his influence over the voting outcome, while demanding a premium from the dispersed shareholders. Thus, the tension between exit and voice (e.g., Hirschman (1970)) also exists in our model, which demonstrates that the incentives to exit can prevail even when the voting premium is positive. Hence, a positive voting premium does not necessarily indicate a more concentrated ownership structure. This analysis is presented in Section C.2 of the Online Appendix.

Vote trading. Our baseline model focuses on the case where one security has voting rights bundled with cash flow rights. In practice, votes can often be traded separately from cash flow rights, for example, through share lending. In Section D.1 of the Online Appendix, we extend the model by adding a second market in which voting rights are traded separately. Since dispersed shareholders are never pivotal for the voting outcome, they are willing to supply their votes for an arbitrarily small price. Hence, the price of a vote is zero in this setting. However, as long as the market for votes provides only limited ability for the blockholder to accumulate voting power, the voting premium is still strictly positive. Thus, the price of a separately traded vote is conceptually different from the voting premium for a share that combines cash flow and voting rights.

Influence premium. In practice, blockholders can exert influence even without having formal control rights if they can influence the company's management. In Section D.2 of the

Online Appendix, we analyze a version of the model in which decisions are taken by management rather than by voting, and management gives some weight to the value of its current shareholder base, represented by the preferences of the post-trade average shareholder. Then the blockholder's trades influence decisions by changing these preferences, and the blockholder values this influence, which may give rise to an "influence premium" on the share price. The influence premium is different from the voting premium and can even be larger. In particular, accumulating more shares always increases the blockholder's influence on management, but it does not always increase the blockholder's impact if decisions are taken by a shareholder vote.

Multiple blockholders. Section D.3 of the Online Appendix generalizes our model to the case with multiple blockholders. We show that if they share the same preferences toward the proposal, then the voting premium declines as the number of blockholders increases. In contrast, if the blockholders are sufficiently heterogeneous, their trades pull the median voter in opposite directions. Then, as the blockholders' biases become more extreme, each blockholder tries harder to gain influence over the voting outcome, which results in a higher voting premium.

Vote participation Implicit to our analysis is that dispersed shareholders participate in the vote even though they do not expect to be pivotal for the outcome. In practice, institutional investors vote their shares even if they are unlikely to be pivotal to avoid being accused for breaching their fiduciary duties to their ultimate investors. Retail investors also vote, but at a relatively low rate (Brav, Cain, and Zytnick (2022)). In general, investors with stronger views about the proposal (i.e., larger $|b|$) are expected to participate and vote with a higher probability. We illustrate how such selective participation can be incorporated into our analysis in Section D.4 of the Online Appendix. We show that it changes the composition of voter base, and as a result, the identity of the median voter. Other than that, our analysis can be performed as in the baseline model and generate similar qualitative results.

8 Empirical implications and measures of the voting premium

There is a large empirical literature that provides measures of the voting premium and analyses of the cross-sectional and time-series variation of the voting premium. The purpose of this section is to locate the model developed above in the context of the existing empirical evidence and, conversely, shed some light on the empirical discussion by exploring the implications of our model. Specifically, it is not the purpose of this section to offer a comprehensive survey of empirical studies and methodologies and their potential strengths and shortcomings.[27]

Broadly, there are five major strategies that have been developed in the literature to measure the voting premium and the economic value of voting power. We survey 40 studies in more detail in Table 1 in the Appendix and provide a summary in the table below. Of these studies, 15 use data on the US, 4 on Germany, 3 on Italy, 3 are cross-country studies, and the rest provide evidence on 11 other countries.

Methodology	Avg. (%)	Median (%)	Number of studies
Dual-class shares	23.59	14.53	23
Block-trade premium	41.50	29.55	9
Option replication	0.20	0.16	5
Equity lending	0.01		2
Record-day trading	0.09	0.12	1

The most salient feature of these studies is that they report very divergent estimates of the voting premium. Below, we first discuss why estimates of the voting premium may vary across methodologies (Section 8.1) and then the cross-sectional variation of voting premiums within methodologies (Section 8.2) and relate them to our model.

[27] Some papers already contain surveys of different strands of this literature. Rydqvist (1992) provides an early survey of studies on dual-class shares and Dittmann (2004), Adams and Ferreira (2008), and Kind and Poltera (2013) provide more recent updates.

8.1 Differences across methodologies

Marginal values vs. block values. Most methods to estimate the voting premium measure the value of a marginal vote. This applies to all methods that rely on stock market prices, i.e., all methods except for the block-trade premium. By contrast, block trades reveal the average valuation of a voting right for the entire block. The table above shows that block trades are associated with significantly larger premiums (average: 41.50%; median: 29.55%) than found in studies of dual-class share premiums (average: 23.59%; median: 14.53%) or those using the three other methods. Based on our model, we would expect the blockholder's willingness to pay for an entire block of shares to be larger than his willingness to pay for an additional voting share. In particular, the equilibrium MPV in our model may equal zero if the blockholder is the median voter at his equilibrium trading amount y^* (Proposition 2), resulting in a zero dual-class share premium (Proposition 6, equation (40)). However, his average, per-share willingness to pay for a block of votes of size y^* may be much larger.

Voting yields and capitalized voting premiums. In addition, it is salient from the table that studies relying on dual-class shares and block-trades obtain much larger estimates than the other three methods. We attribute this to the fact that the former two methods capitalize the value of the voting right over longer time horizons, which span potentially infinitely many future shareholder meetings. In contrast, the three other studies estimate the voting yield, which captures a period of one year or less. In the Online Appendix, we calibrate a simple valuation model and show that once the difference in the time horizon is accounted for, the estimates from these two sets of methods are in fact consistent with each other.

Separate vs. joint trading of cash flow and voting rights. Another important difference between the methodologies is whether they estimate the price of the vote that is traded separately (as in the equity lending market) or the price of the vote that is traded in conjunction with cash flow rights (as in the comparison of two classes of stock with differential voting rights). Our analysis emphasizes that the two types of methodologies could give very different estimates of the price of the vote. Indeed, in the extension to a separate market for votes discussed above, we show that the premium on the price of voting shares could be strictly positive even if the price of a separately traded vote is zero.

8.2 The cross-sectional variation in the voting premium

This section offers observations on the cross-sectional variation of the voting premium and discusses them in the context of our model.

Negative values of the voting premium. One implication from our analysis is that the voting premium can sometimes be negative, which emanates from the free-rider effect and the possibly substantial price impact of the blockholder's trades (see Proposition 5 and the related discussion). Interestingly, while the estimates of the mean and median of the voting premium in the studies surveyed in Table 1 are always positive, many studies report that the voting premium is negative for some companies.[28] These findings are consistent with our model, but are difficult to interpret in the context of extant theories. Empirical studies often explain them by pointing out that voting shares may suffer from a liquidity discount relative to non-voting shares.[29] This explanation is in line with our argument: the negative voting premium in our model arises exactly because the trading of voting shares has a stronger price impact than the trading of non-voting shares. If we define liquidity as price impact, then our model predicts the differential liquidity of voting and non-voting shares, which arises because the blockholder's accumulation of voting rights changes dispersed shareholders' valuations.

Voting premiums, takeovers, and shareholder meetings. One of the standard explanations for how the blockholder's willingness to pay a premium for voting control is translated into higher prices for voting shares is the takeover mechanism, and several empirical studies find support for this explanation.[30] However, this theory has some limitations. First, since the 1990s, many countries have enacted coattail provisions, which mandate equal treatment of all classes of shares in control changes (Maynes (1996); Nenova (2003)). Second, Dittmann (2004) surveys 12 studies of companies with dual-class share structures and shows that if investors would correctly anticipate the ex-post frequencies of takeovers and takeover premiums

[28] E.g., see Rydqvist (1996), Nenova (2003), and Caprio and Croci (2008) for the dual-class share premium and Albuquerque and Schroth (2010) and Albuquerque and Schroth (2015) for the block trading premium.

[29] Odegaard (2007) separates liquidity effects from control effects in Norway, which used to have three classes of shares that differed in their voting rights and the possibility of foreign ownership.

[30] For models, see Grossman and Hart (1988); Harris and Raviv (1988); Bergström and Rydqvist (1992). For empirical evidence see Bergström and Rydqvist (1992); Zingales (1995); Rydqvist (1996); Smith and Amoako-Adu (1995).

paid, then the premium on voting shares in dual-class firms should be smaller by about one order of magnitude compared to the observed premium in most countries. Hence, the takeover explanation is probably only a partial explanation of premiums on voting shares.

Differently from this argument, our analysis shows how the voting premium can arise without contests for majority control, and solely as a result of blockholders' desire to influence the voting outcomes at shareholder meetings. This prediction is consistent with the findings of the more recent literature, which analyzes the time-series variation in the voting premium and finds that the voting premium is largest around shareholder meetings compared to other periods of the year (see Kind and Poltera (2013); Kalay, Karakas, and Pant (2014); Kind and Poltera (2017); Fos and Holderness (2020)).

Voting premiums and ownership structure. Studies on the relationship between the voting premium and ownership concentration show that it is often non-monotonic: the value of voting rights is small both if ownership is very dispersed and if it is very concentrated with one blockholder who has majority control (Kind and Poltera (2013)). Therefore, one common methodology uses the probability of being pivotal inferred from oceanic Shapley values instead of ownership concentration to predict the voting premium.[31] Our analysis in Section D.3 of the Online Appendix suggests a new empirical direction by showing that it is not only the concentration of ownership and the probability of being pivotal that matter, but also the preferences of blockholders. Specifically, if blockholders have similar preferences, then ownership concentration is positively correlated with the voting premium, and if blockholders disagree with each other, the voting premium increases the more they disagree.

9 Conclusion

We develop a theory of voting and trading in which a blockholder and dispersed shareholders trade with each other and then vote on a proposal. We analyze the trading decisions of blockholders, when they would be willing to pay a higher price in order to accumulate voting power, and how their trades translate into a premium for voting shares. The model generates

[31] The method was pioneered by Rydqvist (1987) and is based on the theory of oceanic Shapley values of Milnor and Shapley (1978). For applications, see Zingales (1994), Zingales (1995), Chung and Kim (1999), Caprio and Croci (2008), and Nenova (2003).

a number of insights about the voting premium and the equilibrium ownership structure of the firm.

We find that the voting premium does not reflect the economic value of voting rights to the blockholder, and that it is also unrelated to the voting power of the blockholder. Moreover, common measures of the voting premium may often underestimate the true value of voting rights to their owners. Our analysis also shows that a negative voting premium can arise when dispersed shareholders free-ride on the blockholder's trades, and that liquidity of voting shares can be different from that of non-voting shares. We extend the model to explore the role of the market for votes and the interaction between multiple blockholders. Overall, our analysis emphasizes how asset prices are affected by blockholders' desire to move the voting outcome in their preferred direction when voting and cash flow rights are bundled in shares.

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Appendix - Proofs

Proof of Lemma 1. Given the realization of q, a shareholder indexed by b votes his shares for the proposal if and only if $q > -b$. Denote the fraction of post-trade shares voted to approve the proposal by $\Lambda(q)$. Note that $\Lambda(q)$ is weakly increasing. If we have $\Lambda(\Delta) \leq \tau$ for the highest possible $q = \Delta$, then q^* in the statement of the lemma is equal to Δ. Similarly, if we have $\Lambda(-\Delta) > \tau$ for the lowest possible $q = -\Delta$, , then q^* in the statement of the lemma is equal to $-\Delta$. Finally, if $\Lambda(-\Delta) \leq \tau < \Lambda(\Delta)$, there exists $q^* \in [-\Delta, \Delta)$ such that the fraction of votes voted in favor of the proposal is greater than τ if and only if $q > q^*$. Hence, the proposal is approved if and only if $q > q^*$. ■

Proof of Proposition 1. Recall that $s(q; y, q_e^*)$, given by (13), is increasing in q. Denote

$$\begin{aligned} \beta_l(y, q_e^*) &\equiv -s^{-1}(\tau; y, q_e^*), \qquad (42) \\ \beta_h(y, q_e^*) &\equiv -s^{-1}(\tau - \alpha - y; y, q_e^*), \qquad (43) \end{aligned}$$

As follows from the arguments prior to the proposition, the proposal is approved if and only if $q > -b_{MV}(\beta, y, q_e^*)$, where

$$b_{MV}(\beta, y, q_e^*) = \begin{cases} \beta_l(y, q_e^*) & \text{if } \beta \leq \beta_l(y, q_e^*) \\ \beta & \text{if } \beta_l(y, q_e^*) < \beta \leq \beta_h(y, q_e^*) \\ \beta_h(y, q_e^*) & \text{if } \beta_h(y, q_e^*) < \beta \end{cases} \qquad (44)$$

is the identity of the median voter.

Since shareholders' expectations q_e^* at the trading stage have to be consistent with the actual decision rule at the voting stage, the equilibrium at the voting stage can be characterized as follows: the proposal is approved if and only if $q > q^*(y)$, where

$$\begin{aligned} -q^*(y) &= \begin{cases} \beta_L(y) & \text{if } \beta < \beta_L(y) \\ \beta & \text{if } \beta_L(y) < \beta < \beta_H(y) \\ \beta_H(y) & \text{if } \beta_H(y) < \beta \end{cases} \qquad (45) \\ &= \begin{cases} \max\{\beta, \beta_L(y)\} & \text{if } \beta < \beta^* \\ \min\{\beta, \beta_H(y)\} & \text{if } \beta > \beta^*, \end{cases} \qquad (46) \end{aligned}$$

is the identity of the median voter, $\beta_L(y)$ and $\beta_H(y)$ are the solutions of

$$\begin{aligned} s(-\beta_L; y, -\beta_L) &= \tau, \qquad (47) \\ s(-\beta_H; y, -\beta_H) &= \tau - \alpha - y, \qquad (48) \end{aligned}$$

and $\beta^* \equiv \beta_H(-\alpha) = \beta_L(-\alpha)$.

Condition (47) can be rewritten from (13) as

$$\beta_L = \beta_l(y, -\beta_L) \Leftrightarrow R(\beta_L; y, -\beta_L) = 1 - \frac{\tau}{1 - \alpha - y}. \qquad (49)$$

Similarly, condition (48) can be rewritten as

$$\beta_H = \beta_h(y, -\beta_H) \Leftrightarrow R(\beta_H; y, -\beta_H) = 1 - \frac{\tau - \alpha - y}{1 - \alpha - y}. \qquad (50)$$

From (12), $R(b';y,q^*)$ is a cdf and lies in the unit interval. Moreover,

$$\lim_{\beta\to-\bar{b}} R(\beta;y,-\beta)=0 \text{ and } \lim_{\beta\to\bar{b}} R(\beta;y,-\beta)=1. \tag{51}$$

Hence, solutions to (49) and (50), and, therefore, of (47) and (48), must exist. For $y=-\alpha$, the right hand sides of (49) and (50) are identical and β^* is defined from $R(\beta^*;y,-\beta^*)=1-\tau$.

The derivative of $R(\beta;y,-\beta)$ with respect to β is :

$$\begin{aligned}\frac{\partial R(\beta;y,-\beta)}{\partial\beta} &= g(\beta)\left(1+\frac{\beta-\mathbb{E}[b]}{\gamma}\frac{H(-\beta)}{1-\alpha-y}\right)\\ &\quad -G(\beta)\frac{f(-\beta)}{1-\alpha-y}\frac{\mathbb{E}[b]-\mathbb{E}[b|b<\beta]}{\gamma}.\end{aligned} \tag{52}$$

The first line of (52) equals $r(\beta;y,-\beta)>0$. Since, $\mathbb{E}[b]>\mathbb{E}[b|b<\beta]$, the second line is negative. Hence, $R(\beta;y,-\beta)$ and $s(-\beta;y-\beta)$ may be non-monotonic in β. From (52), $\frac{\partial R(\beta;y,-\beta)}{\partial\beta}>0$ if and only if

$$\frac{\frac{G(\beta)}{g(\beta)}f(-\beta)(\mathbb{E}[b]-\mathbb{E}[b|b<\beta])+H(-\beta)(\mathbb{E}[b]-\beta)}{1-\alpha-y}<\gamma,$$

and thus, there exists $\overline{\gamma}<\infty$ such that if $\gamma>\overline{\gamma}$, then $\frac{\partial R(\beta;y,-\beta)}{\partial\beta}$ for every $y\geq-\alpha$. In this case, (51) implies that the solutions to (47)-(48) exist and are unique. Lemma 2 derives the properties of $\beta_L(y)$, $\beta_H(y)$ in this case, and in particular shows that $\beta_L(y)$ is decreasing and $\beta_H(y)$ is increasing in y.

Combining the properties of $\beta_L(y)$, $\beta_H(y)$ from Lemma 2 with the arguments above, it follows that there exists $\overline{y}$ such that if $y>\overline{y}$, the median voter is the blockholder ($-q^*(y)=\beta$), whereas if $y<\overline{y}$, the median voter is a dispersed shareholder with bias $-q^*(y)\neq\beta$, where $|q^*(y)+\beta|$ is decreasing in y.

Figure A1 plots the median voter (vertical axis) as a function of the blockholder's trade y (horizontal axis). The figure shows that function $\beta_H(y)$ ($\beta_L(y)$) is upward (downward) sloping, starting at β^* for $y=-\alpha$ and reaching a maximum of $\bar{b}$ (minimum of $-\bar{b}$) as y approaches $\tau-\alpha$ $(1-\alpha-\tau)$; this property is shown in the proof of Lemma 2.

The left panel of the figure considers the case in which the blockholder's bias is $\beta>\beta^*$. For any trade $y<y_H\equiv\beta_H^{-1}(\beta)$, the median voter is a dispersed shareholder whose bias is strictly increasing in y, whereas for any $y\geq y_H$, the median voter is the blockholder himself. Hence, the median voter is given by $\min\{\beta,\beta_H(y)\}$, shown as the bold line in the figure. It follows that by choosing the appropriate trade in the interval $[-\alpha,y_H]$, the blockholder can change the identity of the median voter to be any point in the interval $[\beta^*,\beta]$. In particular, by buying more (or selling fewer) shares, the blockholder can push the bias of the median voter closer to β. However, the blockholder cannot choose a median voter outside of the interval $[\beta^*,\beta]$. Intuitively, the median voter cannot be larger than β because the blockholder's optimal voting strategy (to support the proposal whenever $q>-\beta$) prevails when his stake becomes sufficiently large. Likewise, the median voter cannot be lower than β^* because the collective preferences of dispersed shareholders prevail when the blockholder exits his position.

The right panel of the figure considers the case in which the blockholder's bias is $\beta<\beta^*$, which is a mirror image of the left panel. In particular, the blockholder can influence the identity of the median voter to be any point in the interval $[\beta, \beta^*]$ by choosing the appropriate trade in the interval $[-\alpha, y_L]$.

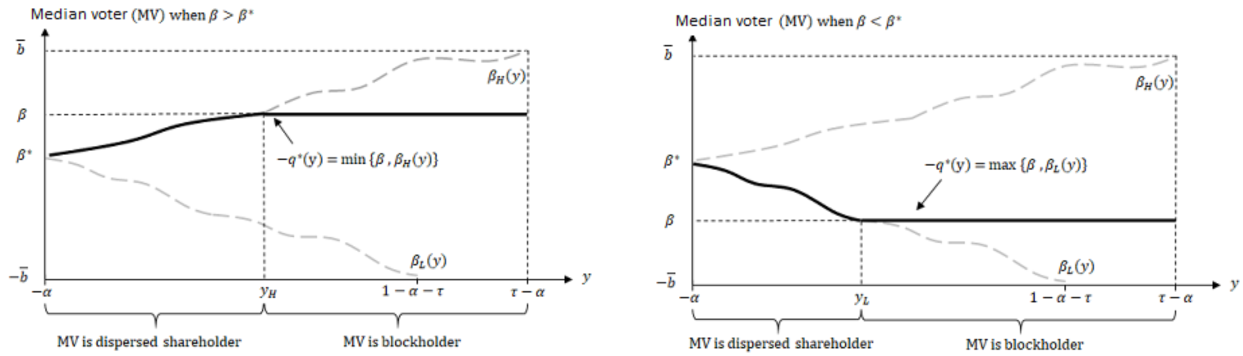

Figure A1 - median voter as a function of the blockholder's trade ■

Lemma 2 (Properties of the median voter) *Suppose that the solutions $\beta_L(y)$ and $\beta_H(y)$ of (47) and (48) are unique for all y. Then:*

(i) $\frac{\partial R(\beta_L; y, -\beta_L)}{\partial \beta_L} > 0$ *and*

$$\frac{\partial \beta_L(y)}{\partial y} = -\frac{\frac{1-G(\beta_L)}{1-\alpha-y}}{\frac{\partial R(\beta_L; y, -\beta_L)}{\partial \beta_L}} < 0. \tag{53}$$

Moreover, $\lim_{y \nearrow 1-\tau-\alpha} \beta_L(y) = -\underline{b}$ *and* $\lim_{y \nearrow 1-\tau-\alpha} \frac{\partial \beta_L}{\partial y} = -\left[\tau \frac{\partial R(\beta_L; y, -\beta_L)}{\partial \beta_L}\right]^{-1}$.

(ii) $\frac{\partial R(\beta_H; y, -\beta_H)}{\partial \beta_H} > 0$ *and*

$$\frac{\partial \beta_H(y)}{\partial y} = \frac{\frac{G(\beta_H)}{1-\alpha-y}}{\frac{\partial R(\beta_H; y, -\beta_H)}{\partial \beta_H}} > 0. \tag{54}$$

Moreover, $\lim_{y \nearrow \tau-\alpha} \beta_H(y) = \underline{b}$ *and* $\lim_{y \nearrow \tau-\alpha} \frac{\partial \beta_H(y)}{\partial y} = \left[(1-\tau) \frac{\partial R(\beta_H; y, -\beta_H)}{\partial \beta_H}\right]^{-1}$.

(iii) *If the blockholder sells all her shares ($y = -\alpha$), then $\beta_L(-\alpha) = \beta_H(-\alpha) \equiv \beta^*$.*

(iv) *As γ becomes large, we have:*

$$\lim_{\gamma \to \infty} \beta_L(y) = G^{-1}\left(1 - \frac{\tau}{1-\alpha-y}\right), \quad \lim_{\gamma \to \infty} \beta_H(y) = G^{-1}\left(\frac{1-\tau}{1-\alpha-y}\right), \tag{55}$$

and

$$\lim_{\gamma\to\infty}\frac{\partial\beta_L(y)}{\partial y} = -\frac{1-G(\lim_{\gamma\to\infty}\beta_L(y))}{g(\lim_{\gamma\to\infty}\beta_L(y))(1-\alpha-y)}, \tag{56}$$

$$\lim_{\gamma\to\infty}\frac{\partial\beta_H(y)}{\partial y} = \frac{G(\lim_{\gamma\to\infty}\beta_H(y))}{g(\lim_{\gamma\to\infty}\beta_H(y))(1-\alpha-y)}, \tag{57}$$

$$\lim_{\gamma\to\infty}\beta^* = G^{-1}(1-\tau). \tag{58}$$

Proof of Lemma 2. To simplify the expressions, define

$$X(b';y,q^*) = \frac{(\mathbb{E}[b]-\mathbb{E}[b\,|b\le b'])\,H(q^*)}{\gamma(1-\alpha-y)}. \tag{59}$$

With this definition, we have

$$R(b';y,q^*) = G(b')(1-X(b';y,q^*)) \Leftrightarrow -G(b')X(b';y,q^*) = R(b';y,q^*)-G(b') \tag{60}$$

and

$$\frac{\partial R(b';y,q^*)}{\partial y} = -\frac{X(b';y,q^*)G(b')}{1-\alpha-y}. \tag{61}$$

(i) If $\beta_L(y)$ is the unique solution to (47), then $\frac{\partial R(\beta_L;y,-\beta_L)}{\partial\beta_L} > 0$. We apply the implicit function theorem to condition (47), which requires

$$\begin{aligned}&\left[(1-\alpha-y)\frac{\partial R(\beta_L;y,-\beta_L)}{\partial y}+1-R(\beta_L;y,-\beta_L)\right]dy \\ +\quad&(1-\alpha-y)\frac{\partial R(\beta_L;y,-\beta_L)}{\partial\beta_L}d\beta_L = 0,\end{aligned} \tag{62}$$

where $\frac{\partial R(\beta_L;y,-\beta_L)}{\partial y}$ is given by the same expression as above and $R(\beta_L;y,-\beta_L)$ is again given from (49) so that

$$1-R(\beta_L;y,-\beta_L) = \frac{\tau}{1-\alpha-y}. \tag{63}$$

Substituting for $1-R(\beta_L;y,-\beta_L)$ and dividing by $1-\alpha-y$ allows us to rewrite (62) as

$$\left[\frac{\partial R(\beta_L;y,-\beta_L)}{\partial y}+\frac{\tau}{(1-\alpha-y)^2}\right]dy+\frac{\partial R(\beta_L;y,-\beta_L)}{\partial\beta_L}d\beta_L = 0. \tag{64}$$

Hence,

$$\frac{\partial\beta_L}{\partial y} = -\frac{\frac{\partial R(\beta_L;y,-\beta_L)}{\partial y}+\frac{\tau}{(1-\alpha-y)^2}}{\frac{\partial R(\beta_L;y,-\beta_L)}{\partial\beta_L}}. \tag{65}$$

We next use (61) and (63) to rewrite the numerator of (65) as

$$\begin{aligned}\frac{\partial R\left(\beta_L; y, -\beta_L\right)}{\partial y} + \frac{\tau}{\left(1-\alpha-y\right)^2} &= \frac{1}{1-\alpha-y}\left(-G\left(\beta_L\right) X\left(\beta_L, y, -\beta_L\right) + 1 - R\left(\beta_L; y, -\beta_L\right)\right) \\ &= \frac{1-G\left(\beta_L\right)}{1-\alpha-y} > 0,\end{aligned}$$

where the second transformation uses (60). Hence, $\frac{\partial \beta_L}{\partial y} < 0$ if $\frac{\partial R(\beta_L; y, -\beta_L)}{\partial \beta_L} > 0$.

For $y \nearrow 1-\tau-\alpha$, almost all dispersed shareholders are required to pass the proposal without the blockholder. Then $R \to 0$ and $\beta_L \to -\bar{b}$, $G\left(\beta_L\right) \to 0$, and $\frac{1-G(\beta_L)}{1-\alpha-y} \to \frac{1}{\tau}$. Then

$$\frac{\partial \beta_L}{\partial y} \to -\frac{1}{\frac{\partial R(\beta_L; y, -\beta_L)}{\partial \beta_L}\tau} < 0. \tag{66}$$

(ii) If $\beta_H\left(y\right)$ is the unique solution to (48), then $\frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H} > 0$. We apply the implicit function theorem to condition (50), which requires

$$\begin{aligned}&\left[\left(1-\alpha-y\right)\frac{\partial R\left(\beta_H; y, -\beta_H\right)}{\partial y} - R\left(\beta_H, y, -\beta_H\right)\right] dy \\ + \quad &\left(1-\alpha-y\right)\frac{\partial R\left(\beta_H; y, -\beta_H\right)}{\partial \beta_H} d\beta_H = 0.\end{aligned} \tag{67}$$

Substituting for $R\left(\beta_H, y, -\beta_H\right)$ from (50)and dividing by $1-\alpha-y$ gives

$$\left[\frac{\partial R\left(_H, y, -\beta_H\right)}{\partial \beta_H} - \frac{1-\tau}{\left(1-\alpha-y\right)^2}\right] dy + \frac{\partial R\left(\beta_H, y, -\beta_H\right)}{\partial \beta_H} d\beta_H = 0. \tag{68}$$

Hence,

$$\frac{\partial \beta_H}{\partial y} = -\frac{\frac{\partial R(\beta_H, y, -\beta_H)}{\partial y} - \frac{1-\tau}{(1-\alpha-y)^2}}{\frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H}}. \tag{69}$$

We use (61) and (50) to rewrite the numerator of (69) as

$$\begin{aligned}\frac{\partial R\left(\beta_H, y, -\beta_H\right)}{\partial y} - \frac{1-\tau}{\left(1-\alpha-y\right)^2} &= \frac{1}{1-\alpha-y}\left(-G\left(\beta_H\right) X\left(\beta_H, y, -\beta_H\right) - R\left(\beta_H, y, -\beta_H\right)\right) \\ &= -\frac{G\left(\beta_H\right)}{1-\alpha-y} < 0,\end{aligned}$$

where the second line uses (60). Hence, $\frac{\partial \beta_H}{\partial y} > 0$ in any equilibrium in which $\frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H} > 0$.

For $y \nearrow \tau - \alpha$, the number of dispersed shareholders needed to pass the proposal becomes negligible. Then (50) implies $R \to 1$ and (51) implies $\beta_H \to \bar{b}$ and $\mathbb{E}\left[b \left| b \leq \beta_h\right.\right] \to \mathbb{E}\left[b\right]$. Then

$\frac{-G(\beta_h)}{1-\alpha-y} \rightarrow \frac{-1}{1-\tau}$ and (69) simplifies to

$$\frac{\partial \beta_H}{\partial y} \rightarrow \frac{1}{\frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H}(1-\tau)} > 0. \tag{70}$$

(iii) If $y = -\alpha$, $\beta_h(y, q_e^*) = \beta_l(y, q_e^*)$ from (42) and (43), hence $\beta_L(y) = \beta_H(y)$, which are both assumed to be unique. Hence, β^* can be obtained as the unique solution to (47).

(iv) From (12), $\lim_{\gamma \to \infty} R(-q^*; y, q^*) = G(-q^*)$. Substituting into (49) and (50) gives (55). From (52), $\lim_{\gamma \to \infty} \frac{\partial R(-q^*; y, q^*)}{\partial(-q^*)} = g(-q^*)$. Substituting into (53) and (54) gives (56). ■

Proof of Proposition 2. We start by noting that given (4) and (9), we can rewrite

$$\begin{aligned}
\Pi(y) &= (\alpha + y) v(\beta, q^*(y)) - yp^*(y) - \frac{\eta}{2} y^2 \\
&= \alpha v(\beta, q^*(y)) + y(\beta - \mathbb{E}[b]) H(q^*) - (\gamma + \eta/2) y^2 \\
&= \alpha v_0 + \alpha \mathbb{E}[\theta | q > q^*(y)] H(q^*) + ((\alpha + y)\beta - yE[b]) H(q^*) - (\gamma + \eta/2) y^2,
\end{aligned}$$

which explains the derivation of

$$\frac{\partial \Pi(y)}{\partial y} = (\beta - \mathbb{E}[b]) H(q^*) - (2\gamma + \eta) y + \frac{\partial(-q^*(y))}{\partial y} [\alpha(q^*(y) + \beta) + y(\beta - \mathbb{E}[b])] f(q^*(y)),$$

as claimed in the main text. Note that $\frac{\partial(-q^*(y))}{\partial y}$ and $\frac{\partial \Pi(y)}{\partial y}$ do not exist when $\beta = \beta_L(y)$ or $\beta = \beta_H(y)$, which correspond to values y_L and y_H in Figure A1. In those cases, we interpret $\frac{\partial(-q^*(y))}{\partial y}$ as the right derivative of $-q^*(y)$, which is zero, and $\frac{\partial \Pi(y)}{\partial y}$ as the right derivative of $\Pi(y)$, which is $MPC(y)$.

Recall that by assumption, the blockholder's trade in equilibrium is in the interval $(-\alpha, 1-\alpha)$. So hereafter we assume $y \in (-\alpha, 1-\alpha)$. We start by giving sufficient conditions under which $\Pi(y)$ is "well-behaved," namely, continuous, concave, and has a unique maximizer. From Proposition 1, there exists a $\overline{\gamma}_1$ such that, if $\gamma > \overline{\gamma}_1$, then $\beta_L(y)$ and $\beta_H(y)$ are uniquely determined and both are continuous functions of y. If so, $\Pi(y)$ is a continuous function of y as well. In addition, Lemma 3 in the Online Appendix shows that there exists $\overline{\gamma}_2 < \infty$ such that if $\gamma > \overline{\gamma}_2$, then $\Pi(y)$ is a concave function. Combined, if $\gamma > \max\{\overline{\gamma}_1, \overline{\gamma}_2\}$, then $\Pi(y)$ is a continuous and concave function, and hence, it has a unique maximizer. We denote the unique maximizer by y^*.

Next, we define y^{**}. If y^* is such that $\beta_L(y^*) < \beta < \beta_H(y^*)$, then it must be $q^*(y^*) = -\beta$ and $\frac{\partial(-q^*(y))}{\partial y} = 0$. Therefore, using (18)-(19), y^* must solve

$$\begin{aligned}
(\beta - \mathbb{E}[b]) \Pr[q > -\beta] - (2\gamma + \eta) y^* &= 0 \Leftrightarrow \\
y^* &= y^{**} \equiv \frac{1}{2\gamma + \eta} (\beta - \mathbb{E}[b])(1 - F(-\beta)).
\end{aligned} \tag{71}$$

Notice that

$$\lim_{\gamma \to \infty} y^{**} = 0.$$

Second, recall that $\beta_L(-\alpha) = \beta_H(-\alpha) = \beta^*$ from Proposition 1, and note that

$$\lim_{\gamma\to\infty} \beta^* = G^{-1}(1-\tau) \in \left(-\bar{b}, \bar{b}\right).$$

Next, we consider two cases:

1. Suppose $\beta \in [-\bar{b}, \beta^*)$. We argue that there exists a unique $\underline{y} \in (-\alpha, 1-\alpha-\tau)$ such that: (i) $\beta = \beta_L(\underline{y})$, (ii) if $y \in (-\alpha, \underline{y})$ then $\beta \in [-\bar{b}, \beta_L(y))$, and (iii) if $y > \underline{y}$ then $\beta \in (\beta_L(y), \beta_H(y))$. To see why, recall that: (1) $\beta_H(y)$ is an increasing function of y, (2) $\beta_L(y)$ is a decreasing function of y, (3) $\beta < \beta^* = \beta_L(-\alpha) = \beta_H(-\alpha)$, and (4) $\lim_{y\nearrow 1-\alpha-\tau} \beta_L(y) = -\bar{b}$. Combined, these four facts prove the arguments above. Moreover, these arguments imply that the median voter is given by

$$-q^*(y) = \begin{cases} \beta_L(y) & \text{if } -\alpha < y < \underline{y} \\ \beta & \text{if } \underline{y} < y < 1-\alpha, \end{cases}$$

Notice that by the definition of $\beta_L(\cdot)$, $\underline{y}$ is given by the solution of

$$\begin{aligned} R(\beta, \underline{y}, -\beta) &= 1 - \frac{\tau}{1-\alpha-\underline{y}} \Leftrightarrow \\ \underline{y} &= 1-\alpha - \frac{\tau}{1-G(\beta)} + \frac{1}{\gamma}\frac{G(\beta)}{1-G(\beta)}\left(\mathbb{E}[b] - \mathbb{E}[b|b<\beta]\right)(1-F(-\beta)), \end{aligned}$$

where $\lim_{\gamma\to\infty} \underline{y} = 1-\alpha-\frac{\tau}{1-G(\beta)}$ and $\lim_{\gamma\to\infty} \underline{y} < 0 \Leftrightarrow \beta > G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$. Also notice that

$$\Pi'(y) = (\beta - \mathbb{E}[b])\Pr[q > q^*(y)] - (2\gamma+\eta)y + \begin{cases} MPV(y) & \text{if } -\alpha < y < \underline{y} \\ 0 & \text{if } y > \underline{y}, \end{cases}$$

and it is not defined for $y = \underline{y}$. For $-\alpha < y < \underline{y}$, we have

$$MPV(y) = \frac{\partial \beta_L(y)}{\partial y} f(-\beta_L(y))[\alpha(\beta - \beta_L(y)) + y(\beta - \mathbb{E}[b])]$$

and

$$\begin{aligned} \lim_{\gamma\to\infty} MPV(y) &= -\frac{1-G(\lim_{\gamma\to\infty}\beta_L(y))}{g(\lim_{\gamma\to\infty}\beta_L(y))} \\ &\quad \times \frac{f(-\lim_{\gamma\to\infty}\beta_L(y))}{1-\alpha-y}\left[\alpha(\beta - \lim_{\gamma\to\infty}\beta_L(y)) + y(\beta-\mathbb{E}[b])\right], \end{aligned} \tag{72}$$

where $\lim_{\gamma\to\infty}\beta_L(y) = G^{-1}(1-\frac{\tau}{1-\alpha-y})$. Thus, $\lim_{\gamma\to\infty} MPV(y)$ is bounded (recall $y < 1-\alpha$). We consider two subcases.

   (a) First, suppose $\beta > G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$. Then for a large γ we have $\underline{y} < 0 \approx y^{**}$. Fix any $\varepsilon \in (0, -\lim_{\gamma\to\infty}\underline{y})$, and notice that for any $y \in (-\alpha, \lim_{\gamma\to\infty}\underline{y}+\varepsilon)/\{\lim_{\gamma\to\infty}\underline{y}\}$ we have $\lim_{\gamma\to\infty}\Pi'(y) = \infty$. Thus, there exists $\overline{\gamma}_3 < \infty$ such that if $\gamma > \overline{\gamma}_3$ then

$\Pi'(y) > 0$ for any $y \in (-\alpha, \underline{y}+\varepsilon)/\{\underline{y}\}$. Therefore, the maximizer of $\Pi(y)$ is greater than $\underline{y}$, that is, $y^* > \underline{y}$. Recall that if $y > \underline{y}$ then $\beta \in (\beta_L(y), \beta_H(y))$, which implies $MPV(y) = 0$. Therefore, it must be $MPV(y^*) = 0$, $-q^*(y^*) = \beta$, and $y^* = y^{**}$.

(b) Second, suppose $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$. Then, for a large γ we have $1-\alpha > \underline{y} > 0 \approx y^{**}$. Fix any $\varepsilon \in (0, \lim_{\gamma\to\infty} \underline{y})$, and notice that for any $y \in (\lim_{\gamma\to\infty} \underline{y} - \varepsilon, 1-\alpha)/\{\lim_{\gamma\to\infty} \underline{y}\}$ we have $\lim_{\gamma\to\infty} \Pi'(y) = -\infty$. Thus, there exists $\overline{\gamma}_4 < \infty$ such that if $\gamma > \overline{\gamma}_4$ then $\Pi'(y) < 0$ for any $y \in (\underline{y}-\varepsilon, 1-\alpha)/\{\underline{y}\}$. Therefore, the maximizer of $\Pi(y)$ is smaller than $\underline{y}$, that is, $y^* < \underline{y}$. In particular, since $\Pi(y)$ is continuous and concave, and since $\Pi'(y)|_{y=-\alpha} > 0$ for a large γ, there is a unique $\hat{y} \in (-\alpha, \underline{y})$ such that $\Pi'(y)|_{y=\hat{y}} = 0$. Therefore, the optimizer of $\Pi(y)$ is $y^* = \hat{y}$ and the median voter is a dispersed shareholder with a bias $\beta_L(\hat{y}) > \beta$.
Since $\lim_{\gamma\to 0} \hat{y} = 0$ and $\lim_{\gamma\to\infty} \beta_L(y) = G^{-1}(1 - \frac{\tau}{1-\alpha-y})$, (72) implies

$$\lim_{\gamma\to\infty} MPV(\hat{y}) = \frac{\tau}{1-\alpha} \frac{f(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))} \frac{\alpha}{1-\alpha}(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) - \beta) > 0. \quad (73)$$

In addition,

$$\lim_{\gamma\to\infty} MPC(\hat{y}) = (\beta - \mathbb{E}[b]) H\left(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})\right) - 2\lim_{\gamma\to\infty}[\gamma\hat{y}].$$

Since $\Pi'(y)|_{y=\hat{y}} = 0$ in this case, it must be $\lim_{\gamma\to\infty} MPC(\hat{y}) + \lim_{\gamma\to\infty} MPV(\hat{y}) = 0$, that is

$$\lim_{\gamma\to\infty}[2\gamma\hat{y}] = \lim_{\gamma\to\infty} MPV(\hat{y}) + (\beta - \mathbb{E}[b]) H\left(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})\right).$$

This implies that $\hat{y}$ and $MPV(\hat{y})$ could have different signs. For example, if α is small, $\beta < \mathbb{E}[b]$ (which is likely given $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$), then $\lim_{\gamma\to\infty} MPV(\hat{y}) > 0$ but is close to zero, whereas $(\beta - \mathbb{E}[b]) H\left(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})\right) < 0$ and is bounded from zero, so $\lim_{\gamma\to\infty}[2\gamma\hat{y}] < 0$, which implies that $\hat{y}$ converges to 0 from below, i.e., $\hat{y} < 0$ while $MPV(\hat{y}) > 0$.

2. Suppose $\beta \in (\beta^*, \overline{b}]$. We argue that there exists a unique $\overline{y} \in (-\alpha, \tau-\alpha)$ such that: (i) $\beta = \beta_H(\overline{y})$, (ii) if $y \in (-\alpha, \overline{y})$ then $\beta \in (\beta_H(y), \overline{b}]$, and (iii) if $y > \overline{y}$ then $\beta \in (\beta_L(y), \beta_H(y))$. To see why, recall that: (1) $\beta_H(y)$ is an increasing function of y, (2) $\beta_L(y)$ is a decreasing function of y, (3) $\beta > \beta^* = \beta_L(-\alpha) = \beta_H(-\alpha)$, and (4) $\lim_{y\nearrow\tau-\alpha} \beta_H(y) = \overline{b}$. Combined, these four facts prove the arguments above. Moreover, these arguments imply that the median voter is given by

$$-q^*(y) = \begin{cases} \beta_H(y) & \text{if } -\alpha < y < \overline{y} \\ \beta & \text{if } \overline{y} < y < 1-\alpha, \end{cases}$$

Notice that by the definition of $\beta_H(\cdot)$, $\overline{y}$ is given by the solution of

$$\begin{aligned} R(\beta, \overline{y}, -\beta) &= 1 - \frac{\tau - \alpha - \overline{y}}{1 - \alpha - \overline{y}} \Leftrightarrow \\ \overline{y} &= 1 - \alpha - \frac{1 - \tau}{G(\beta)} - \frac{1}{\gamma}\left(\mathbb{E}[b] - \mathbb{E}[b|b < \beta]\right)(1 - F(-\beta)), \end{aligned}$$

where $\lim_{\gamma\to\infty} \overline{y} = 1 - \alpha - \frac{1-\tau}{G(\beta)}$ and $\lim_{\gamma\to\infty} \overline{y} < 0 \Leftrightarrow \beta < G^{-1}(\frac{1-\tau}{1-\alpha})$. Also notice that

$$\Pi'(y) = (\beta - \mathbb{E}[b]) \Pr[q > q^*(y)] - (2\gamma + \eta)y + \begin{cases} MPV(y) & \text{if } -\alpha < y < \overline{y} \\ 0 & \text{if } y > \overline{y}, \end{cases}$$

and it is not defined for $y = \overline{y}$. For $-\alpha < y < \overline{y}$, we have

$$MPV(y) = \frac{\partial \beta_H(y)}{\partial y} f(-\beta_H(y))[\alpha(\beta - \beta_H(y)) + y(\beta - \mathbb{E}[b])]$$

and

$$\begin{aligned} \lim_{\gamma\to\infty} MPV(y) &= \frac{G(\lim_{\gamma\to\infty}\beta_H(y))}{g(\lim_{\gamma\to\infty}\beta_H(y))} \\ &\quad \times \frac{f(-\lim_{\gamma\to\infty}\beta_H(y))}{1 - \alpha - y}\left[\alpha(\beta - \lim_{\gamma\to\infty}\beta_H(y)) + y(\beta - \mathbb{E}[b])\right], \end{aligned} \tag{74}$$

where $\lim_{\gamma\to\infty}\beta_H(y) = G^{-1}(\frac{1-\tau}{1-\alpha-y})$. Thus $\lim_{\gamma\to\infty} MPV(y)$ is bounded (recall $y < 1 - \alpha$). We consider two subcases.

(a) First, suppose $\beta < G^{-1}(\frac{1-\tau}{1-\alpha})$. Then for a large γ we have $\overline{y} < 0 \approx y^{**}$. Fix any $\varepsilon \in (0, -\lim_{\gamma\to\infty}\overline{y})$, and notice that for any $y \in (-\alpha, \lim_{\gamma\to\infty}\overline{y} + \varepsilon)/\{\lim_{\gamma\to\infty}\overline{y}\}$ we have $\lim_{\gamma\to\infty}\Pi'(y) = \infty$. Thus, there exists $\overline{\gamma}_5 < \infty$ such that if $\gamma > \overline{\gamma}_5$ then $\Pi'(y) > 0$ for any $y \in (-\alpha, \overline{y} + \varepsilon)/\{\overline{y}\}$. Therefore, the maximizer of $\Pi(y)$ is greater than $\overline{y}$, that is, $y^* > \overline{y}$. Recall that if $y > \overline{y}$ then $\beta \in (\beta_L(y), \beta_H(y))$, which implies $MPV(y) = 0$. Therefore, it must be $MPV(y^*) = 0$, $-q^*(y^*) = \beta$, and $y^* = y^{**}$. Moreover, since $y > \overline{y} \Rightarrow MPV(y) = 0$, it must be $-q^*(y^*) = \beta$, $MPV(y^*) = 0$, and $y^* = y^{**}$.

(b) Second, suppose $\beta > G^{-1}(\frac{1-\tau}{1-\alpha})$. Then, for a large γ we have $1-\alpha > \overline{y} > 0 \approx y^{**}$. Fix any $\varepsilon \in (0, \lim_{\gamma\to\infty}\overline{y})$, and notice that for any $y \in (\lim_{\gamma\to\infty}\overline{y} - \varepsilon, 1 - \alpha)/\{\lim_{\gamma\to\infty}\overline{y}\}$ we have $\lim_{\gamma\to\infty}\Pi'(y) = -\infty$. Thus, there exists $\overline{\gamma}_6 < \infty$ such that if $\gamma > \overline{\gamma}_6$ then $\Pi'(y) < 0$ for any $y \in (\overline{y} - \varepsilon, 1 - \alpha)/\{\overline{y}\}$. Therefore, the maximizer of $\Pi(y)$ is smaller than $\overline{y}$, that is, $y^* < \overline{y}$. In particular, since $\Pi(y)$ is continuous and concave, and since $\Pi'(y)|_{y=-\alpha} > 0$ for a large γ, there is a unique $\hat{y} \in (-\alpha, \overline{y})$ such that $\Pi'(y)|_{y=\hat{y}} = 0$. Therefore, the optimizer of $\Pi(y)$ is $y^* = \hat{y}$ and the median voter is a dispersed shareholder with a bias $\beta_H(\hat{y}) < \beta$.

Since $\lim_{\gamma\to 0}\hat{y}=0$ and $\lim_{\gamma\to\infty}\beta_H(y)=G^{-1}(\frac{1-\tau}{1-\alpha-y})$, (74) implies

$$\lim_{\gamma\to\infty} MPV(\hat{y})=\frac{1-\tau}{1-\alpha}\times\frac{f(-G^{-1}(\frac{1-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\tau}{1-\alpha}))}\frac{\alpha}{1-\alpha}(\beta-G^{-1}(\frac{1-\tau}{1-\alpha}))>0. \tag{75}$$

In addition,

$$\lim_{\gamma\to\infty} MPC(\hat{y})=(\beta-\mathbb{E}[b])\,H\left(-G^{-1}(\frac{1-\tau}{1-\alpha})\right)-2\lim_{\gamma\to\infty}[\gamma\hat{y}].$$

Since $\Pi'(y)|_{y=\hat{y}}=0$ in this case, it must be $\lim_{\gamma\to\infty} MPC(\hat{y})+\lim_{\gamma\to\infty} MPV(\hat{y})=0$, that is

$$\lim_{\gamma\to\infty}[2\gamma\hat{y}]=\lim_{\gamma\to\infty} MPV(\hat{y})+(\beta-\mathbb{E}[b])\,H\left(-G^{-1}(\frac{1-\tau}{1-\alpha})\right).$$

Potentially, $\hat{y}$ and $MPV(\hat{y})$ could again have different signs. For example, if α is small, $\beta<E[b]$ (which is possible if $G^{-1}(\frac{1-\tau}{1-\alpha})<E[b]$), then $\lim_{\gamma\to\infty} MPV(\hat{y})>0$ but is close to zero, whereas $(\beta-\mathbb{E}[b])\,H\left(-G^{-1}(\frac{1-\tau}{1-\alpha})\right)<0$ and is bounded from zero, so $\lim_{\gamma\to\infty}[2\gamma\hat{y}]<0$, which implies that $\hat{y}$ converges to 0 from below, i.e., $\hat{y}<0$ while $MPV(\hat{y})>0$.

Notice that $\lim_{\gamma\to\infty}\beta^*=G^{-1}(1-\tau)\in\left(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}),G^{-1}(\frac{1-\tau}{1-\alpha})\right)$. According to part 1.a and 2.a, for a large γ, if $G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})<\beta<\beta^*$ or $\beta^*<\beta<G^{-1}(\frac{1-\tau}{1-\alpha})$, then $-q^*(y^*)=\beta$, $MPV(y^*)=0$, and $y^*=y^{**}$. This establishes part (i) in the statement. According to part 1.b, if $\beta<\min\{G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}),\beta^*\}=G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$, then the median voter is a dispersed shareholder with bias $\beta_L(y^*)>\beta$. According to part 2.b, if $\beta>\max\{G^{-1}(\frac{1-\tau}{1-\alpha}),\beta^*\}=G^{-1}(\frac{1-\tau}{1-\alpha})$, then the median voter is a dispersed shareholder with bias $\beta_H(y^*)<\beta$. Letting

$$\underline{\beta}\equiv G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right) \tag{76}$$

$$\overline{\beta}\equiv G^{-1}\left(\frac{1-\tau}{1-\alpha}\right) \tag{77}$$

establishes part (ii).

To see expression (23) for the share price in (iii), we simply plug in y^* and $q^*(y^*)$ into (16).

Given (73) and (75), there exists $\overline{\gamma}_7>0$ such that if $\gamma>\overline{\gamma}_7$, then $MPV(y^*)>0$ for $\beta\notin\left(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}),G^{-1}(\frac{1-\tau}{1-\alpha})\right)$.

Part (iv) follows from Lemma 4 in the Online Appendix.

Letting $\overline{\gamma}=\max\{\overline{\gamma}_1,\overline{\gamma}_2,\overline{\gamma}_3,\overline{\gamma}_4,\overline{\gamma}_5,\overline{\gamma}_6,\overline{\gamma}_7\}$, completes the proof. ■

Proof of Corollary 2. If $b=E[b]$ for all dispersed shareholders, then their trades are $x=-y$ from equation (10). Then $r(b;y,q_e^*)=g(b)$ and $R(b';y,q_e^*)=G(b')$ from (11) and (12). Dispersed shareholders vote in favor if and only if $q\geq -b$ and $q^*=-b$, because the median voter is always a dispersed shareholder, independently of how the blockholder votes, as long as $\alpha+y<\tau$. Hence, $\frac{\partial(-q^*)}{\partial y}=0$, so the voting premium is zero from (27). ■

Proposition 5 is a special case of the following result.

Proposition 7. *Suppose $\alpha = 0$. There exist $\overline{\gamma} < \infty$ such that if $\gamma > \overline{\gamma}$, then the equilibrium exists and is unique. In equilibrium, the median voter is a dispersed investor with bias*

$$-q^*(y^*) = \begin{cases} \beta_L(y^*) > \beta & \text{if } \beta < G^{-1}(1-\tau) \\ \beta_H(y^*) < \beta & \text{if } \beta > G^{-1}(1-\tau). \end{cases} \tag{78}$$

Moreover:

(i) If $\mathbb{E}[b] < \beta$, then the blockholder's equilibrium trade satisfies $y^* > 0$ and

$$y^* = \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])H(q^*(y^*)) + \frac{1}{2\gamma+\eta}MPV(y^*), \tag{79}$$

the share price is given by

$$p^* = v(b^*, q^*(y^*)) + \frac{\gamma}{2\gamma+\eta}MPV(y^*), \tag{80}$$

where $MPV(y^*) < 0$ if and only if $\beta < G^{-1}(1-\tau)$.

(ii) If $\beta < \mathbb{E}[b]$, then the blockholder does not trade in equilibrium (i.e., $y^* = 0$), the no-short-selling constraint binds, $MPV(0) = 0$, and

$$\begin{aligned} p^* &= v(\mathbb{E}[b], q^*(0)) \\ &= v(b^*, q^*(0)) + \frac{\gamma}{2\gamma+\eta}(\mathbb{E}[b] - \beta)H(q^*(0)). \end{aligned}$$

Proof. We build on the proof of Proposition 2, and adjust to the special case with $\alpha = 0$. Showing the existence of a unique maximizer, which we denote by y^*, follows the same arguments and hence is omitted. Recall that $\beta_L(0) = \beta_H(0) = \beta^*$ from Proposition 1, and note that

$$\lim_{\gamma\to\infty} \beta^* = G^{-1}(1-\tau) \in \left(-\bar{b}, \bar{b}\right).$$

Next, we consider two cases:

1. Suppose $\beta \in [-\bar{b}, \beta^*)$. As in the proof of Proposition 2, there exists a unique $\underline{y} \in (0, 1-\tau)$ such that: (i) $\beta = \beta_L(\underline{y})$, (ii) if $y \in (0, \underline{y})$ then $\beta \in [-\bar{b}, \beta_L(y))$, and (iii) if $y > \underline{y}$ then $\beta \in (\beta_L(y), \beta_H(y))$. Thus, the median voter is given by

$$-q^*(y) = \begin{cases} \beta_L(y) & \text{if } 0 < y < \underline{y} \\ \beta & \text{if } \underline{y} < y < 1. \end{cases}$$

   Moreover, as in the proof of Proposition 2, and by the definition of $\beta_L(\cdot)$ we have $\lim_{\gamma\to\infty} \underline{y} = 1 - \frac{\tau}{1-G(\beta)}$, and notice that $\lim_{\gamma\to\infty} \underline{y} > 0 \Leftrightarrow \beta < G^{-1}(1-\tau)$. Also notice that

$$MPV(y) = \begin{cases} \frac{\partial\beta_L(y)}{\partial y} f(-\beta_L(y)) y (\beta - \mathbb{E}[b]) & \text{if } 0 < y < \underline{y} \\ 0 & \text{if } y > \underline{y}, \end{cases}$$

and it is not defined for $y = \underline{y}$. Notice that

$$\lim_{\gamma\to\infty} MPV(y) = -\frac{1 - G(\lim_{\gamma\to\infty}\beta_L(y))}{g(\lim_{\gamma\to\infty}\beta_L(y))}\frac{f(-\lim_{\gamma\to\infty}\beta_L(y))}{1-y} y(\beta - \mathbb{E}[b]),$$

where $\lim_{\gamma\to\infty}\beta_L(y) = G^{-1}(1 - \frac{\tau}{1-y})$. Thus, $\lim_{\gamma\to\infty} MPV(y)$ is bounded (recall $y < 1$). Since $\beta < \beta^*$ and $\lim_{\gamma\to\infty}\beta^* = G^{-1}(1-\tau)$, for a large γ we have $1 > \underline{y} > 0$, and for the same reasons as in the proof of Proposition 2, the maximizer of $\Pi(y)$ is smaller than $\underline{y}$ and given by the solution of FOC subject to the no-short-selling constraint that we impose below. That is, the optimal trade solves

$$y^* = \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b]) H(q^*(y^*)) + \frac{1}{2\gamma+\eta} MPV(y^*),$$

subject to being non-negative. In particular, notice that $\lim_{\gamma\to\infty} y^* = 0$, and hence $\lim_{\gamma\to\infty} MPV(y^*) = 0$. Suppose the no-short-selling constraint does not bind in the limit, that is, y^* converges to zero from above. Then, the FOC implies

$$\begin{aligned}\lim_{\gamma\to\infty}(\beta - \mathbb{E}[b]) H\left(q^*\left(\lim_{\gamma\to\infty} y^*\right)\right) - \lim_{\gamma\to\infty} 2\gamma\hat{y} + \lim_{\gamma\to\infty} MPV(y^*) &= 0 \Leftrightarrow \\ (\beta - \mathbb{E}[b]) H(q^*(0)) &= \lim_{\gamma\to\infty} 2\gamma y^*.\end{aligned}$$

Thus, y^* converges to zero from above if and only if $\beta > \mathbb{E}[b]$. If $\beta < \mathbb{E}[b]$, then the no-short-selling constraint must bind in the limit, and in that case, the blockholder does not trade (i.e., $y^* = 0$). If $\beta > \mathbb{E}[b]$, then the no-short-selling constraint does not bind in the limit, and having y^* converging to zero from above implies that $MPV(y^*)$ converges to zero from below. That is, if $\beta > \mathbb{E}[b]$, then for large γ it must be

$$MPV(y^*) < 0 < y^*.$$

Then, the share price is $p^* = v(b^*, q^*(y^*)) + \frac{\gamma}{2\gamma+\eta} MPV(y^*)$, where $MPV(y^*) < 0$.

2. Suppose $\beta \in (\beta^*, \overline{b},]$. As in the proof of Proposition 2, there exists a unique $\overline{y} \in (0,\tau)$ such that: (i) $\beta = \beta_H(\overline{y})$, (ii) if $y \in (0,\overline{y})$ then $\beta \in (\beta_H(y), \overline{b}]$, and (iii) if $y > \overline{y}$ then $\beta \in (\beta_L(y), \beta_H(y))$. Thus, the median voter is given by

$$-q^*(y) = \begin{cases} \beta_H(y) & \text{if } 0 < y < \overline{y} \\ \beta & \text{if } \overline{y} < y < 1, \end{cases}$$

Moreover, as in the proof of Proposition 2, and by the definition of $\beta_H(\cdot)$ we have $\lim_{\gamma\to\infty}\overline{y} = 1 - \frac{1-\tau}{G(\beta)}$, and notice that $\lim_{\gamma\to\infty}\overline{y} > 0 \Leftrightarrow \beta > G^{-1}(1-\tau)$. Also notice that

$$MPV(y) = \begin{cases} \frac{\partial\beta_H(y)}{\partial y} f(-\beta_H(y)) y(\beta - \mathbb{E}[b])] & \text{if } 0 < y < \overline{y} \\ 0 & \text{if } y > \overline{y}, \end{cases}$$

and it is not defined for $y = \overline{y}$. Notice that

$$\lim_{\gamma\to\infty} MPV(y) = \frac{G(\lim_{\gamma\to\infty}\beta_H(y))}{g(\lim_{\gamma\to\infty}\beta_H(y))}\frac{f(-\lim_{\gamma\to\infty}\beta_H(y))}{1-y}y(\beta - \mathbb{E}[b]),$$

where $\lim_{\gamma\to\infty}\beta_H(y) = G^{-1}(\frac{1-\tau}{1-y})$. Thus, $\lim_{\gamma\to\infty} MPV(y)$ is bounded (recall $y < 1$). Since $\beta > \beta^*$ and $\lim_{\gamma\to\infty}\beta^* = G^{-1}(1-\tau)$, for a large γ we have $1 > \overline{y} > 0$, and for the same reasons as in the proof of Proposition 2, the maximizer of $\Pi(y)$ is smaller than $\overline{y}$ and given by the solution of FOC subject to the no-short-selling constraint that we impose below. That is, the optimal trade solves

$$y^* = \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])H(q^*(y^*)) + \frac{1}{2\gamma+\eta}MPV(y^*),$$

subject to being non-negative. In particular, notice that $\lim_{\gamma\to\infty} y^* = 0$, and hence $\lim_{\gamma\to\infty} MPV(y^*) = 0$. Suppose the no-short-selling constraint does not bind in the limit, that is, y^* converges to zero from above. Then, the FOC implies

$$\begin{aligned}\lim_{\gamma\to\infty}(\beta - \mathbb{E}[b])H\left(q^*\left(\lim_{\gamma\to\infty} y^*\right)\right) - \lim_{\gamma\to\infty} 2\gamma\hat{y} + \lim_{\gamma\to\infty} MPV(y^*) &= 0 \Leftrightarrow \\ (\beta - \mathbb{E}[b])H(q^*(0)) &= \lim_{\gamma\to\infty} 2\gamma y^*.\end{aligned}$$

Thus, y^* converges to zero from above if and only if $\beta > \mathbb{E}[b]$. If $\beta < \mathbb{E}[b]$, the no-short-selling constraint must bind in the limit, and in that case, the blockholder does not trade ($y^* = 0$). If $\beta > \mathbb{E}[b]$, the no-short-selling constraint does not bind in the limit, and having y^* converging to zero from above implies that $MPV(y^*)$ converges to zero from above. That is, if $\beta > \mathbb{E}[b]$, then for large γ it must be $0 < MPV(y^*)$ and $0 < y^*$. Then, the share price is $p^* = v(b^*, q^*(y^*)) + \frac{\gamma}{2\gamma+\eta}MPV(y^*)$, where $MPV(y^*) > 0$.

■

Proof of Proposition 4. Notice that

$$\begin{aligned}
u(b) &= (e + x^*(b))\, v(b, q^*) - x^*(b)\, p^* - \frac{\gamma}{2} x^*(b)^2 \\
&= ev(b, q^*) + x^*(b)\left[v(b, q^*) - p^*\right] - \frac{\gamma}{2} x^*(b)^2 \\
&= ev(b, q^*) + \frac{v(b, q^*) - p^*}{\gamma}\left[v(b, q^*) - p^*\right] - \frac{\gamma}{2}\left[\frac{v(b, q^*) - p^*}{\gamma}\right]^2 \\
&= ev(b, q^*) + \frac{1}{2\gamma}\left[v(b, q^*) - p^*\right]^2 \\
&= ev(b, q^*) + \frac{1}{2\gamma}\left[v(b, q^*) - \gamma y^* - v(\mathbb{E}[b], q^*)\right]^2 \\
&= ev(b, q^*) + \frac{1}{2\gamma}\left[(b - \mathbb{E}[b])\, H(q^*) - \gamma y^*\right]^2 \\
&= ev(b, q^*) + \frac{(b - \mathbb{E}[b])^2 H(q^*)^2 - 2(b - \mathbb{E}[b])\, H(q^*)\, \gamma y^* + (\gamma y^*)^2}{2\gamma}
\end{aligned}$$

and

$$\begin{aligned}
W^* &= \int_{-\bar{b}}^{\bar{b}} u(b)\, g(b)\, db \\
&= e\int_{-\bar{b}}^{\bar{b}} v(b, q^*)\, g(b)\, db + \int_{-\bar{b}}^{\bar{b}}\left[\frac{(b - \mathbb{E}[b])^2 H(q^*)^2 - 2(b - \mathbb{E}[b])\, H(q^*)\, \gamma y^* + (\gamma y^*)^2}{2\gamma}\right] g(b)\, db \\
&= ev(\mathbb{E}[b], q^*) + \frac{H(q^*)^2}{2\gamma}\int_{-\bar{b}}^{\bar{b}} (b - \mathbb{E}[b])^2 g(b)\, db + \frac{\gamma}{2}(y^*)^2 \\
&= ev(\mathbb{E}[b], q^*) + \frac{\sigma_b^2}{2\gamma} H(q^*)^2 + \frac{\gamma}{2}(y^*)^2
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{\partial W^*}{\partial \beta} &= e\frac{\partial v(\mathbb{E}[b], q^*)}{\partial q^*}\frac{\partial q^*}{\partial \beta} - \frac{\sigma_b^2}{\gamma} H(q^*)\, f(q^*)\, \frac{\partial q^*}{\partial \beta} + \gamma y^* \frac{\partial y^*}{\partial \beta} \\
&= -e(\mathbb{E}[b] + q^*)\, f(q^*)\, \frac{\partial q^*}{\partial \beta} - \frac{\sigma_b^2}{\gamma} H(q^*)\, f(q^*)\, \frac{\partial q^*}{\partial \beta} + \gamma y^* \frac{\partial y^*}{\partial \beta} \\
&= \left[e(\mathbb{E}[b] + q^*) + \frac{\sigma_b^2}{\gamma} H(q^*)\right] f(q^*)\, \frac{\partial(-q^*)}{\partial \beta} + \gamma y^* \frac{\partial y^*}{\partial \beta}.
\end{aligned}$$

We consider two cases:

1. Suppose $\beta > G^{-1}(\frac{1-\tau}{1-\alpha})$. Based on the proof of Lemma 4, for large γ we have $-q^*(y^*) = \beta_H(y^*)$, $MPV(y^*) > 0$, and $y^* = \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])\, H(-\beta_H(y^*)) + \frac{1}{2\gamma+\eta} MPV(y^*)$. More-

over, $\frac{\partial(-q^*(y^*))}{\partial\beta} > 0$, $\frac{\partial MPV(y^*)}{\partial\beta} > 0$, and $\frac{\partial y^*}{\partial\beta} > 0$. Thus,

$$\frac{\partial W^*}{\partial\beta} = \left[e\left(\mathbb{E}\left[b\right] - \beta_H\left(y^*\right)\right) + \frac{\sigma_b^2}{\gamma} H\left(-\beta_H\left(y^*\right)\right)\right] f\left(-\beta_H\left(y^*\right)\right) \frac{\partial\beta_H\left(y^*\right)}{\partial\beta} + \gamma y^* \frac{\partial y^*}{\partial\beta}.$$

Since $\beta_H\left(y^*\right)$ does not depend on β directly, we can write $\frac{\partial\beta_H(y^*)}{\partial\beta} = \frac{\partial\beta_H(y^*)}{\partial y}\frac{\partial y^*}{\partial\beta}$. Notice $\frac{\partial\beta_H(y^*)}{\partial\beta}, \frac{\partial y^*}{\partial\beta} > 0$ implies $\frac{\partial\beta_H(y^*)}{\partial y} > 0$. Thus,

$$\begin{aligned}\frac{\partial W^*}{\partial\beta} &= \left[e\left(\mathbb{E}\left[b\right] - \beta_H\left(y^*\right)\right) + \frac{\sigma_b^2}{\gamma} H\left(-\beta_H\left(y^*\right)\right)\right] f\left(-\beta_H\left(y^*\right)\right) \frac{\partial\beta_H\left(y^*\right)}{\partial y}\frac{\partial y^*}{\partial\beta} + \gamma y^* \frac{\partial y^*}{\partial\beta} \\ &= \left(\left[e\left(\mathbb{E}\left[b\right] - \beta_H\left(y^*\right)\right) + \frac{\sigma_b^2}{\gamma} H\left(-\beta_H\left(y^*\right)\right)\right] f\left(-\beta_H\left(y^*\right)\right) \frac{\partial\beta_H\left(y^*\right)}{\partial y} + \gamma y^*\right) \frac{\partial y^*}{\partial\beta}.\end{aligned}$$

Notice $\lim_{\gamma\to\infty}\beta_H\left(y^*\right) = G^{-1}(\frac{1-\tau}{1-\alpha})$, $\lim_{\gamma\to\infty}\frac{\partial\beta_H(y^*)}{\partial y} \in (0,\infty)$, and $\lim_{\gamma\to\infty}\gamma y^* = \frac{1}{2}\left(\beta - \mathbb{E}\left[b\right]\right) H(-G^{-1}($ $\frac{1}{2}\lim_{\gamma\to\infty} MPV\left(y^*\right)$. Thus,

$$\lim_{\gamma\to\infty}\frac{\partial W^*}{\partial\beta} = \left(\begin{array}{c} e\left(\mathbb{E}\left[b\right] - G^{-1}(\frac{1-\tau}{1-\alpha})\right) f\left(-G^{-1}(\frac{1-\tau}{1-\alpha})\right) \lim_{\gamma\to\infty}\frac{\partial\beta_H(y^*)}{\partial y} \\ +\frac{1}{2}\left(\beta - \mathbb{E}\left[b\right]\right) H(-G^{-1}(\frac{1-\tau}{1-\alpha})) + \frac{1}{2}\lim_{\gamma\to\infty} MPV\left(y^*\right)\end{array}\right) \times \lim_{\gamma\to\infty}\frac{\partial y^*}{\partial\beta}$$

Recall $\beta > G^{-1}(\frac{1-\tau}{1-\alpha})$ implies $\lim_{\gamma\to\infty} MPV\left(y^*\right) > 0$. Thus, if $\beta > \mathbb{E}\left[b\right] \geq G^{-1}(\frac{1-\tau}{1-\alpha})$ then $\frac{\partial W^*}{\partial\beta} > 0$ for large γ. In this case, both $\frac{\partial W^*}{\partial\beta} > 0$ and $\frac{\partial MPV(y^*)}{\partial\beta} > 0$ as required.

From Lemma 2 and expression (75) in the proof of Proposition 2, we have

$$\begin{aligned}\lim_{\gamma\to\infty} MPV\left(y^*\right) &= \frac{1-\tau}{1-\alpha} \times \frac{f(-G^{-1}(\frac{1-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\tau}{1-\alpha}))}\frac{\alpha}{1-\alpha}(\beta - G^{-1}(\frac{1-\tau}{1-\alpha})) \\ &= \frac{1-\tau}{1-\alpha} \times \frac{f(-\overline{\beta})}{g(\overline{\beta})}\frac{\alpha}{1-\alpha}(\beta - \overline{\beta}) \\ \lim_{\gamma\to\infty}\frac{\partial\beta_H\left(y^*\right)}{\partial y} &= \frac{G\left(G^{-1}\left(\frac{1-\tau}{1-\alpha}\right)\right)}{g\left(G^{-1}\left(\frac{1-\tau}{1-\alpha}\right)\right)(1-\alpha)} \\ &= \frac{G\left(\overline{\beta}\right)}{g\left(\overline{\beta}\right)(1-\alpha)}\end{aligned}$$

Therefore

$$\lim_{\gamma\to\infty}\frac{\partial W^*}{\partial\beta} = \left(\left(\mathbb{E}\left[b\right] - \overline{\beta}\right)\frac{1-\tau}{1-\alpha}\frac{f\left(-\overline{\beta}\right)}{g\left(\overline{\beta}\right)} + \frac{1}{2}\frac{1-\tau}{1-\alpha}\frac{f(-\overline{\beta})}{g(\overline{\beta})}\frac{\alpha}{1-\alpha}(\beta - \overline{\beta}) + \frac{1}{2}\left(\beta - \mathbb{E}\left[b\right]\right) H(-\overline{\beta})\right) \times \lim_{\gamma\to\infty}$$

2. Suppose $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$. Based on the proof of Lemma 4, for large γ we have $-q^*\left(y^*\right) = \beta_L\left(y^*\right)$, $MPV\left(y^*\right) > 0$, and $y^* = \frac{1}{2\gamma+\eta}\left(\beta - \mathbb{E}\left[b\right]\right) H(-\beta_L\left(y^*\right)) + \frac{1}{2\gamma+\eta} MPV\left(y^*\right)$. Moreover, $\frac{\partial MPV(y^*)}{\partial\beta} < 0$, and there is $\underline{\alpha} > 0$ such that if $\alpha > \underline{\alpha}$, then $\frac{\partial y^*}{\partial\beta} < 0$ and

$\frac{\partial \beta_L(y^*)}{\partial y} < 0$. Notice at $\lim_{\gamma\to\infty} \beta_L(y^*) = G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$, $\lim_{\gamma\to\infty} \frac{\partial \beta_L(y^*)}{\partial y} \in (-\infty, 0)$, and $\lim_{\gamma\to\infty} \gamma y^* = \frac{1}{2}(\beta - \mathbb{E}[b]) H(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})) + \frac{1}{2}\lim_{\gamma\to\infty} MPV(y^*)$. Thus, we can write

$$\lim_{\gamma\to\infty} \frac{\partial W^*}{\partial \beta} = \begin{pmatrix} e\left(\mathbb{E}[b] - G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})\right) f\left(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})\right) \lim_{\gamma\to\infty} \frac{\partial \beta_L(y^*)}{\partial y} \\ +\frac{1}{2}(\beta - \mathbb{E}[b]) H(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})) + \frac{1}{2}\lim_{\gamma\to\infty} MPV(y^*) \end{pmatrix} \times \lim_{\gamma\to\infty} \frac{\partial y^*}{\partial \beta}$$

Notice $\frac{\partial W^*}{\partial \beta} < 0$ if $\beta - \mathbb{E}[b] > 0$ and $\mathbb{E}[b] - G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) < 0$. Recall $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ implies $\lim_{\gamma\to\infty} MPV(y^*) > 0$. Thus, if $\mathbb{E}[b] \le \beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ then $\frac{\partial W^*}{\partial \beta} < 0$ for large γ. In this case, both $\frac{\partial W^*}{\partial \beta} < 0$ and $\frac{\partial MPV(y^*)}{\partial \beta} < 0$ as required.

From Lemma 2 and expression (73) in the proof of Proposition 2, we have

$$\begin{aligned} \lim_{\gamma\to\infty} MPV(y^*) &= \frac{\tau}{1-\alpha} \frac{f(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))} \frac{\alpha}{1-\alpha}(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) - \beta) \\ &= \frac{\tau}{1-\alpha} \frac{f(-\underline{\beta}))}{g(\underline{\beta})} \frac{\alpha}{1-\alpha}(\underline{\beta} - \beta) \\ \lim_{\gamma\to\infty} \frac{\partial \beta_L(y^*)}{\partial y} &= -\frac{1 - G(G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right))}{g(G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right))(1-\alpha)} \\ &= -\frac{1 - G(\underline{\beta})}{g(\underline{\beta})(1-\alpha)} \end{aligned}$$

Therefore,

$$\lim_{\gamma\to\infty} \frac{\partial W^*}{\partial \beta} = \left(-\left(\mathbb{E}[b] - \underline{\beta}\right) \frac{f\left(-\underline{\beta}\right)}{g(\underline{\beta})} \frac{\tau}{1-\alpha} + \frac{1}{2}(\beta - \mathbb{E}[b]) H(-\underline{\beta}) + \frac{1}{2}\frac{\tau}{1-\alpha} \frac{f(-\underline{\beta}))}{g(\underline{\beta})} \frac{\alpha}{1-\alpha}(\underline{\beta} - \beta)\right) \times \mathrm{li}_{\gamma-}$$

Notice that if $\alpha < \underline{\alpha}$ then $\lim_{\gamma\to\infty} \frac{\partial y^*}{\partial \beta} > 0$. Thus, if $\alpha \approx 0$ and $\mathbb{E}[b] < \beta < \underline{\beta}$ then the first term is positive, the second term is positive, and the third term is arbitrarily small. In this case, $\frac{\partial W^*}{\partial \beta} > 0 > \frac{\partial MPV(y^*)}{\partial \beta}$.

■

Proof of Proposition 6. The objective $\Pi(y, \hat{y})$ of the blockholder with dual-class shares can be rewritten as:

$$\begin{aligned} \max_{y,\hat{y}} \Pi(y, \hat{y}) &= (\alpha + y) v(\beta, q^*(y)) - y p^*(y) - \frac{\eta}{2} y^2 + (\hat{\alpha} + \hat{y}) v(\beta, q^*(y)) - \hat{y}\hat{p}^*(\hat{y}) - \frac{\eta}{2}\hat{y}^2 \\ &= \alpha v(\beta, q^*(y)) + y(\beta - \mathbb{E}[b]) \Pr[q > q^*(y)] - (\gamma + \eta/2) y^2 \\ &\quad + \hat{\alpha} v(\beta, q^*(y)) + \hat{y}(\beta - \mathbb{E}[b]) \Pr[q > q^*(y)] - (\gamma + \eta/2)\hat{y}^2 \\ &= (\alpha + \hat{\alpha}) v(\beta, q^*(y)) + (y + \hat{y})(\beta - \mathbb{E}[b]) \Pr[q > q^*(y)] - (\gamma + \eta/2)\left(y^2 + \hat{y}^2\right) \\ &= (\alpha + \hat{\alpha}) v_0 + (\alpha + \hat{\alpha}) \Pr[q > q^*(y)] \mathbb{E}[\theta | q > q^*(y)] \\ &\quad + ((\alpha + \hat{\alpha} + y + \hat{y})\beta - (y + \hat{y})\mathbb{E}[b]) \Pr[q > q^*(y)] - (\gamma + \eta/2)\left(y^2 + \hat{y}^2\right). \end{aligned}$$

We rewrite the first-order condition with respect to y, $\frac{\partial \Pi(y,\hat{y})}{\partial y} = 0$, as

$$\left[\begin{array}{c} \underbrace{(\beta - \mathbb{E}[b]) \Pr[q > q^*(y)] - (2\gamma + \eta) y}_{\text{marginal payoff from buying cash flow rights}} \\ + \underbrace{\frac{\partial(-q^*(y))}{\partial y} f(q^*(y)) [(\alpha + \hat{\alpha})(q^*(y) + \beta) + (y + \hat{y})(\beta - \mathbb{E}[b])]}_{\text{marginal payoff from buying voting rights } MPV(y,\hat{y})} \end{array} \right] = 0 \Leftrightarrow$$

$$(\beta - \mathbb{E}[b]) \Pr[q > q^*(y)] - (2\gamma + \eta) y + MPV(y, \hat{y}) = 0 \Leftrightarrow$$

$$y^* = \frac{1}{2\gamma + \eta} (\beta - \mathbb{E}[b]) \Pr[q > q^*(y)] + \frac{1}{2\gamma + \eta} MPV(y, \hat{y}).$$

We rewrite the first-order condition with respect to $\hat{y}$, $\frac{\partial \Pi(y,\hat{y})}{\partial \hat{y}} = 0$, as

$$\underbrace{(\beta - \mathbb{E}[b]) \Pr[q > q^*(y)] - (2\gamma + \eta) \hat{y}}_{\text{marginal payoff from buying cash flow rights in non-voting shares}} = 0 \Leftrightarrow$$

$$\hat{y}^* = \frac{1}{2\gamma + \eta} (\beta - \mathbb{E}[b]) \Pr[q > q^*(y)],$$

which implies (40). ■