Collective Brands

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Abstract

We analyze the effect of a shared brand name, such as geographical names, on incentives of otherwise autonomous firms to establish a reputation for product quality. On the one hand, brand membership provides consumers with more information about past quality and therefore can motivate investment when the scale of production is too small to motivate stand alone firms to invest. On the other hand, a shared brand name may motivate free riding on the group's reputation, reducing incentives to invest. We identify conditions under which collective branding may deliver higher quality than stand alone firms can achieve.

1 Introduction

There are many instances in which otherwise autonomous firms, which make independent business decisions and retain their own profits, market their products under a shared brand name. Often, the shared brand name is perceived as a badge of superior quality by consumers, who are willing to pay premium prices for them (e.g. Landon and Smith, 1998, and Loureiro and McCluskey, 2000, 2003). Examples include regional agricultural brands protected by designation of origin (PDO) and geographical indication (PGI) status in the EU such as champagne bubbly wine, Parma ham and cheese, Roquefort Cheese. In countries where such Protected Geographical Status laws are enforced, only products genuinely originating in that region are allowed to be identified as such in commerce. Similarly, the Jaffa label is shared by many independent Israeli orange growers and exporters.

Another important example is franchising which in 2007 accounted for 9.2 percent of total U.S. GDP (Kosova and Lafontaine, 2012) and which spans the range from fast food restaurants to accounting and law firms. In a typical business-format franchising arrangement, franchisees sell under the common franchise logo, but are otherwise independent businesses which retain their own profits after paying the chain the corresponding fees (typically based on the outlet's sales).

Some premium food products, though sold by individual producers, share a common logo. For example, many of Germany's top wine producers are members of the VDP wine association and carry the VDP logotype. VDP members must adhere to more stringent standards than those set down in the German wine law. Similarly, otherwise independent members of many prestigious professional organizations share a common logo.

The fact that collective brand labels are associated with superior quality suggests that firms which are members of these brands invest more to maintain brand quality (or at least are perceived to do so by consumers) and earn higher profits than they would as stand alone firms. This seems surprising. If consumers' perception of the collective brand label's quality is jointly determined by their experience with the qualities provided by different individual members, and if the provision of high quality requires costly investment, it would seem that each member has an incentive to free ride on the investments of fellow members. If so, why are these brand labels perceived as badges of quality?

It is true that in some cases, the perception of superior quality may be partly attributable to exogenous advantages such as climate, soil quality, access to superior inputs, technology and so on. However, even when such natural advantages are present the achievement of superior quality presumably also requires the requisite investment of effort and other resources. The free riding problem might also be mitigated to some extent by monitoring the efforts and investments of individual members to maintain quality standards. However, monitoring is costly and imperfect and is therefore unlikely to eliminate free riding altogether. Thus it would seem that producers have less of an incentive to invest in quality as members of a collective brand than they would as stand alone firms.

The purpose of this paper is to show that collective branding may lead to higher quality in the market and increase welfare by incentivizing brand members to invest in quality, when they would not do so as stand alone firms. This may have important policy implications. For example, critics of marketing boards and state trading enterprises contend that these institutions reduce efficiency and welfare by fostering collusion[1]. By contrast, our analysis suggests that by enhancing reputational incentives, such institutionalized collective brands may actually increase efficiency and welfare by enabling higher product quality than would be attainable in their absence.

The idea is the following. When product quality is difficult to observe before purchase and is revealed to consumers only after consuming the product ('experience goods'), their perception of quality and the amount they are willing to pay for the

[1] An alternative view in defense of STE's is that they provide economies of scale in production and promotion.

product is based on past experience with the product - its reputation. Thus the extent to which a firm is able to receive a good return on its investment in quality depends on how much information consumers have about its past performance. If firms are small, relative to the size of the market, consumers may not have much information about the past quality of any individual firm. In that case, an individual firm may be unable to effectively establish a robust reputation for quality on its own and consequently has little incentive to invest in quality. Here collective branding may come to the rescue and serve as a vehicle for reputation formation by increasing the relevant information available to consumers. Specifically, suppose small individual firms market their products under a collective brand name, sharing a collective reputation, while otherwise retaining full autonomy. Since the collective brand name covers a larger share of the market than any individual member firm, consumers are better able to assess the reputation of the brand than of individual members. This in turn increases the value of a good brand reputation for each member, and may thus incentivize members to invest in quality when they would otherwise not do so. This is the 'reputation effect' of collective branding.

But as noted above, branding may also have an opposing effect on investment incentives. Unless the brand is able to effectively monitor individual investment, sharing a collective reputation may encourage individual members to free ride on the efforts of other members. Therefore the full effect of collective branding on investment in quality is determined by the interaction of these two opposing factors - the fact that, on the one hand, a good collective reputation is more valuable than a stand alone reputation, against the incentive to free ride, on the other.

Accordingly, we analyze the effects of collective branding under two regimes. In the first, called 'perfect monitoring', free riding on the brand's reputation is deterred because members which fail to invest are detected and excluded from using the brand name. Since then only the reputation effect is operative, a brand member's incentive to invest is always greater than that of a stand alone firm. Moreover, the incentive to invest increases with brand size (the number of firms which are members of the brand) - the larger the brand, the greater the incentive of each member to invest and therefore the more profitable membership is. Thus in this case "bigger is better". We show that this feature also applies if brand membership requires costly authentication of investment.

We find that for appropriate parameters this pro - investment effect of collective branding also applies in the alternative ‘no-monitoring’ regime, in which failure to invest cannot lead to exclusion from the brand. Specifically, collective branding can still facilitate investment if investment is a sufficiently important ingredient for the attainment of high quality - that is, if the difference between the expected product quality of a firm which invests in quality and one which doesn't is sufficiently large. However, in contrast to the case of perfect monitoring, here "bigger is better" only up to a point. Once the brand is sufficiently large, the marginal contribution of an individual member's investment to the brand's reputation becomes too small to override free riding, reducing the brand's incentive to invest relative to stand alone firms. Thus, in this case the brand size which maximizes firms' profits is large enough to enable successful reputation building but small enough to discourage individual free riding. Thus one might speculate that a regional brand like Champagne wine owes its success not only to unique soil and climatic conditions but also to fortuitous natural boundaries which encompass "just the right" number of producers under its brand label.

Empirical Evidence

Casual observation suggests that collective branding is often observed in situations where consumers are unlikely to have much information about individual producers. Thus, for example, the export of agricultural products is often managed by marketing boards and state trading enterprises rather than by the individual producers as foreign consumers are unlikely to recognize individual producers. Similarly, restaurants on highway stops, where there is little repeat business, almost always belong to well known chains. In the franchising context, Jin and Leslie (2009) provide evidence that chain restaurants - which share a collective brand name - maintain better hygiene than non-chain restaurants.

In an econometric study of the determinants of reputation in the Italian wine industry, Castriota and Delmastro (2008) show that brand reputation is increasing in the number of bottles produced by the brand and decreasing in the number of individual producers in the brand. This is consistent with our analysis. Keeping output fixed, an increase in the number of individual producers has no reputation effect since the number of units whose quality consumers observe is unchanged. However, it does increase the incentive for free riding (which increases with the number of members), and hence lowers investment incentives and reduces the brand's reputation. Relatedly, in an experimental study, Huck and Lűncer (2009) find that more sellers invest in quality when buyers are informed about the average past quality of all sellers - which corresponds to a collective brand in our model - than when they only know the record of the seller from whom they actually buy. However, consistent with our analysis, when the number of sellers increases, the average quality declines.

Online hiring markets also provide evidence for reputational effects of collective branding. Stanton and Thomas (2010) find that employers are willing to pay more to inexperienced online workers (which have yet to establish individual reputation) affiliated with outsourcing agencies than to inexperienced independent contractors and that this advantage dissipates over time as employers learn about individual productivity.

Related Literature

The centrality of *individual* firms' reputation for quality for their success is the theme of a very large literature (see the survey article of Bar Issac and Tadelis (2008)). By contrast our concern is to understand the role of a collective reputation on the fortunes of otherwise autonomous firms. Tirole (1996) analyzes how group behavior affects individual incentives to invest (behave honestly) when the group size is fixed exogenously. By contrast, our focus is precisely on the role of the group size on individual investment incentives.

Our analysis is most closely related to a substantial literature on brand extension and umbrella branding. This refers to the practice of multiproduct firms to use the same brand name on otherwise unrelated products in order to signal quality of experience goods to consumers.[2] (Andersson (2002), Cabral (2000, 2009), Cai and Obara (2009), Choi (1998), Choi, J. and D.S Jeon. (2007), Hakenes and Peitz (2008,2009), Miklos-Thal (2012), Rasmusen (2011), Wernerfelt (1988)). Both collective branding and umbrella branding provide firms with greater incentives to invest in quality than if products are branded separately. The main difference is that in an umbrella brand a central authority makes investment decisions for each of the brand's products and internalizes the effect of each individual product's quality on the reputation of the entire brand. By contrast, in a collective brand, individual members are concerned only with the effect of their investment decisions on the value of their own product. Therefore, umbrella branding incentivizes investment more than collective branding, but the latter can nevertheless support higher quality than stand alone firms.

Our analysis can also contribute to understanding the role of cooperatives. While the conventional approach (e.g., Sexton and Sexton, 1987) views cooperatives as a means of joint integration allowing for the exploitation of scale economies, market power and risk pooling, our analysis suggests an additional important function of cooperatives—joint signaling of information. [3]

2 The Model: Stand Alone Firms

We consider a market for an experience good - consumers observe quality only after buying, but not at the time of purchase. There are two periods, N risk neutral firms and we normalize the number of consumers per firm to be 1. There are two possible product qualities, low (l) and high (h). Firms are of two types, H and L, which are distinguished by their technological ability to produce high quality. An L firm produces high quality

[2] Relatedly Rob and Fishman (2005) show that a firm's investment in quality increases with size and Guttman and Yacouel (2007) show that larger firms benefit more from a good reputation.

[3] Another literature which addresses related issues is the common trait literature (e.g., Benabou and Gertner, 1993, Fishman 1996), in which an individual's behavior reveals information about a common trait that she shares with other group members.

with probability b at each period whether or not it invests. An H firm produces high quality with probability b if it does not invest but if it invests, it produces high quality with probability g at each period, where $0 < b < g \leq 1$. In either case the realized quality at period 2 is independent of its realization at period 1. The cost of investment is fixed at e and investment is "once and for all": Prior to period 1, each firm decides whether or not to invest and that, along with its type, determines the probability with which it produces high quality at periods 1 and 2. We denote by N_H and N_L the total number of H and L firms respectively, $N_L \geq N_H$, and by $r = \frac{N_H}{N_H + N_L}$ the proportion of H firms in the market.

Each consumer is in the market for one period, demands at most one (discrete) unit, and exits the market at the end of the period. Her utility from a low quality unit is zero, from one high quality unit is 1 and her utility from any additional unit is zero. A consumer buys if her expected utility from a unit is greater or equal to the price she pays.

In order to focus on the reputational effects of collective branding on investment incentives in the most direct way, it is convenient to assume that firms have monopolistic market power and can make take it or leave it offers to consumers. Specifically, a consumer is randomly matched with a firm and can either buy from that firm or not buy at all. Thus, if consumers' expected utility from a unit of firm i is v_i, firm i's price is assumed to be v_i. Thus branding has no effect on firms' pricing power or market share, and can only affect firms' investment incentives via reputational considerations.[4]

Consumers face both adverse selection and moral hazard; they cannot directly observe a firm's type (H or L) and also do not observe if it has invested. Firms learn their type after investing. The sequence of events is as follows. First each firm decides

[4]This could be because consumers have high transportation costs which effectively endows firms with local monopoly pricing power. Alternatively, consider a standard search model: A consumer knows only the price distribution but not which firm charges what price, is randomly and costlessly matched with one firm and can either buy from that firm or sequentially search for other firms, incurring a positive search cost at each search. As is well known, these assumptions imply that firms have monopoly pricing power (Diamond, 1971).

whether or not to invest. After investing (or not) each firm learns its type. Then the market opens at period 1. At this period consumers decide whether or not to buy from the firm with which they are matched when their only information about firms is r. At the beginning of period 2, before buying, consumers learn the realized quality of *each* firm at the preceding period (e.g., by interacting with consumers of the previous generation) and update their beliefs. Then the market opens at period 2. We assume that $2(rg - b) \geq e$, so that investment is efficient.

Remark 1: The assumption that firms learn their type after investing seems natural. For example, it is difficult to know if one has the aptitude to succeed in academic research before attempting to write a dissertation. Technically, this assumption enables us to use the relatively simple updating rule by which firms calculate their profit from investment derived below (1). Intuitively, our results should also hold, qualitatively, if firms know their type before investing, but the technical analysis would be considerably more daunting. We also note that if N_L and N_H are large, (1) is also approximately correct if firms know their type before investing.

Let $s_i = 0$ if firm i produced a low quality unit at period 1 and $s_i = 1$ if firm i produced a high quality unit at period 1. Let $S = (s_1, s_2, ..., s_N)$ be the industry profile of realized qualities. A consumer's belief about firm i is the probability with which she believes that the firm is an H firm and has invested.[5] As was mentioned above, at period 2 consumers are perfectly informed about S and thus their beliefs at period 2 may depend on S. Let B_1 denote consumers' belief at period 1, $B_1 \in [0,1]^N$, and $B_2(S)$ be consumers' belief at period 2, where

$$B_2 : S \longrightarrow [0,1]^N.$$

A firm's profit is the sum of its revenues at periods 1 and 2 less the investment cost, e, if it invests. A firm's strategy is whether or not to invest and is denoted by

[5] As far as a consumer is concerned, an H firm which has not invested is equivalent to an L firm since both produce high quality with the same probability.

$f \in \{I, NI\}$, where I means "invest" and NI means "don't invest".[6]

An *equilibrium* is a strategy f for each firm and consumer beliefs B_1 and $B_2(S)$ such that:

- Each firm's strategy f maximizes its profit, given the strategies of all other firms and consumer beliefs.
- B_1 and $B_2(S)$ are consistent with firms' strategies.
- Consumers maximize their expected utility (i.e., they buy if and only if the price is less or equal to the expected value of the good).

We seek to characterize symmetric pure strategy equilibria. Trivially, there always exists an equilibrium in which no firm invests.[7] The more interesting possibility is the existence of an 'investment equilibrium' (*IE*) in which firms invest. Since firms invest before learning their type, in a symmetric pure strategy IE, all firms invest.

Suppose there is an IE. Since at period 1 firms have no history and since firms invest, consumers believe that any firm is an H firm which invests with probability r. Therefore at period 1 the expected utility from any firm - and hence its price - is $rg + (1-r)b$.

At period 2, consumers are informed about S and update their beliefs. Let $\Pr(H \mid s_i, S_{-i})$ be the posterior probability - and hence consumers' belief[8] - at period 2 that a randomly selected firm i is type H when its realized quality at period 1 is s_i and those of the other firms is $S_{-i} \equiv (S \backslash s_i)$. Then the actual price of firm i at period 2 is $g \Pr(H \mid s_i, S_{-i}) + b(1 - \Pr(H \mid s_i, S_{-i}))$. However, since S is of course unknown at the time of investment, what is relevant for firms' investment strategy is the *expected* price, as

[6] We do not formally include a firm's price as part of its strategy since we assume that its price always equals consumers' expected utility.

[7] In this equilibrium consumers believe that no firm invests, which makes it optimal for firms not to invest.

[8] For any realization of s_i, S_{-i} consistent with firms' strategy, consumers' equilibrium beliefs must be consistent with Bayesian updating.

evaluated at the time of investment. This is calculated as follows. Let $E_{S_{-i}} \Pr(H \mid s_i, S_{-i})$ be the *expected* (with respect to S_{-i}) consumer belief at period 2 - *as evaluated by firm i at the time of investment, before learning its type* - that firm i is type H, given that its realized quality will be s_i. Thus:

$$
\begin{aligned}
E_{S_{-i}} \Pr(H \mid s_i, S_{-i}) &= \sum_{S_{-i}} \Pr(H \mid s_i, S_{-i}) \Pr(S_{-i} \mid s_i) = \sum_{S_{-i}} \frac{\Pr(H, s_i, S_{-i})}{\Pr(s_i, S_{-i})} \frac{\Pr(s_i, S_{-i})}{\Pr(s_i)} \\
&= \sum_{S_{-i}} \frac{\Pr(H, s_i, S_{-i})}{\Pr(s_i)} = \frac{\Pr(H, s_i)}{\Pr(s_i)} = \Pr(H \mid s_i). \qquad (1)
\end{aligned}
$$

That is, while consumers' actual belief at period 2 will depend on the realization of S_{-i}, their *expected* belief at the time of investment does not.

Thus if $p(s_i)$ is a firm's expected - as evaluated at the time of investment - second period price, conditional on its realized quality being s_i,

$$
\begin{aligned}
p(s_i) &= g E_{S_{-1}} \Pr(H \mid s_i, S_{-i}) + b(1 - E_{S_{-1}} \Pr(H \mid s_i, S_{-i})) \\
&= g \Pr(H \mid s_i) + b(1 - \Pr(H \mid s_i))
\end{aligned}
$$

Since an H firm which invests produces high quality with probability g and an L firm produces high quality with probability b, Bayes' rule gives (henceforth we omit subscript i):

$$
\Pr(H \mid h) = \frac{gr}{gr + b(1-r)}
$$

$$
\Pr(H \mid l) = \frac{(1-g)r}{(1-g)r + (1-b)(1-r)}
$$

and thus:

$$
\begin{aligned}
p(h) &= g \Pr(H \mid h) + b(1 - \Pr(H \mid h)) \qquad (2) \\
&= b + (g-b) \Pr(H \mid h) = b + \frac{(g-b)gr}{gr + b(1-r)}
\end{aligned}
$$

and similarly:

$$
\begin{aligned}
p(l) &= g \Pr(H \mid l) + b(1 - \Pr(H \mid l)) \qquad (3) \\
&= b + (g-b) \Pr(H \mid l) = b + \frac{(g-b)(1-g)r}{(1-g)r + (1-b)(1-r)}.
\end{aligned}
$$

Let R and R_{-1} be the expected second period revenues of a firm that invests and doesn't invest respectively:

$$R = r\left[gp(h) + (1-g)p(l)\right] + (1-r)\left[bp(h) + (1-b)p(l)\right] \tag{4}$$

and

$$R_{-} = bp(h) + (1-b)p(l) \tag{5}$$

Thus an H firm's expected gain from investment is $e^* \equiv R - R_{-}$ and thus by (2) - (5):

$$e^* = r(g-b)^2\left[\frac{gr}{gr + b(1-r)} - \frac{(1-g)r}{(1-g)r + (1-b)(1-r)}\right].$$

Thus:

Proposition 1 *When firms stand alone an IE exists if and only if* $e \leq e^*$*.*

In the 'stand alone' setting, firms have only a limited opportunity to establish a reputation for quality, since consumers' information is limited to one observation per firm. Hence if $e > e^*$, an *IE* does not exist because the cost of investment exceeds the individual firm's expected return from acquiring a good reputation.

3 Collective Branding

In this section we extend the setup of the previous section to allow otherwise autonomous firms to market their products under a shared brand name and show that, in contrast to the stand alone setting, investment equilibria may exist even when $e > e^*$. The idea is that when the products of two or more firms share a common brand name, consumers may condition their beliefs about a specific firms' type based on the past performance of *all* the brand's members rather than on the firm's individual performance alone. Thus, branding may provide consumers with better information which may in turn increase the incentive of firms to invest.

The timing of events is now modified as follows. After firms invest and learn their types, and before the market opens at period 1, brands are formed as described below.

In order to facilitate the comparison of collective brands with stand alone firms, it is convenient to assume that consumers are aware of firms' brand affiliation only at the second period, so that at period 1 consumers' beliefs and firms' revenue are the same in both settings. Thus any effect of branding on investment incentives can now only be due to its effect on second period revenues.

Formally, a *collective brand assignment* is a partition of the N firms. Let $\wp$ be the set of all the possible partitions of the N firms and let $P \in \wp$. Each element $Q \in P$ is called a collective brand and each firm $i \in Q$ assigned to Q by P is called a member of brand Q. The rule which determines how individual firms are assigned to brands is called a *brand assignment rule.* Let $\pi_i(Q)$ denote firm $i's$ profit as a member of brand Q and let π_i be its profit if it stands alone.

In this setting firms' strategies and consumers' beliefs at period 2 may depend not only on S but also on P. That is,

$$f : \wp \longrightarrow \{I, NI\}$$

$$B_2 : \wp \times S \longrightarrow [0,1]^N$$

We define a BE (Brand Equilibrium) by $P \in \wp, f, B_1$ and B_2 such that:

E.1 Each firm's strategy f maximizes its profit, given the strategies of all other firms and consumer beliefs.

E.2 B_1 and $B_2(\wp, S)$ are consistent with firms' strategies.

E.3 (individual rationality) For each $Q \in P$ and $i \in Q$, $\pi_i(Q) \geq \pi_i$. That is, if a firm is assigned to brand Q by P, membership in Q must be at least as profitable as standing alone.

E.4 $\nexists i, Q \in P$ s.t. $: \forall j \in Q,\ i \notin Q,\ i \in Q' \in P,\ \pi_j(Q \cup \{i\}) \geq \pi_j(Q), \pi_i(Q \cup \{i\}) \geq \pi_i(Q')$, with the inequality strict for at least one j or i. That is, adding an additional member to brand $Q \in P$ can not increase both its profit and the profit of existing (assigned) members of Q.

For any $m \in \{1, ..., N_H\}$, such that $\frac{N_H}{m}$ and $\frac{N_L}{m}$ are integers, let $n_H^m = \frac{N_H}{m}$ and $n_L^m = \frac{N_L}{m}$. Define an m *partition* as a partition consisting of n_H^m brands, each of which has exactly m type H members - henceforth called H brands - and n_L^m brands each of which has exactly m type L members - henceforth called L brands.

While the above definition of branding equilibria does not include an explicit description of the brand formation process, the assumption that brands are differentiated by firm type implies that firms can detect each others' type, while consumers cannot. This seems reasonable, as firms are market "professionals", while consumers are not.

We refer to the number of firms which are members of a brand as the *brand size* and define a BIE as a BE in which all firms invest. Let $q = \frac{n_H^m}{n_H^m + n_L^m}$ be the proportion of H brands.

Remark 2: Note that the "m *partition*" branding equilibria that we consider have the property that L and H firms are "pooled" in brands of the same size. If the brand sizes of L and H firms differed, or if all L firms stood alone, brand size would perfectly reveal the firms' type to consumers. But then firms would have no incentive to invest.

We analyze branding equilibria under two alternative regimes. Under *perfect monitoring*, H firms which don't invest may be excluded from membership in H brands. The interpretation is that the "brand" can detect if a firm has invested and exclude those which don't. By contrast, in the *no-monitoring* regime, membership in an H brand cannot be conditioned on investment. The interpretation is that failure to invest is undetectable and cannot jeopardize brand membership.

3.1 Perfect Monitoring

In this section we analyze collective branding under perfect monitoring. Let e_m be the largest value of e for which a BIE exists for an m partition under perfect monitoring.

Proposition 2 *Corresponding to every* $m \in \{2, ..., N_H\}$ *such that* $\frac{N_H}{m}$ *and* $\frac{N_L}{m}$ *are integers:*

(i) $e_m > e^*$.

(ii) e_m *is strictly increasing in* m.

Proof of proposition: The proof is by construction. Let the brand assignment rule be: Each H firm which invests is assigned to an H brand of size m and each L firm is assigned to an L brand of size m. If an H firm doesn't invest, it is assigned to one of the L brands (recall that under perfect monitoring such exclusion from H brand membership is feasible) and one L firm is assigned to an H brand in its place (so that in this case one of the H brands ends up with $m-1$ type H members and one type L member, and one L brand ends up with $m-1$ type L members and one type H member)[9]. Let consumer beliefs (at period 2) be: a stand alone firm or a firm which is a member of a brand of size $\neq m$ is either type L or has not invested.

Thus if all firms invest there are n_H^m H brands, each member of which is type H and n_L^m L brands, each member of which is type L. Let a brand's *record* be the total number of high quality units produced by all the members of the brand at period 1. Denote the record of brand i of size m as $s_i^m \in \{0, 1, ..., m\}$, let $S^m = (s_1^m, ..., s_{n_H^m + n_L^m}^m)$, and let $S_{-i}^m \equiv (S^m \backslash s_i^m)$. Let $\Pr(H^m \mid s_i^m, S_{-i}^m)$ be the posterior probability, and therefore consumers' belief at period 2, that, given S_{-i}^m, and s_i^m, brand i of size m, is an H brand. To simplify notation, in the remainder of the proof we omit the subscript and superscript of s_i^m when this does not lead to any ambiguity. By a completely analogous argument

[9]This rule ensures that the 'threat' to exclude H firms which fail to invest from membership in H brands does not change brand sizes and hence does not affect consumer beliefs which depend on brand size.

to (1), consumers' *expected* (with respect to S^m_{-i}) belief - *as evaluated at the time of investment* - that a brand with record s is an H brand is given by:

$$\Pr(H^m \mid s) = \frac{qg^s(1-g)^{m-s}}{qg^s(1-g)^{m-s} + (1-q)b^s(1-b)^{m-s}} \tag{6}$$

Thus, conditional on the brand's realized record being s, the expected revenue (price) of each member of an brand of size m at period 2 is given by $p^m(s)$:

$$\begin{aligned} p^m(s) &= g\Pr(H^m \mid s) + b(1 - \Pr(H^m \mid s)) \\ &= b + (g-b)\Pr(H^m \mid s) \end{aligned} \tag{7}$$

Let R^m_L be a firm's expected revenue at period 2 - as evaluated at the time of investment - conditional on turning out to be type L and a member of an L brand of size m. Then

$$R^m_L \equiv \sum_{s=0}^{m} \binom{m}{s} b^s(1-b)^{m-s}p^m(s) \tag{8}$$

Similarly, let R^m_H be a firm's expected revenue at period 2 - as evaluated at the time of investment - conditional on turning out to be type H and a member of an H brand of size m. Then

$$R^m_H = \sum_{s=0}^{m} \binom{m}{s} g^s(1-g)^{m-s}p^m(s) \tag{9}$$

Thus, at the time of investment, the expected revenue of a firm which invests is:

$$R^m = rR^m_H + (1-r)R^m_L \tag{10}$$

Given that all other firms invest, a firm's expected profit if it invests is $R^m - e$ while if it doesn't invest its expected profit is R^m_L. Thus investment is optimal if $R^m - R^m_L \geq e$. The following lemma is proved in the Appendix.

Lemma 1 *For every $m \geq 1$, $R^m - R^m_L$ is increasing with m .*

Let $\varepsilon_m \equiv R^m - R^m_L$. By equations (4) and (8) - (10), $R^1 = R$, and by (5) and (8), $R^1_L = R_-$. Hence by Lemma 1, and the definition of e^* it follows that for $m \geq 2$:

$$\varepsilon_m = R^m - R^m_L > R^1 - R^1_L = R - R_- = e^*.$$

Let $e_m = \varepsilon_m$. Thus, if $m \geq 2$, $R^m - R^m_L > e^*$ and thus investment is optimal if $e > e^*$

Since by (6) - (8), $R^m_L \geq b$, and since, given consumer beliefs, a stand alone firm's profit is b (whether or not it invests), it follows that brand membership is more profitable for an L firm, and a fortiori for an H firm, than standing alone, and thus condition *E.3* is satisfied. It is also obvious that condition *E.4* is satisfied. This completes the proof of part (i) of the proposition[10]. Part (ii) then follows directly from Lemma 1.■

Thus under collective branding with perfect monitoring, there are multiple brand sizes which can support investment equilibria when $e > e^*$. As the preceding proposition establishes, these may be ranked in terms of their effect on investment: The larger is the brand size, m, the greater is the range of investment costs for which investment is sustainable in equilibrium. In particular, the largest investment cost under which investment is sustainable corresponds to the brand size N_H where all H firms are in the same brand. Thus "bigger is better" in the sense that the larger is N_H, the larger the range of investment costs which can support equilibrium investment. The same observation applies to the relationship between brand size and firm profits: As the proof of the proposition makes clear, the larger the equilibrium brand size, the greater the H firms' profit and the lower the L firms' profit[11].

[10] The above equilibrium was constructed under the assumption that there exists m such that $\frac{N_H}{m}$ and $\frac{N_L}{m}$ are integers. However, such equilibria exist more generally. Specifically, for any m such that $\frac{N_H}{m}$ is an integer (which always the case for $m = N_H$), let $I\{\frac{N_L}{m}\}$ be the largest integer $\leq \frac{N_L}{m}$, let there be $\frac{N_H}{m}$ H brands, $I\{\frac{N_L}{m}\}L$ brands and $N_L - I\{\frac{N_L}{m}\}m$ stand alone L firms. Then, although the construction is more complicated, a similar equilibrium to that of proposition 2 may be constructed in which the profit of stand alone L firms is b.

[11] This suggests that the equilibrium brand size $m = N_H$ is supported by more plausible consumer beliefs than $m < N_H$. Specifically, as is shown in the proof of the proposition, equilibria in which $m < N_H$ require that consumers believe that a brand of size larger than m is either type L or type H which doesn't invest. But, it is precisely the H firms which would profit, while L firms would lose, if the brand size increased, as long as consumers believed that a brand size $> m$ with a record greater or equal to that of a brand of size m is at least as likely to invest. Thus, consumer beliefs which associate larger brand size with lower quality seem somewhat unpalatable. By contrast, consumers appropriately associate a brand size larger than N_H with lower quality because such a brand must include at least some L firms.

Although the preceding argument suggests a theory of 'mega' brands, this conclusion must be tempered once the assumption of perfect monitoring is relaxed, as the following section shows.

3.2 No-Monitoring

We now turn to examine the extent to which the analysis of the previous section applies in the case of no-monitoring. In this setting failure to invest cannot prevent a firm from using the brand label and thus firms have less of an incentive to invest than in the perfect monitoring regime. Nevertheless, the following proposition establishes that if g is sufficiently large, collective branding can still incentivize investment when stand alone firms will not invest. Let $\widetilde{e}_m$ be the largest value of e for which a BIE exists for an m partition under no-monitoring.

Proposition 3 *Under no-monitoring, for every $m \in \{2, ..., N_H\}$ such that $\frac{N_H}{m}$ and $\frac{N_L}{m}$ are integers there is $g(m) < 1$ such that if $g \geq g(m)$, $\widetilde{e}_m > e^*$.*

Proof: Let the brand assignment rule be that every H firm is assigned to an H brand of size m and every L firm is assigned to an L brand of size m (in contrast to the perfect monitoring setting, here the assignment rule cannot condition brand membership on investment). Suppose that all firms invest, and let s_i^m, S^m, S_{-i}^m , $p^m(s)$, R_H^m, R_L^m and R^m and consumer beliefs be the same as in the proof of proposition 2. Thus, at the time of investment, the expected revenue of a firm which invests is R^m. Let R_{-1}^m be the expected revenue of a firm which doesn't invest. If it turns out to be type H, then whether or not it invested, it will be assigned to an H brand (in which all $m-1$ other members invest) and if it turns out to be type L it will assigned to an L brand. Thus [12]

$$R_{-1}^m = r \sum_{s=0}^{m-1} \binom{m-1}{s} g^s (1-g)^{m-1-s} \left[(1-b)p^m(s) + bp^m(s+1)\right] + (1-r)R_L^m \quad (11)$$

[12] $\binom{m-1}{s} g^s(1-g)^{m-1-s}$ is the probability that the other, $m-1$ investing firms, produce s high quality units. With probability $1-b$ the firm which doesn't invest produces low quality in which case the brand produces s high quality units and each member receives the price $p^m(s)$. With probability b the m-th firm produces high quality and the price is $p^m(s+1)$.

Let $\widetilde{\varepsilon}_m \equiv R^m - R^m_{-1}$. Thus investment is optimal if $e \leq \widetilde{\varepsilon}_m$.

The following lemma, proved in the appendix, shows that an analogous result to Lemma 1 applies under no monitoring if $g = 1$.

Lemma 2 *Under no- monitoring, if $g = 1$, $\widetilde{\varepsilon}_m$ is strictly increasing in m for $m \geq 1$.*

By equations (4) and (8) - (10), $R^1 = R$, and by (5), (8) and (11), $R^1_{-1} = R_-$. Hence $\widetilde{\varepsilon}_1 = R^1 - R^1_{-1} = e^*$. Thus it follows from the lemma that if $g = 1$, then $\widetilde{\varepsilon}_m > e^*$ for all $m > 1$. By equations (6) - (11), $\widetilde{\varepsilon}_m$ is continuous in g, implying that there is $g(m) < 1$, such that for $g \geq g(m)$, $\widetilde{\varepsilon}_m > e^*$. Finally, let $\widetilde{e}_m = \widetilde{\varepsilon}_m$.

Given consumer beliefs, the revenue of a firm which stands alone is $b < R^m_{-1}$ where the inequality follows from (7) and (11). Thus conditions E.3 and E.4 are satisfied. This completes the proof. ■

The intuition behind Proposition 3 is that the incentive to free ride on the investment of other brand members reflects the adverse effect of a single low quality observation on the brand's reputation. If g is sufficiently large, even a single low quality unit sufficiently tarnishes the brand's reputation to deter free riding.

However, this is true only as long as the brand size is not "too large". Once the brand size is sufficiently large, the effect of a single low observation on the brand's reputation is too small to deter free riding. Therefore, in contrast to the case of perfect monitoring, under no-monitoring it is not generally true that 'bigger is better'. In fact, the following proposition shows that under no-monitoring, for sufficiently large m, a *BIE* for an m partition does not exists for $e \geq e^*$.

Proposition 4 *Under no-monitoring, for every $g < 1$, there is $m(g)$ such that for $m \geq m(g)$, $\widetilde{e}_m \leq e^*$.*

Proof: In the Appendix.

Thus, if $\widetilde{m}$ denotes the brand size which maximizes $\widetilde{e}_m$ - the brand size for which a BIE exists for the largest range of investment costs - then, $\widetilde{m} < N_H$ if N_H is sufficiently large, in contrast to the case of perfect monitoring. [13]

3.3 Relationship to Umbrella Brands

Umbrella branding is the practice by which multiproduct firms market otherwise unrelated products under the same brand name in order to signal quality. How do the incentives of collective brands to invest in reputation compare with those of umbrella brands? To address this question in our setting, consider an m partition each element of which is now a multiproduct firm which makes investment decisions for, bears the investment costs of and owns the the profits of each 'member' (product). Thus, if the umbrella brand is size m, and the price of each of its members (products) is p, the brand's revenue is pm. We compare the umbrella brand's investment incentives with those of the collective brand under no-monitoring[14].

In the case of collective brands under no-monitoring, the highest investment cost for which a BIE exists for an m partition is $\widetilde{e}_m = R^m - R^m_{-1}$. If the umbrella brand of size m invests in all its members, then its second period expected profit is $m(R^m - e)$. For the same reason, if it invests in only $m-1$ of its products, its profit is $m(R^m_{-1} - e) + e$. Thus an umbrella brand of size m invests in all its products if:

$$m(R^m - e) - m(R^m_{-1} - e) - e = m(R^m - R^m_{-1}) - e = m\widetilde{e}_m - e \geq 0$$

Thus, while a BIE exists for collective brands only if $e \leq \widetilde{e}_m$, in the case of umbrella brands it exists if $e \leq m\widetilde{e}_m$. Thus umbrella branding incentivizes investment more than collective branding.

The intuition for this is straightforward. In the cases of both collective brands and umbrella brands, a low quality realization of one member reduces the reputation of the

[13] Also, in contrast to perfect monitoring, the equilibrium brand size which is most profitable for H firms $> \widetilde{m}$ if N_H is sufficiently large. This is because $\widetilde{m}$ maximizes $R^m - R^m_{-1}$ while the most profitable brand size for a given e is the largest m for which $R^m - R^m_{-1} \geq e$.

[14] The appropriate comparison is to no-monitoring because under perfect monitoring brand members have no discretion with respect to investment decisions while the owner of the umbrella brand can decide in which products to invest.

entire brand. In the case of the collective brand, individual members are only concerned about how this affects the value of their own product. By contrast, the umbrella brand internalizes the effect of its investment in each of its products on the reputation of its entire product line.

3.4 Costly Monitoring

We have considered two polar regimes; *perfect monitoring*, in which only H firms which invest join H brands, and *no monitoring*, in which non - investors cannot be excluded from membership in H brands and therefore invest only if investment is individually optimal. Consider an intermediate case in which the brand cannot detect failure to invest and, accordingly, membership in an H brand requires a firm to incur a fixed cost of c to verify that it invests - for example by hiring a reliable external auditor to certify its investment[15]. Then, a brand member's profit is $R^m - e - c$ while the profit from standing alone is b. Thus, a BIE exists for the m partition if $R^m - (e + c) > b$. Thus, since R^m increases with m, investment incentives and H firms' profit increase with m, just as in the case of perfect monitoring without monitoring costs. This also suggests that under monitoring costs there is a minimal brand size - the brand must be large enough for reputational gains associated with increased size to cover monitoring costs in addition to investment costs.

3.5 Franchising

Franchising shares some features of umbrella brands and some features of collective brands. The franchisor collects a share of each franchisee's revenues - and thus benefits from the investment of each outlet - but franchisees bear investment costs. In practice, franchisors tend to monitor franchisees quite closely, by contractually requiring that the service be in accordance with the pattern determined by the franchisor, through field support, external service audits, peer review and consumer feedback (Spinelli Jr,

[15] Alternatively and equivalently, the cost c is shared by all brand members.

Rosenberg, Birley, 2004), all of which suggests that quality assurance is costly to the franchisor. Thus, if the franchisor incurs a monitoring cost c for each franchisee that it monitors and gets a fraction α of franchisees' revenue, its profit is $m(\alpha R^m - c)$ which, since R^m is increasing, increases with m. This suggests that, as in the case of collective brands with perfect or costly monitoring, the franchisor's profit increases with the number of franchisees and that the number of franchisees must be large enough for reputational gains to cover monitoring costs.

Indeed, leading franchise chains are huge and seem to strive for unlimited growth. For example, in the US alone, there are over 20,000 Subway, 14,000 McDonalds , 7000 Pizza Hut, 11000 Starbucks and 13000 H&R Block tax preparation locations. However, it should be noted that the number of chain outlets or locations can greatly exaggerate the number of "brand members" since franchisees often own multiple units. Indeed, the policy of many large chains is to actively encourage franchisees to take on multiple outlets. For example, Domino's Pizza and Subway offer reduced fees for franchisees that acquire further units (see https://www.businessfranchise.com/special-features/multiple). According to NatWest/BFA Franchise Survey 2008, one fifth of franchisees own multiple units, with an average of seven units each. This policy might be designed to reduce monitoring costs,. First, owners of multiple units have more of an incentive to internalize the effects of their investment on the brand reputation than owners of single units (particularly if the outlets owned are in close geographical proximity). Second, it may be cheaper for the franchisor to monitor owners of multiple units than single unit owners. For example, the former can be effectively monitored by retaliating against all its units if the quality of one randomly sampled unit is defective, while in the case of single unit owners, it is necessary to monitor each unit individually.[16]

[16] Moreover, there is some evidence that monitoring by franchisors is less than perfect, possibly to save on monitoring costs. For example, Jin and Leslie (2008) show that within a chain, company owned restaurants tend to have better hygiene than franchisee owned restaurants, suggesting at least some free riding by franchisees on the chain reputation. Relatedly, Ater and Rigby (2012) show that chain outlets at locations in which repeat business is infrequent tend to be company owned, possibly to save on monitoring costs at locations in which individual incentives to free ride are particularly strong.

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4 Appendix

4.1 Proof of Lemma 1

By equations (6), (7) and (9)

$$\begin{aligned} R_H^m &= b+(g-b)\sum_{s=0}^{m}\binom{m}{s}g^s(1-g)^{m-s}\frac{qg^s(1-g)^{m-s}}{g^s(1-g)^{m-s}q+b^s(1-b)^{m-s}(1-q)} \\ &= b+(g-b)\sum_{s=0}^{m}\binom{m}{s}g^s(1-g)^{m-s}\frac{q}{q+(1-q)x_s^m} \\ &= b+(g-b)\sum_{s=0}^{m}\binom{m}{s}g^s(1-g)^{m-s}k_s^m \end{aligned}$$

where

$$x_s^m \equiv \frac{b^s(1-b)^{m-s}}{g^s(1-g)^{m-s}} \quad \text{and } k_s^m \equiv \frac{q}{q+(1-q)x_s^m}$$

Let S be a binomial random variable with the parameters (m, g). Let

$$X^m \equiv \frac{b^S(1-b)^{m-S}}{g^S(1-g)^{m-S}} \quad \text{and } K^m \equiv \frac{q}{q+(1-q)X^m}$$

Note that

$$E(X^{m+1} \mid X^m) = g\frac{b^{S+1}(1-b)^{m-S}}{g^{S+1}(1-g)^{m-S}} + (1-g)\frac{b^S(1-b)^{m+1-S}}{g^S(1-g)^{m+1-S}} = bX^m + (1-b)X^m = X^m$$

implying that $X^1, X^2, X^3, ...$ is a martingale. Since $X^m \geq 0$, K^m is a strictly convex function of X^m, then by Jensen's Inequality, $EK^{m+1} > EK^m$. Hence,

$$R_H^{m+1} = b+(g-b)\sum_{s=0}^{m+1}\binom{m+1}{s}g^s(1-g)^{m+1-s}k_s^{m+1} > b+(g-b)\sum_{s=0}^{m}\binom{m}{s}g^s(1-g)^{m-s}k_s^m = R_H^m$$

which proves that R_H^m is increasing with m.

Substitute equations (6) and (7) into (8) yielding

$$\begin{aligned} R_L^m &= b+(g-b)\sum_{s=0}^{m}\binom{m}{s}b^s(1-b)^{m-s}\frac{qg^s(1-g)^{m-s}}{g^s(1-g)^{m-s}q+b^s(1-b)^{m-s}(1-q)} \\ &= b+(g-b)\sum_{s=0}^{m}\binom{m}{s}g^s(1-g)^{m-s}\frac{qx_s^m}{qx_s^m+(1-q)} \end{aligned}$$

Since $\frac{qX^m}{qX^m+1-q}$ is a concave function of X^m, by Jensen's Inequality

$$E\frac{qX^{m+1}}{qX^{m+1}+1-q} < E\frac{qX^m}{qX^m+1-q}$$

implying

$$\begin{aligned} R_L^{m+1} &= b+(g-b)\sum_{s=0}^{m+1}\binom{m+1}{s}g^s(1-g)^{m+1-s}\frac{qx_s^{m+1}}{qx_s^{m+1}+1-q} \\ &< b+(g-b)\sum_{s=0}^{m}\binom{m}{s}g^s(1-g)^{m-s}\frac{qx_s^m}{qx_s^m+1-q} = R_L^m \end{aligned}$$

which proves that R_L^m is decreasing with m. Thus and since by (10) $R^m - R_L^m = r(R_H^m - R_L^m)$, it is increasing with m. ■

4.2 Proof of Lemma 2

Proof: When $g = 1$, the $m - 1$ investing firms produce high quality with certainty. If the mth firm doesn't invest it produces high quality with probability b, in which case its revenues (and that of every other member of the brand) are R^m. With probability $1 - b$ it produces low quality in which case $s = m - 1$ and, by equations (6) and (7) $\Pr(H^m \mid m - 1) = 0$ and $p^m(s) = b$. Hence,

$$R^m_{-1} = r\left[bR^m_H + (1-b)b\right] + (1-r)R^m_L$$

Hence, and by equation (10) if $g = 1$,

$$R^m - R^m_{-1} = rR^m_H - r\left[bR^m_H + (1-b)b\right] = r(1-b)(R^m_H - b)$$

It follows that

$$\widetilde{\varepsilon}_m = R^m - R^m_{-1} = r(1-b)(R^m_H - b)$$

Since by Lemma 1 R^m_H is increasing with m, it follows that $\widetilde{\varepsilon}_m$ is increasing with m. ■

4.3 Proof of Proposition 4

The proof is using the following Claim.

Claim

$$R^m_H = \sum_{s=0}^{m-1} \binom{m-1}{s} g^s (1-g)^{m-1-s} \left[gp^m(s+1) + (1-g)p^m(s)\right] \tag{12}$$

Proof of the Claim: Let s' be the number of high quality units produced by any given group of $m-1$ members of an H brand of size m. Since the mth firm invests, it produces high quality with probability g and low quality with probability $1-g$. Hence, the brand produces $s'+1$ high quality units and receives a price of $p^m(s'+1)$ with probability g and produces s' high quality units and receives a price of $p^m(s')$ with probability $1-g$. Since the probability that $m-1$ members produce s' high quality units is $\binom{m-1}{s'} g^{s'} (1-g)^{m-1-s'}$

it follows that

$$R_H^m = \sum_{s=0}^{m-1} \binom{m-1}{s} g^s (1-g)^{m-1-s} \left[g p^m(s+1) + (1-g) p^m(s) \right]$$

which proves the Claim.

Using equations (10) - (12)

$$\begin{aligned} \widetilde{\varepsilon}_m &= R^m - R_{-1}^m = rR_H^m - r\sum_{s=0}^{m-1} \binom{m-1}{s} g^s (1-g)^{m-1-s} \left[(1-b) p^m(s) + b p^m(s+1) \right] \\ &= r(g-b) \sum_{s=0}^{m-1} \binom{m-1}{s} g^s (1-g)^{m-1-s} \left[p^m(s+1) - p^m(s) \right] \end{aligned}$$

Substituting for $p^m(s)$ from equations (6) and (7) and recalling from the proof of Lemma 1 that $x_s^m \equiv \frac{b^s(1-b)^{m-s}}{g^s(1-g)^{m-s}}$:

$$\widetilde{\varepsilon}_m = r(g-b)^2 \sum_{s=0}^{m-1} \binom{m-1}{s} g^s (1-g)^{m-1-s} \frac{q(1-q)(x_s^m - x_{s+1}^m)}{\left[q + (1-q)x_{s+1}^m \right] \left[q + (1-q)x_s^m \right]}.$$

Substituting

$$x_s^m - x_{s+1}^m = \frac{b^s(1-b)^{m-s}}{g^s(1-g)^{m-s}} - \frac{b^{s+1}(1-b)^{m-s-1}}{g^{s+1}(1-g)^{m-s-1}} = \frac{b^s(1-b)^{m-s-1}}{g^s(1-g)^{m-s-1}} \left(\frac{1-b}{1-g} - \frac{b}{g} \right)$$

yields

$$\widetilde{\varepsilon}_m = r(g-b)^2 \sum_{s=0}^{m-1} \binom{m-1}{s} b^s (1-b)^{m-s-1} \frac{q(1-q)\left(\frac{1-b}{1-g} - \frac{b}{g} \right)}{\left[q + (1-q)x_{s+1}^m \right] \left[q + (1-q)x_s^m \right]}.$$

Hence, and since $\lim_{m\to\infty} x_s^m = \infty$ and $\sum_{s=0}^{m-1} \binom{m-1}{s} b^s (1-b)^{m-s-1} = 1$ it follows that

$$\lim_{m\to\infty} \widetilde{\varepsilon}_m = 0$$

and the lemma follows immediately. ■