The Interdisciplinary Center, Herzlia
Efi Arazi School of Computer Science
M.Sc. program - Research Track

Bitcoin+: Cheap Support For Complex Spending Conditions in a UTXO Ledger

by
Yaron Kaner

M.Sc. dissertation, submitted in partial fulfillment of the requirements
for the M.Sc. degree, research track, School of Computer Science
The Interdisciplinary Center, Herzliya

September 2020

This work was carried out under the supervision of Dr. Tal Moran from the Efi Arazi School of Computer Science, The Interdisciplinary Center, Herzliya.

Acknowledgements

I would like to express my deepest appreciation and gratitude to my advisor, Dr. Tal Moran from the Interdisciplinary Center (IDC), for the great part he had in this work, for his uncompromising mission towards quality and accuracy, and for the countless hours he devoted to mentoring me. His patience and never-ending willingness to teach are what made this journey both possible and enjoyable.

Abstract

Cryptocurrencies can be used merely to transfer value between identities, but many of the more interesting uses of cryptocurrencies require contracts, e.g, “a transfer of X coins from party S to party R should occur only if conditions A and B hold”. Bitcoin (and related cryptocurrencies) place strict limits on the language in which these conditions can be phrased. In particular, they have limited length and don’t allow loops.

In this work, we show how to augment the Bitcoin scripting language with a single “innocuous” operation to that allows us to create “meta conditions” with much more expressive power (e.g., as defined by arbitrarily-sized circuits).

We construct a protocol to compile such meta-conditions into a set of (augmented) Bitcoin transactions. We then show how to use this compiler to realize a full “meta-ledger” functionality, which we show is secure in the universal composability framework.

Contents

1 Introduction

Introduced by Satoshi Nakamoto in 2008 [8], Bitcoin is the first decentralized cryptocurrency, and still the most popular in terms of market cap and transaction volume (as measured in USD).

Bitcoin's design is based on a *transaction ledger*. Loosely, a transaction ledger accepts *transactions* as input from honest parties and outputs an ordered list of transactions (which all honest parties agree on).

Bitcoin Transactions and the UTXO Ledger

The transaction is the primary abstraction in Bitcoin; there are no separate notions of "accounts" or "balances". Instead, each Bitcoin transaction has inputs and outputs. Every transaction output (TXO) specifies an amount (in Bitcoins) and a *spending condition*, and every input references a single TXO and has some additional "proof data".

A transaction output can be thought of as a "coin" whose value is the amount specified by the output. "Spending" the coin is done by publishing a new transaction whose input references the corresponding TXO. The spending condition specifies who is allowed to spend the coin.

In order to verify the validity of a transaction, nodes need to store only the set of unspent transaction outputs—the *UTXO database*, for which this type of ledger is named.

Scripts and Smart Contracts

The most common type of spending condition is verification of a signature with respect to a specific public key: an input can spend such a TXO if its proof data contains a valid signature (under the corresponding public key) of the spending transaction's ID (a collision-resistant hash of the spending transaction).

While signature verification is sufficient for simple transfer of funds, one of the innovations introduced by Bitcoin is a *scripting language* for specifying output conditions. This allows a novel use of cryptocurrencies: the creation of *smart contracts*—cryptographic "boxes" that contain value and "automatically" unlock it if certain conditions are met.

Bitcoin scripting conditions can define more complex "ownership structures" (e.g., funds can be spent if any k out of n public keys sign the spending transaction), can postpone spending until some point in the future ("time-locked transactions") or even require some cryptographic puzzle to be solved before an output can be spent.

Limited Expressiveness

The Bitcoin scripting language is kept purposefully simple. A major motivation for this decision is that the cost of evaluating the spending conditions is borne by *every* Bitcoin node—every node must verify all transactions in the ledger to ensure that they are in consensus on the state of the UTXO database. This cost is offset by a *transaction fee*. However, because the fee is deducted from the spent TXOs, there is no way to charge fees for invalid transactions (which are not "allowed" to spend the TXO at all). If an expensive computation is required to determine transaction validity, this would open the door to cheap denial-of-service (DoS) attacks.

Thus, Bitcoin script is designed to ensure that the execution cost is always known and bounded. In particular, Bitcoin scripts are not Turing complete; in fact, the language does not allow loops of any kind, and scripts have bounded length. Moreover, the set of basic arithmetic operations is also quite limited.

While the existing functionality already allows for interesting use-cases, there are many for which Bitcoin's scripting language is insufficiently expressive. (One interesting example, and a motivation for this work, is the notion of secure computation "with penalties", in which parties run a secure multiparty computation protocol *without* an honest majority, but with the guarantee that if the adversary aborts the computation the honest parties will be compensated financially.)

1.1 Our Contributions

This work aims to increase the expressiveness of the Bitcoin scripting language, without sacrificing the simplicity and efficiency of the UTXO ledger.

Cheap Support for Arbitrary Circuit Conditions

Our main concrete contribution is a protocol that allows a TXO to be bound to an arbitrary boolean circuit—i.e., the output can be spent only by publishing a satisfying assignment for the circuit.

The protocol relies on adding a single new opcode to the Bitcoin scripting language, one that allows a spending condition to test whether a given TXO is in the UTXO database. We refer to this opcode as IS-TXO-UNSPENT, and to the Bitcoin scripting language augmented by the opcode as *Bitcoin+*.

Because executing this opcode only requires access to the existing UTXO database, in exactly the same way as testing for a double-spend, adding it to existing Bitcoin code is very simple, and it has minimal execution overhead (we verified this by modifying the Bitcoin core code to include the new opcode; this required changes to less than 40 lines of code, including additional comment lines).

Universally Composable Overlay Ledgers

Existing protocols that require complex spending conditions are often written assuming an ideal ledger functionality. To allow modular use of our compiler in existing such protocols we formally define a generalized ideal UTXO Ledger functionality, $\mathcal{F}_{ledger}^{\mathcal{L}_{spend}}$, which is parameterized by the language in which spending conditions may be specified.

The ideal functionality preserves "in spirit" the guarantees of the simple UTXO ledger (i.e., the guarantees relied on by most high-level ledger protocols), but at the same time allows a single transaction in the meta-ledger to be represented by multiple transactions in the base-ledger. Defining such a functionality turns out to be quite tricky, since some of the properties of a simple UTXO cannot be guaranteed in this setting. (In chapter 3, we provide an overview of the challenges we faced in defining the generalized ledger functionality and the intuitions behind the relaxations we made to the simple UTXO ledger.)

Our model is in the same spirit as that of Badertscher, Maurer, Tschudi and Zikas [1]. However, we focus specifically on a *UTXO* ledger, and the required relaxations to support meta (overlay) ledgers as instances of the same functionality.

To simplify the protocol descriptions and proofs, we model the ledger as a sequence of transactions, completely ignoring their grouping into *blocks*—this grouping is irrelevant for most higher-level protocols but adds complexity to the model.

We construct a protocol in the $\mathcal{F}_{ledger}^{\mathcal{L}}$-hybrid model (i.e., with access to an ideal UTXO ledger whose spending conditions may be specified in the language $\mathcal{L}$) that realizes the $\mathcal{F}_{ledger}^{\mathcal{L}\cup\mathcal{L}_{\text{Circuit}}}$ ledger functionality, where $\mathcal{L}_{\text{Circuit}}$ is the language of boolean circuits. Our protocol works for every "base" language $\mathcal{L}$ that is sufficiently expressive (the Bitcoin scripting language, augmented with IS-TXO-UNSPENT, is an example).

Our proof of security is in the Externalized UC model [2], where we model the clock as an external shared functionality. This allows any protocol that is secure when using the ideal ledger functionality $\mathcal{F}_{ledger}^{\mathcal{L}\cup\mathcal{L}_{\text{Circuit}}}$ to be transparently "compiled" into a protocol that is secure using only the base-ledger, and composed with other protocols sharing the same global clock.

In addition to being useful in itself, and as a template for constructing other "meta-ledgers", our protocol demonstrates the usefulness of our generalized ledger definition.

1.2 Related Work

Account-based Ledgers

The limitations of the Bitcoin scripting language have been clear from the start. One solution, exemplified by the Ethereum protocol, is to replace the UTXO model with an *account-based* system. At a very high level, the first-class object in such a system is an account. Each account has an associated value, corresponding to the funds held by the account, and a *contract*. The contract is essentially a program that accepts inputs and funds, and can send funds and inputs to other accounts as part of its execution. A transaction in this model is a set of inputs to a specific account's contract.[1]

Ethereum's account-based model is extremely expressive—it allows contracts to be written in a Turing-complete language. However, the cost is paid in higher complexity. Since all miners in the system must execute every transaction to maintain consensus, they must all run more complex code (increasing the probability of bugs and security vulnerabilities) and expend more CPU/memory resources (leading to higher costs and more complex incentive schemes).

Both UTXO and account-based schemes are deployed today, and it does not appear that either one will supplant the other. Thus, it makes sense to study how to increase the expressiveness of UTXO-based ledgers *without* making the leap to a fully account-based scheme, with its corresponding tradeoffs.

Bitcoin Covenants

Möser, Eyal and Gün Sirer proposed extending Bitcoin script to include "covenants" [7]. This extension allows an output condition to specify constraints on the spending transaction—for example, to enforce that the spending transaction's outputs have a specific format. They show some specific use-cases allowed by covenants (such as mitigating the results of private-key theft using "vaults"). Several following works explored ways to implement covenants in different ways, and the use of covenants for additional tasks [9, 11, 12].

In contrast, we focus on allowing *general* computation rather than ad-hoc use-cases. Moreover, we formally prove the security of our protocols (the existing works on covenants do not include formal security proofs at all), and introduce a new, generic framework for formally analyzing overlay ledgers. (Analyzing a covenant-based overlay in this framework is an interesting open question.)

Extending UTXO

Chakravarty, Chapman, MacKenzie, Melkonian, Peyton and Wadler formalize an "Extended UTXO Model" [3], whose purpose appears very similar to that of this work: to support more expressive computation while retaining the advantages of the UTXO model. However, while the high-level goals are the same, the strategies are very different.

[1] This is a gross oversimplification of how Ethereum's account model is constructed, of course, but suffices for the purposes of comparing to the UTXO model.

Our protocol focuses on building a more expressive ledger using a simpler ledger as a substrate—and in particular making a minimal change to the current Bitcoin scripting language (both in code and in runtime complexity) that would allow complex conditions to be supported. In contrast, Chakravarty et al. define a more powerful model (e.g., transaction outputs can have extra *state*, and output conditions can take into account the *contents* of the transaction that spends them), but at the cost of losing compatibility with existing UTXO-based ledgers—effectively, implementing their modifications would require a completely new ledger.

Ledger Formalizations

There are several works trying to formalize UTXO ledgers and to reconcile UTXO and account-based ledgers.

Garay, Kiayias and Leonardos gave the first formal security definitions for a blockchain ledger [4], in the context of analyzing the security of the Bitcoin protocol. Pass, Seeman and Shelat extended the analysis to a more realistic setting (with communication delays) and proposed slightly different security definitions [10]. Both of these works defined security as a set of *properties*.

Kiayias, Zhou and Zikas defined the ledger functionality for the first time as an ideal functionality in the UC framework [6], with the goal of making it usable in other higher-level protocols. However, their functionality was not realizable using existing blockchain protocols (e.g., it did not allow for a "rushing" adversary that can prevent an honest transaction from entering the state by inserting a conflicting transaction before it). Badertscher, Maurer, Tschudi and Zikas defined a slightly weaker ideal functionality [1] that addresses these problems and is still strong enough to be useful in high-level protocols.

Our ideal functionality has the same flavor as [1], but tailored to UTXO ledger overlays. The novelty in our definition is encapsulated in the equivalent of the "validation rules" from [1]: we carefully construct these rules to achieve a ledger that is both powerful enough to be useful, but "weak" enough to allow it to be realized as an overlay. (In particular, our ledger functionalities can be "stacked", to construct meta-meta-ledgers, etc.)

Combining UTXO and account-based ledgers

Zahnentferner defines "Chimeric Ledgers" [14]. He formalizes the two types of ledgers and *translations* between them—e.g., for a given transaction in a UTXO-based ledger, what is the set of transactions in an account-based ledger that have an equivalent effect. However, this paper deals only with "accounting", explicitly ignoring "cryptographic authorization" (i.e., scripts). A follow-up paper by the same author provides definitions for UTXO ledgers with scripts [13].

Both of these papers deal with formalization rather than protocols (there are algorithms for "transforming" from one type of ledger to another, but these are external to the ledgers themselves). Their definitions are in the formal-logic style, which is useful for interfacing with automated theorem-proving tools. In contrast, our formal definitions are ideal functionalities, which are easier to use in protocol constructions, and allow us to prove the security of our protocols in the UC model.

2 Compiling Arbitrary Circuits

We refer to output conditions that are not supported by the underlying ledger as a "meta-conditions". We refer to conditions supported by the underlying ledger as "base-conditions". In this section we construct a compiler from an arbitrary boolean circuit meta-condition to a sequence of Bitcoin+ transactions (the underlying ledger) with base output conditions. The compilation guarantees that if the output of a special *keystone* transaction is spent, it is always possible to extract a satisfying assignment to the circuit.

More concretely, let C be an arbitrary circuit, and let tx^m_C be a transaction with a meta-output txo^m_C whose condition is C (a meta-condition). In section 2.1 we construct a compiler that takes as input a transaction such as tx^m_C and outputs a tuple $(fragments_C, tx_{C-keystone})$, where $fragments_C$ is a set of *fragment* transactions and $tx_{C-keystone}$ is the special final transaction whose output will be spendable only by publishing a satisfying assignment for C. All of the $fragments_C$ transactions and $tx_{C-keystone}$ are valid ledger transactions.

Section 2.3 proves the main soundness guarantee achieved by our circuit compiler — if $fragments_C$ and $tx_{C-keystone}$ are compiled and published using the methods described in section 2.1, and the output of $tx_{C-keystone}$ is successfully spent in the base-ledger, it is always possible to extract a satisfying assignment for C.

We refer to input data that are not supported by the underlying ledger as a "meta-data". We refer to data supported by the underlying ledger as "base-data". In section 2.2, we show the completeness of our compiler. That is, given an assignment A of boolean values that satisfies the circuit C, and a transaction tx^m_A with a meta-input txi^m_A that "meta-spends" txo^m_C, we can compile tx^m_A into a sequence of base transactions $(fragments_A, tx_{A-keystone})$. If published in order, $tx_{A-keystone}$ will successfully spend the output of $tx_{C-keystone}$.

We refer to tx^m_C and tx^m_A as meta-transactions. For simplicity, we restrict the discussion to meta-transactions with either a single meta-input and a single base-output, or a single base-input and single meta-output (our techniques extend to more complex transactions in a straightforward way). We also assume, w.l.o.g., that the circuits are composed solely of NAND gates.

We note that the compiler described in this chapter does not, by itself, realize a composable ideal ledger functionality. In chapters 4 and 5 we show how it can be used to construct such a ledger (whose ideal functionality is explained in chapter 3), as a component in a generic ledger protocol.

2.1 Compiling a Circuit Spending Condition

Consider a transaction whose single meta output's condition is a circuit. $tx^m_{circuit}$ shown at the top of fig. 2.1.1 is one such transaction. It has a single output $txo^m_{circuit}$, whose condition is "$\neg(a \wedge b) = \mathbf{true}$".

Compilation of a circuit C yields four types of base transactions — "input-bit transactions" and "gate transactions" corresponding to the circuit's input-bits and the circuit's logical gates, a single "keystone transaction" whose output represents the circuit and "splitter transactions" that are used to fund the former three types.

To reduce verbiage, we will say x is a *component* if x is an input-bit, a gate or the keystone.

2.1.1 Component Transactions

Input-bit Transactions

Each of C's input-bits has a corresponding *input-bit transaction.* In fig. 2.1.1 input-bits a and b's corresponding transactions are tx_a and tx_b. An input-bit transaction, like any other transaction, should have at least one transaction input, and contain a transaction fee, made up of the difference between incoming and outgoing coins. Each input-bit transaction has two outputs, one for each boolean value (txo_{a+}, txo_{a-}, txo_{b+} and txo_{b-} in fig. 2.1.1).

Gate Transactions

For each internal boolean gate a *gate transaction* is created. In fig. 2.1.1 gate $\neg(a \wedge b)$'s corresponding transaction is $tx_{\neg(a\wedge b)}$. Generally, like the input-bit transaction, each gate transaction has at least one input and two outputs, one for each boolean value.

Circuit-keystone Transaction

An exception to the "two-outputs-per-transaction" rule is the transaction created for the "circuit-keystone", the output gate of the circuit. This transaction only has one output and it holds the reward from $txo^m_{circuit}$. In fig. 2.1.1 $tx_{\neg(a\wedge b)}$ is also the circuit-keystone transaction.

2.1.2 Wiring the Gates

Labels

As described above, tx_x, the transaction corresponding to component x, has a positive output txo_{x+} and a negative output txo_{x-}. We label each transaction with a value in $\{\mathbf{true}, \mathbf{false}, \mathbf{unset}\}$. The label of tx_x is denoted $\textsc{label}(tx_x)$, or simply as $\textsc{label}(x)$. $\textsc{label}(tx_x) = \mathbf{true}$ if only txo_{x+} is spent, $\textsc{label}(tx_x) = \mathbf{false}$ if only txo_{x-} is spent, and otherwise $\textsc{label}(tx_x) = \mathbf{unset}$. Note that the label of a component is a function of the transaction ledger state (it can change as new transactions are appended to the ledger).

Gate Functions

Each gate is dependent upon its (two) inputs, and its value is a Boolean (binary) function of their labels. Let tx_g be gate g's transaction and $f_x(\cdot, \cdot)$ its Boolean function. Also, let a, b be g's two inputs, and tx_a and tx_b their corresponding transactions. The conditions at the outputs of tx_g should enforce the following:

- txo_{g+} can be spent iff $f_x(\textsc{label}(a), \textsc{label}(b)) = \mathbf{true}$,
- txo_{g-} can be spent iff $f_x(\textsc{label}(a), \textsc{label}(b)) = \mathbf{false}$.

For example, for the $\neg(a \wedge b)$ gate, the positive output's condition will be $\neg(\textsc{label}(a) = \mathbf{true} \wedge \textsc{label}(b) = \mathbf{true})$.

2.1.3 Computing Labels in Bitcoin+

In order to implement the gate conditions as described in section 2.1.2, the function $\textsc{label}$ must be expressible in the base-ledger's spending condition language.

Bitcoin's current scripting language is not sufficient for this purpose. We propose to address this by augmenting the Bitcoin scripting language with a single new opcode, `OP_IS_TXO_UNSPENT` (creating the extended language Bitcoin+). `OP_IS_TXO_UNSPENT` accepts the ID of an output and returns **true** iff the TXO is present in the UTXO database, i.e unspent. More concretely, it pops the ID of the transaction and then the index of the output from the stack and pushes the result back. We will use a shorter syntax in this work, denoting IS-TXO-UNSPENT(*txo*) the result of the computation above with respect to TXO *txo*.

Querying whether a transaction output is *spent* may seem more natural. We chose `OP_IS_TXO_UNSPENT` because it is straightforward to implement given the UTXO database that is already maintained by Bitcoin nodes. Note that in itself, `OP_IS_TXO_UNSPENT` doesn't answer whether an output is *spent*. The absence of an output from UTXO does not necessarily imply that it was spent; it could also mean that the transaction was never published.

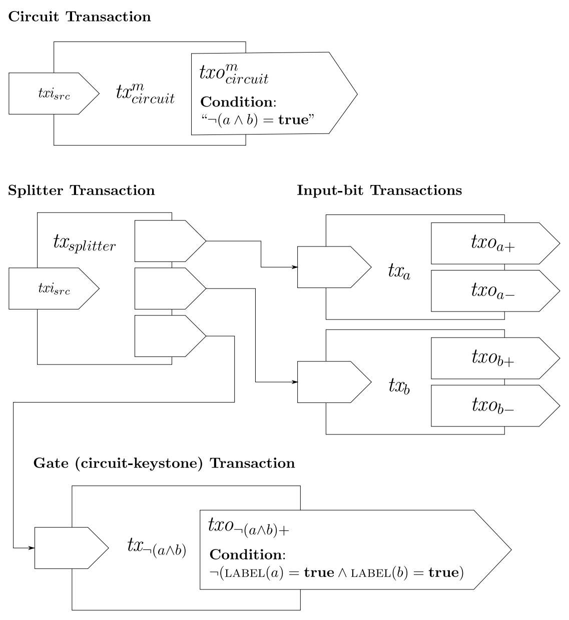
To bridge this gap we use the fact that transaction outputs enter the Bitcoin ledger atomically (i.e., either all transaction outputs of a single transaction are inserted into UTXO or none are). Consider a component transaction tx_x that has two outputs txo_{x+} and txo_{x-} (the same argument applies for input-bits as well). We compute LABEL(x) in the following manner:

1. If both txo_{x+} and txo_{x-} are in UTXO, tx_x wasn't labeled yet, and so LABEL(x) is **unset**.
2. If neither txo_{x+} nor txo_{x-} is in UTXO, we can't know if tx_x was accepted by the ledger, and so LABEL(x) is **unset**.
3. But, if txo_{x+} is in UTXO, and txo_{x-} isn't, then txo_{x-} must have been spent, and therefore LABEL(x) = **false**.
4. Similarly, if txo_{x-} is in UTXO, and txo_{x+} isn't, then txo_{x+} must have been spent, and therefore LABEL(x) = **true**.

Thus, the condition of $txo_{\neg(a \wedge b)+}$ in fig. 2.1.1 that is supposed to implement $\neg$(LABEL(a) $\wedge$ LABEL(b)) can be implemented in Bitcoin+ by applying the following replacements:

$$\begin{aligned}
\textsc{label}(a) = \textbf{true} &\iff \neg\textsc{is-txo-unspent}(txo_{a_+}) \wedge \textsc{is-txo-unspent}(txo_{a_-}) \\
\textsc{label}(b) = \textbf{true} &\iff \neg\textsc{is-txo-unspent}(txo_{b_+}) \wedge \textsc{is-txo-unspent}(txo_{b_-})
\end{aligned}$$

Figure 2.1.1: $tx^m_{circuit}$ Compilation



2.1.4 Splitter Transactions

We also create a transaction to claim the output referenced by the $tx^{m}_{circuit}$ and distribute the coins between the component transactions ($tx_{splitter}$ in fig. 2.1.1). We refer to it as the *splitter transaction*. An output should be created for each funded transaction (e.g., tx_a, tx_b, $tx_{\neg(a \wedge b)}$ in fig. 2.1.1), and funding should cover transaction fees and the reward.

To simplify the description below, we assume a single splitter transaction suffices; however, the compiler can easily be extended to the case where the number of components is larger than the maximum number of outputs the ledger allows per transaction. If this occurs, the compiler uses a tree of transactions to get to the required output count. In this case, the root of the tree is the splitter transaction. It has one output reserved for the circuit keystone, and the rest for the transactions that comprise the internal nodes of the tree. All internal nodes have one input (spending one of their parent's outputs) and the maximal number of outputs allowed by the underlying ledger. The leaf nodes' outputs are spent by the other component transactions.

2.1.5 Loss of Atomicity and Publishing Considerations

Since we use several transactions to implement $tx^{m}_{circuit}$, and their insertion into the ledger is not atomic, an attacker may try to "hijack" the insertion while it is happening. For example, the attacker might claim an output of a splitter transaction that is intended to fund one of the gate transactions, rendering the sender unable to complete the insertion. To prevent this attack, the spending condition for internal TXOs verifies a signature with respect to a freshly generated signature key.

Component transactions spend splitter transaction outputs, and so the splitter transaction must be published first in order for subsequent transactions to be included in the ledger.

2.1.6 Efficiency and Optimizations

Since the compilation creates a transaction for each component, as the circuit grows larger, the total fee required to publish the transactions grows. Our compiler emphasizes simplicity and clarity over efficiency—it is not optimized to use the smallest number of transactions. However, we briefly mention some strategies to lower the transaction count.

For example, it is possible to group the outputs of the input-bit transactions to fewer transactions. Grouping outputs of gate transactions is also possible, but more tricky—a condition of one such output may need to query about the label of an output that is part of the same transaction, and it is not trivial to expect the ledger to provide this ability (this is impossible in Bitcoin, for example). It is possible, however, to merge transactions for gates of the same circuit depth, as they do not reference one another.

The described construction creates one transaction per gate. Another possible optimization is to build transactions for sub-circuits, or allow gates of fan-in larger than two (making each transaction reference more than two inputs).

2.2 Compiler Completeness: Compiling a Circuit Assignment

Consider a transaction whose single meta input specifies as the targeted output an output of a circuit transaction and provides an assignment of boolean values to the input-bits of the circuit.

$tx^{m}_{assignment}$, appearing at the top of fig. 2.2.1, is one such transaction. Its meta input $txi^{m}_{assignment}$ specifies $txo^{m}_{circuit}$ (shown in fig. 2.1.1) as the targeted output and provides the assignment "$a \leftarrow \mathbf{true}, b \leftarrow \mathbf{true}$" to the input-bits of $txo^{m}_{circuit}$'s circuit.

2.2.1 Component-Labeling Transactions

Our compiler produces three types of base transactions — “input-bit-labeling transactions” and “gate-labeling transactions” corresponding to the circuit’s input-bits and the circuit’s logical gates, and a special assignment-keystone transaction, that will spend the circuit output.

Input-bit-labeling Transactions

Input-bit-labeling transactions implement the assignment ($tx_{a \leftarrow \mathbf{true}}$, and $tx_{b \leftarrow \mathbf{false}}$ in fig. 2.2.1). One transaction is created per input-bit, and therefore per input-bit transaction. Input-bit transactions (tx_a, tx_b in fig. 2.1.1), have two outputs each, one positive and one negative. An input-bit-labeling transaction spends one of the outputs, according to the assignment ($tx_{a \leftarrow \mathbf{true}}$’s input spends txo_{a+} and $tx_{b \leftarrow \mathbf{false}}$’s input spends txo_{b+}).

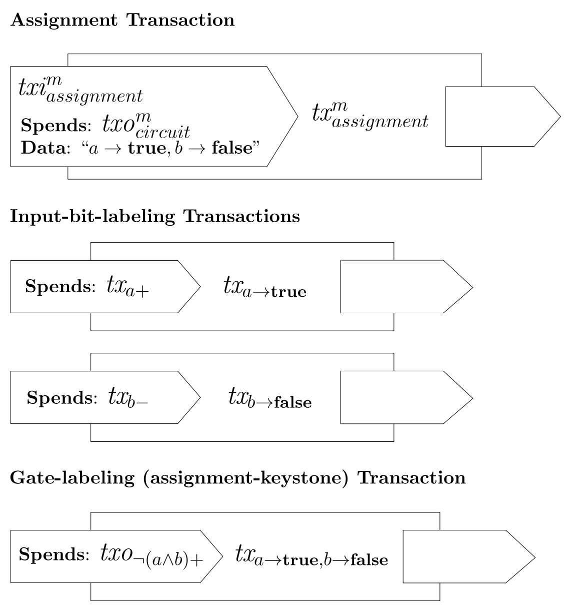
Gate-labeling Transactions

Gate-labeling transactions “evaluate” the circuit given the assignment ($tx_{a \leftarrow \mathbf{true}, b \leftarrow \mathbf{false}}$ in fig. 2.2.1). One is created per gate, and therefore per gate transaction. Gate transactions ($tx_{\neg(a \wedge b)}$ in fig. 2.1.1), compiled from a circuit output, have two outputs each, one positive and one negative (an exception is the circuit-keystone transaction, that only has one output). A non-keystone gate-labeling transaction spends one of the outputs of the corresponding gate transaction, according to the gate values that are computed from the assignment ($tx_{a \leftarrow \mathbf{true}, b \leftarrow \mathbf{false}}$’s input spends $txo_{\neg(a \wedge b)+}$).

Assignment-keystone Transaction

One transaction is created that spends the circuit-keystone transaction’s output; this transaction is called the *assignment-keystone transaction.* Its output will be identical to $tx^m_{assignment}$’s output. Transaction fees of labeling transactions should come from the outputs they claim.

Figure 2.2.1: $tx^m_{assignment}$ Compilation


Assignment Transaction
$txi^m_{assignment}$
Spends: $txo^m_{circuit}$
Data: “$a \rightarrow$ **true**, $b \rightarrow$ **false**”
$tx^m_{assignment}$
Input-bit-labeling Transactions
Spends: tx_{a+}
$tx_{a \rightarrow \mathbf{true}}$
Spends: tx_{b-}
$tx_{b \rightarrow \mathbf{false}}$
Gate-labeling (assignment-keystone) Transaction
Spends: $txo_{\neg(a \wedge b)+}$
$tx_{a \rightarrow \mathbf{true}, b \rightarrow \mathbf{false}}$

2.3 Compiler Soundness: Extracting a Satsifying Assignment

The soundness requirement from our circuit compiler is that the output of the circuit keystone cannot be spent unless a satisfying assignment is published to the ledger. Formally, we require the existence of an extraction algorithm that, given a ledger in which the $tx_{C-keystone}$ is spent, can output a satisfying assignment for C. This extraction algorithm is described in algorithm 2.1, and theorem 2.3.1 guarantees that it works (for the formal statement of the theorem, we need to define some additional notation, which we do in section 2.3.1).

Essentially, the extractor finds the input-bit transactions by traversing the DAG rooted at the circuit-keystone transaction, and by observing state of the ledger can recover the labels of the input-bits. The proof of theorem 2.3.1 is by induction on the depth of the circuit.

2.3.1 Soundness Proof Details

Notation

Let x be an arbitrary component. We define the following terms:

- $\textsc{label}^t(x)$ — the value of $\textsc{label}(x)$ at time t.
- t^*_x — the time $\textsc{label}(x)$ first transitions from **unset**, or $\perp$ if it never does
- $\textsc{label}^*(x)$ — $\textsc{label}^{t^*_x}(x)$ if $t^*_x \neq \perp$, or **unset** otherwise.
- $\textsc{inp}(x)$ — the set of input bits corresponding to x (i.e., if x is an input-bit then $\textsc{inp}(x) = x$, otherwise if x is the gate $x = \neg(y \wedge z)$ then $\textsc{inp}(x) = \textsc{inp}(y) \cup \textsc{inp}(z)$.
- $\textsc{assign}^*(x)$ — $\{(z, \textsc{label}^*(z)) | z \in \textsc{inp}(x)\}$ (the assignment to x's inputs.)

We can now state our soundness theorem:

Theorem 2.3.1. *Let C be an arbitrary circuit, and let tx^m_C be a single-base-input, single-meta-circuit-output transaction, where C is the condition in the output. Assume tx^m_C was compiled and the resulting transactions were published successfully.*

*For every component x, if $\textsc{label}^*(x) \neq$ **unset** then $\textsc{label}^*(x) = C_x(\textsc{assign}^*(x))$ where C_x is the sub-circuit rooted at x.*

In order to prove the theorem, we will require some helpful claims:

Claim 2.3.2. *For every component x and $\alpha \in \{$**true**, **false**$\}$: if $\textsc{label}^*(x) = \alpha$, then for every $t \neq t^*_x$ it holds that $\textsc{label}^t(x) \in \{\alpha,$ **unset**$\}$.*

Proof. For $t < t^*_x$, $\textsc{label}^t(x) =$ **unset** by definition. For $t > t^*_x$, suppose, in contradiction, that $\textsc{label}^t(x) = \neg\alpha$. By definition, this means that at time t, txo_{x^α} is unspent. However, $\textsc{label}^*(tx_x) = \alpha$ means that at time t^*_x, txo_{x^α} is spent. This contradicts the ledger's guarantee that a transaction output cannot re-enter the UTXO. □

Corollary 2.3.3. *For every gate g with inputs a and b and $\alpha \in \{$**true**, **false**$\}$, if $\textsc{label}^*(g) = \alpha$ then $\neg(\textsc{label}^*(a) \wedge \textsc{label}^*(b)) = \alpha$.*

Proof. Since $\textsc{label}^*(g) = \alpha$, txo_{g^α} was spent at t^*_g, and therefore at that time its condition $\neg(\textsc{label}(a) \wedge \textsc{label}(b)) = \alpha$ was satisfied. Since $\alpha \neq$ **unset**, it must be that both $\textsc{label}^{t^*_g}(a) \neq$ **unset** and $\textsc{label}^{t^*_g}(b) \neq$ **unset**. Thus, by claim 2.3.2, it holds that: $\neg(\textsc{label}^*(a) \wedge \textsc{label}^*(b)) = \neg(\textsc{label}^{t^*_g}(a) \wedge \textsc{label}^{t^*_g}(b)) = \alpha$ □

The proof of theorem 2.3.1 follows:

Algorithm 2.1: EXTRACT-ASSIGNMENT

```
1: procedure EXTRACT-ASSIGNMENT($tx_{keystone}$)
2:     return EXTRACT-ASSIGNMENT-INTERNAL($tx_{keystone}$)
3: end procedure

4: procedure EXTRACT-ASSIGNMENT-INTERNAL($tx$)
5:     if $tx$ corresponds to input-bit $i$ then
6:         LABEL*($i$) ← COMPUTE-LABEL*($tx$)
7:         return $\{(i, \text{LABEL}^*(i))\}$
8:     else // $tx$ corresponds to gate $g$
9:         $tx_a, tx_b$ ← the transactions referenced by $g$'s output conditions using IS-TXO-UNSPENT
10:        ASSIGN*($a$) ← EXTRACT-ASSIGNMENT-INTERNAL($tx_a$)
11:        ASSIGN*($b$) ← EXTRACT-ASSIGNMENT-INTERNAL($tx_b$)
12:        return ASSIGN*($a$) ∪ ASSIGN*($b$)
13:    end if
14: end procedure

15: procedure COMPUTE-LABEL*($tx_i$)
16:     $txo_{i^+}, txo_{i^-}$ ← $tx$ outputs
17:     if neither $txo_{i^+}$ nor $txo_{i^-}$ is spent then
18:         return unset
19:     else if both $txo_{i^+}$ and $txo_{i^-}$ are spent then
20:         $txo_{i^\alpha}$ ← Scan the ledger to learn which was spent first
21:         return $\alpha$
22:     else // only $txo_{i^\alpha}$ is spent
23:         return $\alpha$
24:     end if
25: end procedure
```

Proof. The proof is by induction on d the depth of component x. When x is a gate, denote its inputs as a and b.

1. For $d = 0$, x is an input-bit. Then by definition $\text{LABEL}^*(x) = C_g(\text{ASSIGN}^*(x))$.

2. Assume the induction hypothesis holds for all $d \leq n$.

3. For $d = n+1$, sub-circuit C_x is made up of gate x of depth $n+1$ whose two inputs a and b are the roots of two sub-circuits C_a, C_b of depth at most n.

   Since $\text{LABEL}^*(x) \neq \textbf{unset}$, by corollary 2.3.3, $\neg(\text{LABEL}^*(a) \wedge \text{LABEL}^*(b)) \neq \textbf{unset}$. Thus, both $\text{LABEL}^*(a) \neq \textbf{unset}$ and $\text{LABEL}^*(b) \neq \textbf{unset}$, so by the induction hypothesis, $\text{LABEL}^*(a) = C_a(\text{ASSIGN}^*(a))$, $\text{LABEL}^*(b) = C_b(\text{ASSIGN}^*(b))$ and thus

$$\begin{aligned} \text{LABEL}^*(x) &= \neg(\text{LABEL}^*(a) \wedge \text{LABEL}^*(b)) \\ &= \neg(C_a(\text{ASSIGN}^*(a)) \wedge C_b(\text{ASSIGN}^*(b))) \end{aligned}$$

   Since $\text{ASSIGN}^*(x) = \text{ASSIGN}^*(a) \cup \text{ASSIGN}^*(b)$

$$\begin{aligned} &= \neg(C_a(\text{ASSIGN}^*(x)) \wedge C_b(\text{ASSIGN}^*(x))) \\ &= C_g(\text{ASSIGN}^*(x)) \end{aligned}$$

$\square$

3 An Ideal Ledger Functionality

One of the contributions of this paper is a new, formal definition for an ideal ledger functionality. Our motivation for the new definition is twofold. On the one hand, it should be *useful*, conforming to the intuitive notions of security we expect from a UTXO ledger, and simple enough to use in higher-level protocols. At the same time, it should be flexible enough to capture the functionality and security of a "meta-ledger" that is realized as a protocol overlaying a "base" ledger.

A note about UC (in)formalism

To make our pseudocode shorter and more readable, we define all ideal functionalities using functional interfaces rather than message passing. In the fully formal UC specification, an ideal functionality is an ITM that receives messages from external parties, and responds by sending messages. We replace sending a message of the form $(\textbf{Msg-Name}, x_1, x_2, \dots)$ to functionality $\mathcal{F}$ with calling the function $\mathcal{F}.\textsc{MsgName}(x_1, x_2, \dots)$. We also omit the session ID parameters (those are implied in all calls to a functionality).

Finally, we model the clock as an external functionality (as defined in [2]). There are multiple subtleties in modeling global clocks in the UC setting (e.g., see [5]). However, we make use of the clock only for defining liveness (rather than round synchronization). Thus, while our construction is compatible with the fully formal definition, in this paper we simplify by having the ideal functionality itself provide a TICK function, which can only be called by the environment to announce that the clock has advanced.

3.1 Basic Ledger Security

Most definitions for ledger security include the following properties, which we consider base requirements:

1. *Consensus*: the ledger outputs the same list of transactions, in the same order, to all honest parties.
2. *Validity*: all published transactions satisfy the ledger's validity conditions.
3. *Liveness*: if an honest party sends a transaction to the ledger, it will be published by the ledger within some bounded time (which we call the *liveness delay*), unless it becomes invalid within that time. The liveness delay is a function of the transaction contents and the state of the ledger, and liveness might not be guaranteed at all for some valid transactions (e.g., if the transaction fee is too low).

We phrase our consensus guarantee as being about the ledger's *output* to honest parties. This is a stronger guarantee than is generally given for blockchain formalizations; for example, it doesn't allow disagreement on a suffix of the ledger or probabilistic agreement. We justify this by noting that most high-level protocols implicitly assume that the ledger actually does behave in this way, and when implemented over a real blockchain will wait until a transaction becomes "finalized" in order to use it (e.g., in Bitcoin a transaction could be considered "finalized" if it's at least six blocks in the past).

For similar reasons, we do not consider properties such as chain quality that are important for analysis of the ledger protocol itself, but, as far as most higher-level protocols are concerned, are captured in one of the base guarantees (e.g., liveness in the case of chain quality).

UTXO Validity

In the case of a UTXO ledger, the validity conditions for every transaction include the following:

- **Valid sources**: For every transaction input, the transaction output it points to exists on the ledger and was not spent by a previous transaction.
- **Output condition is satisfied**: For every transaction input, the spending condition of the transaction output pointed to by that input is satisfied by the proof data.
- **Non-inflationary**: The sum of the output amounts is no more than the sum of the input amounts.[1]

Adversarial Capabilities

As is usual in cryptography, we prefer our model to be conservative with regards to the adversary. In our modeling, we consider an adversary that has the following capabilities:

- The adversary can arbitrarily reorder transactions, as long as the liveness guarantee isn't violated. This is justified by the fact that the adversary may have partial control over the network, and because an adversarial miner can determine the selection and order of transactions in a published block.
- The adversary can arbitrarily use the transaction fees from published transactions. This is justified by the fact that in most existing blockchains, the transaction fees are awarded to the block miner, who may be adversarial. Although we do strengthen the adversary by allowing it to use the fee *immediately*, we don't believe this reduces the usefulness of the ledger when its output is fully immutable (as it is in our modeling).

Note that while the liveness delay is an *upper* bound on the time between the submission of a transaction and the time at which the transaction is published by the ledger, the adversary can choose to publish transactions earlier. This includes publishing transactions for which the liveness delay is *infinite* (i.e., they are not guaranteed to appear at all), such as transactions that have zero transaction fees.

3.2 Formalizing Bitcoin's UTXO Ledger

Before describing our full functionality, we describe a simpler functionality that models Bitcoin's UTXO ledger more closely. This serves as base for ours, and explaining its limitations will motivate the modifications we made to arrive at our generalized functionality.

The full pseudocode for the Bitcoin-UTXO functionality appears in functionality 3.1 (the "external" APIs appear in functionality 3.1 (part 1) while the internal functions appear in functionality 3.1 (part 2)).

For the honest parties, the functionality consists of a single API call: SEND-TX(tx). Calling this function does not result in immediate publication; it merely inserts the transaction into a *mempool*.[2]

[1]To simplify the discussion, we're ignoring "coinbase" transactions, which are special transactions that do not have any inputs.

[2]The mempool corresponds to the "buffer" in [1], but we use the mempool nomenclature as it is more familiar to non-theorists in the blockchain space.

The adversary may call PROCESS-TX(*tx*) at any time. This will result in the *tx* being published if it's valid, or being removed from the mempool if it's not. This interface corresponds to the ability of the adversary to select which transactions will be included in a block.

Finally, the clock tick is used to guarantee liveness: when the TICK method is called, the ledger will *automatically* process any transactions that have been in the mempool for longer than their liveness delay. This ensures liveness because the call to TICK is not under adversarial control—it is called directly by the environment. (This is motivated by the fact that, w.h.p., after sufficient time any valid transaction that hasn't previously been included in a block will be included by an honest miner.)

Internally, the Bitcoin ledger functionality uses the VALIDATE-AND-APPLY-TX function to verify transaction validity. In this case, the validity functions exactly correspond to Bitcoin's validity. The important point to note is that we abstract the scripting language to a generic "condition" that is specified for every transaction output, which must be satisfied by input data. The condition function receives not only the input data, but also the ID of the spending transaction and the UTXO database as inputs. The ID is required because Bitcoin's signature verification opcode checks for a signature on the spending transaction's ID. We added the UTXO as input to allow use of our new IS-TXO-UNSPENT opcode as well.

3.3 Important Properties of the Ledger

The Bitcoin ledger follows Bitcoin's actual implementation. Thus, while Bitcoin actually realizes this (or something similar in spirit), not all of the properties of the ideal ledger are actually important for building higher-level protocols. In this section, we note some of the properties that *are* important to preserve when defining a generalized ledger. In sections 3.4 and 3.5, on the other hand, we discuss the properties we are willing to (or must) sacrifice.

Simple Signatures

As defined, the Bitcoin ledger functionality itself is not aware of signatures, except as part of the spending condition script. This means there are no restrictions on the properties of the signature scheme (or even a requirement that signatures be verified in spending conditions). An additional important property is that creation of signatures is performed externally to the functionality, rather than using the functionality itself.

Client-Agnostic

The ledger functionality is *client agnostic*—the behavior of the ledger in response to honest parties depends only on the contents of the messages it receives, not their source.

This is also an important property in terms of implementing such a ledger: the gossip networks underlying most existing ledgers don't record the source of a message.

Together with the fact that signatures are external to the functionality, this allows use-cases such as receiving an unsigned transaction, signing it, and sending the signed transaction through other, "off-chain" communication channels—once the transaction is signed, it doesn't matter which party forwards it to the ideal ledger.

3.4 Challenges for a Meta-Ledger

Our goal is to realize a meta-ledger functionality with a richer condition language by compiling meta-transactions to multiple transactions on a base-ledger.

Ideally, we would like to simply realize the Bitcoin functionality described in functionality 3.1. Unfortunately, this is not always possible.

3.4.1 Spending Is Not Atomic

The main reason is that in the Bitcoin ledger functionality, once a transaction output enters the UTXO database, it can only be in one of two states: either it is (1) unspent (in the UTXO), in which case a transaction whose input points to that output and satisfies its condition is valid, or (2) it is spent—in which case the ledger contains a subsequent transaction that spends the output (and hence contains data that satisfies its spending condition).

However, if we compile a meta-transaction to multiple base-transactions using the IS-TXO-UNSPENT opcode, in order to satisfy the spending condition of the "meta-output" a user might have to publish multiple "fragment" transactions. If it's possible for this process to be interrupted by the adversary (e.g., by publishing a transaction that spends one of the fragment outputs), then the publishing of the meta-transaction might fail (e.g., the subsequent fragments are no longer valid).

Thus, a meta-output could end up in an "intermediate" state in which spending is not possible (even if the meta-output condition could be satisfied), but no satisfying assignment can be extracted from the ledger.

In our compiler (from chapter 2), this can occur, for example, if an adversary spends the outputs of the input-bit-labeling transactions —in this case an honest user might not be able to compile an assignment meta-transaction, even if she knows a satisfying assignment.

3.4.2 Outputs and Inputs Are Not Atomic

In Bitcoin's ledger, the inputs and outputs of a transaction affect the ledger atomically—if a transaction spends a set I of TXOs and generates a new set of TXOS O, either all TXOs in I were spent and all TXOs in O are in the UTXO, or neither happens.

However, requiring input/output atomicity would greatly restrict the class of realizeable meta-ledgers—it would require every meta-transaction to correspond to a specific base-transaction that has exactly the same inputs and outputs.

If this is not ensured, a transaction that spends TXOs I and creates TXOs O may correspond to a set of base transactions in which one transaction, tx_I, spends the TXOs in I while another, tx_O,creates the outputs O. Since the adversary is allowed to delay and reorder transactions, there may be a point in time at which tx_I was published in the base-ledger while tx_O was not.

This situation does, in fact, occur in our compiler, when compiling a meta-transaction that has an output with a circuit condition as described in chapter 2. The meta-transactions inputs are spent by the splitter transaction (c.f. section 2.1.4) while the outputs are generated by the circuit-keystone transaction (c.f. section 2.1.1).

Thus, we must relax the input/output atomicity requirement if we wish to support meta-ledger constructions such as ours.

3.4.3 Signatures Are Hard To Forge

Bitcoin's spending-condition language supports signature verification—an output condition can require the input data to contain a valid signature of the spending transaction's ID.

When we compile a meta-transaction into base transactions, we also have to translate the input data (including signatures) into input data that is valid in the base-ledger. Since the meta-transaction is (necessarily) different from any base transaction, the security of the signature scheme would prevent any attempt to naïvely transfer a signature from a meta-transaction to a signature on a base transaction.

3.4.4 Fragments Must Appear in Meta-Ledger Too

The compiler takes a single meta-transaction and creates multiple base transactions. However, these base transactions cannot be completely hidden from the meta-ledger. First, the spending condition language can query the contents of the UTXO using IS-TXO-UNSPENT (otherwise the ledger functionality would not be powerful enough to support our own compiler). This means that, at the very least, unspent outputs in the base-ledger must be reflected in the meta-ledger.

Second, fragment outputs may have monetary value. If they are hidden from the meta-ledger, an adversarial transaction that spends them in the base-ledger may also have to be hidden (since this input would not exist in the meta-ledger, such a transaction could violate the non-inflation rule); over time, this could cause a significant fraction of the base-ledger to be hidden.

The problem this creates is that the fragments in the meta-ledger must have a source of funding, but since they are the result of compilation (and do not appear in the input), their funding must somehow be “siphoned off” from the input of meta-transaction. Conceptually, this is fine—we treat the creation of fragments as part of the fee for the meta-transaction. However, the ledger guarantees that the input transactions appear in the output unmodified—and we cannot require the input transaction to have a specific format, such as an extra “fragment-funding” output (since the input to the ledger is set by the adversarial environment).

3.4.5 Fragments Must Be Recognizable

Finally, a technical point: if the meta-ledger can contain base transactions as well as meta transactions, honest parties must be able to differentiate between a base transaction that is a the result of a compiled meta-transaction and a regular base transaction. Otherwise, an honest party could receive *as input* the output of a valid compilation, in which case the ledger output would differ from its input (it would contain a meta transaction instead).

3.5 Our Generalized Ledger

The full pseudocode for our ledger functionality appears in functionality 3.2. In this section, we describe the differences between our generalized ledger and the simpler Bitcoin ledger, with intuitive justifications for the modifications.

3.5.1 Addition of a locked Output State and a Locking Condition

In order to handle non-atomic spends (cf. section 3.4.1), we introduce a **locked** state for outputs. A **locked** output is still considered unspent (e.g., by the IS-TXO-UNSPENT opcode), but any transaction that spends this output is not guaranteed liveness (this is equivalent to saying that honest users cannot spend **locked** outputs, but the adversary can).

Of course, allowing the adversary to lock outputs at will means that *no* transaction is guaranteed liveness. To restrict this, we add the concept of a *locking condition* (the locking condition language becomes an additional parameter of the ledger functionality).

Every output has both a locking condition (specified in the locking-condition language) and a spending condition (specified in the spending-condition language). In order to *spend* an output, both the locking condition and the spending condition must be satisfied by the input. In order to *lock* an output, it's sufficient to satisfy the locking condition (note that locking can only be done by the adversary, and does not allow the adversary to transfer value from the locked output).

The addition of a locking condition is a generalization (rather than weakening) of the Bitcoin ledger. To see this, consider the case of identical locking and spending condition languages. Then honest users can simply specify the *same* condition for both locking and spending, in which case any adversary who could lock an output could also spend it (so locking does not confer extra capabilities).

In our protocol, we will use *different* spending and locking languages, allowing us to capture the “intermediate” state that occurs when fragments are spent in an adversarial way: the *spending* language will consist of circuits, and the *locking* language will be the locking language of the base-ledger. Thus, an honest user can use the base-ledger language to prevent the adversary from locking outputs (e.g., by requiring a specific signature to satisfy the locking condition).

3.5.2 Addition of pending and pending-locked Output States

Our relaxation of input/output atomicity (c.f., section 3.4.2) is to allow the ledger to express an intermediate state where the inputs of a transaction have been applied to the state, but the outputs are not yet available for spending. Thus, we separate the publication of a transaction from the "release of its outputs".

In the intermediate period, the outputs are in a new state: **pending**. The ledger does not allow spending of outputs in state **pending**. To preserve liveness guarantees, **pending** outputs automatically transition into state **unspent** after the maximum liveness delay has elapsed. (The adversary may choose to release the outputs of a transaction before that time.)

If, before a meta output is released, one of its corresponding fragment outputs was spent or locked, we require the meta output to transition into state **locked**, rather than state **unspent**. To mark it as such we introduce an additional state: **pending-locked**.

3.5.3 Allowing the Adversary to Choose Transaction IDs

We sidestep the problem of signature translation (cf. section 3.4.3) by allowing IDs to be "translated" instead—i.e., letting IDs from the base-ledger be "copied" to transactions in the meta-ledger (and vice versa), so that the signatures can be used unmodified.

We implement this in our ledger functionality by making two changes:

1. Whenever the ledger functionality encounters a new transaction, it asks the adversary for its ID (rather than using a fixed hash function). The ledger verifies that the IDs returned by the adversary are unique (otherwise it picks a unique ID itself).

2. We separate the ID of the transaction from the IDs of its outputs (instead of treating the TXO ID as a simple index relative to the transaction ID).

While these changes are a weakening of the ledger functionality (we give the adversary additional capabilities), they do preserve *collision-resistance* of IDs, which is the critical property used by most higher-level protocols.

Transactions may have multiple IDs

The changes above are not quite sufficient for our purposes, however. The reason lies in the way we use this capability in our security proof. In the proof, the simulator (ideal-world adversary) uses the ID-generation capability to copy an ID from a fragment output by the compiler to the meta-transaction. Once the meta-transaction is signed (this is done by the environment in our proof), the signature can then be used, unmodified as a signature of the fragment in the base-ledger.

However, the compiler is *randomized*—for example, our compiler generates a fresh signature key for every compilation, which is used in creating the fragment spending conditions. Thus, each call of the compiler on the same meta-transaction can produce different fragment transactions, with different IDs. To reflect this, our ledger allows a transaction to have multiple IDs—every request for a transaction ID will be answered by a fresh ID, even if it is for the same transaction.

IDs with Auxiliary Data

Allowing multiple IDs per transaction leads to a new problem, however: in order to allow the ledger to be client-agnostic (i.e., it doesn't matter *who* sent a transaction, only what the transaction contains), the simulator needs additional information that would allow it to "invert" the transaction ID into the appropriate fragments.

We handle this by having the ledger return not just IDs but also some opaque "auxiliary data", which must be sent together with the transaction. This auxiliary data is used internally by the ledger, but does not appear in the ledger output. (In our case, the auxiliary data is simply the entire result of the compilation)

To make things a little clearer, the following describes the interactions of an honest party, Alice, with the ledger in order to send a transaction:

1. Alice creates the transaction "core", $\widehat{tx}$. This contains the inputs and output conditions, but without IDs or any signature data (Alice needs the IDs in order to generate signatures).

2. Alice sends $\widehat{tx}$ to the ledger and requests its IDs.

3. The ledger forwards $\widehat{tx}$ to the adversary, Eve, to request IDs, and waits for her response. Eve responds with the transaction ID and the IDs of the transaction outputs. She also sends the auxiliary data (The ledger ensures that all the IDs sent by Eve are fresh).

4. The ledger sends the IDs to Alice, together with the auxiliary data.

5. Alice computes the full transaction tx, which includes both the IDs and signatures.

6. Alice sends tx together with the auxiliary data to the ledger to be published. (The ledger verifies that the IDs match the transaction and auxiliary data.)

7. The ledger output will contain tx (including the IDs and signatures), but not the auxiliary data.

The ledger *doesn't* verify that Alice is the one who requested the transaction IDs; Alice could send tx and the auxiliary data to Bob, who could later send it to the ledger. This is an important property, since there are ledger protocols that rely on off-chain transmission of transactions.

Note that our protocol uses an ideal *base*-ledger, and the IDs output in the meta-ledger are a fixed function of the base-ledger IDs (which, if the base-ledger is Bitcoin, are set according to the Bitcoin rules). However, the protocol *is still secure* if based on the weaker ledger functionality we define.

3.5.4 Allowing (Adversarial) Partial Spending of a TXO

We address the problem of funding fragments in the meta-ledger (cf. section 3.4.4) by allowing the adversary to spend transaction fees arbitrarily. This is implemented in our generalized ledger by:

1. Letting the adversary decide, for every published transaction, an *input amount* for each of its inputs. The input amount specifies how much of the corresponding output's value is used by that input. The sum of the input amounts must still be greater or equal to the sum of the output amounts.

2. Recording in the UTXO how much of an output amount remains. If an output is spent (i.e., its state in the UTXO is **spent**), but the remaining amount is greater than zero, we consider that output *partially-spent*.

3. A partially-spent output counts as spent for the purpose of IS-TXO-UNSPENT, and transactions that attempt to spend it are not guaranteed liveness (i.e., honest parties cannot spend it). However,
4. *Any* input that spends a partially-spent output is valid, as long as the input value is not more than the remaining output amount. Thus, the adversary can use any remaining value without having to generate new signatures, for example.

Essentially, partially-spent outputs represent transaction fees. In our protocol, we use the meta-transaction fees to fund the fragment transactions in the meta-ledger by having both the meta-transaction and a fragment partially-spend the same source TXO.

3.5.5 "Fragment" and "Keystone" Marks on TXs

We require base transactions to be marked in some identifiable way that isn't likely to occur in "natural" transactions. For simplicity, we assume that there are two types of marks: "Fragment" marks and "Keystone" marks.

The ledger functionality recognizes fragment and keystone transactions, and does not guarantee liveness for these transactions (this solves the problem from section 3.4.5).

We assume these marks are ledger-specific (i.e., a transaction that the meta-ledger recognizes as a fragment would be considered a standard transaction by the base-ledger)

In practice, these marks can be implemented using a special "unlikely" spending condition (e.g., fixing a publicly-known value x, and checking that x appears in the input data).

3.5.6 Notation Notes

Syntactic Sugar

To make our proofs simpler, we added a REMOVE-TX(tx) call to the functionality, which can be called by the adversary to remove the transaction tx from the mempool—as long as tx is invalid or liveness is not guaranteed when REMOVE-TX(tx) is called. This is merely syntactic sugar, as the adversary would get the same result by calling PROCESS-TX(tx) if tx is invalid, or by letting tx remain in the mempool and never processing it if its liveness is not guaranteed (since, in this case, the ledger functionality will never call PROCESS-TX(tx) itself).

Ledger States and Ledger Output

In our ledger, a *state* contains an extension of a *UTXO*—for each output it includes its state, and also its output conditions and remaining amount.

The ledger state contains only the transaction outputs, not the transactions themselves. However, it is a deterministic function of the *ledger output*—which consists of the ordered list of published transactions and transaction output release events. (For ease of notation, when we use a ledger output *out* in place of a ledger state, this is simply shorthand for "the unique state s computed from *out*".)

Functionality 3.1 (part 1): Bitcoin UTXO Ledger: APIs

The functionality maintains:

- *mempool*: a set of transactions pending processing
- *UTXO*: set of unspent TXOs

Honest API

```
procedure SEND-TX($tx$)
    Add $tx$ to $mempool$
    Send (Tx-Received, $tx$) to $\mathcal{A}$
end procedure
```

Adversarial API

```
procedure PROCESS-TX($tx$)
    if $tx$ is in $mempool$ then
        Remove $tx$ from $mempool$
    end if

    $UTXO' \xleftarrow{fresh\ copy} UTXO$
    $is\_success \leftarrow$ VALIDATE-AND-APPLY-TX($tx$)
    if $is\_success$ then
        Send (Tx-Published, $tx$) to all parties
    else
        $UTXO \leftarrow UTXO'$  // Restore “backup”
    end if
end procedure
```

External (Environment) API

```
procedure TICK()  // Can only be called by $\mathcal{E}$
    Output (Tick) only to $\mathcal{A}$
    for all $tx \in mempool$ do
        if
            $tx$ was added to $mempool$ more than $\delta_{bitcoin}$ ticks ago and
            IS-LIVE$^{bitcoin}$($tx$)
        then
            PROCESS-TX($tx$)
        end if
    end for
end procedure
```

...Continued in functionality 3.1 (part 2)...

Functionality 3.1 (part 2): Bitcoin UTXO Ledger: Internals

...Continued from functionality 3.1 (part 1)...

```
procedure VALIDATE-AND-APPLY-TX($tx$)
    $(inputs_{tx}, outputs_{tx}) \xleftarrow{parse} tx$
    $incoming\_coins \leftarrow$ the sum of the $amount$s associated with $inputs_{tx}$
    $outgoing\_coins \leftarrow$ the sum of the $amount$s associated with $outputs_{tx}$
    if $outgoing\_coins > incoming\_coins$ and $tx$ is not a coin-minting transaction then
        return false
    end if

    for all $txi \in inputs_{tx}$ do:
        $(id_{txo}, data^{spend}_{txi}) \xleftarrow{parse} txi$
        if $id_{txo} \notin UTXO$ then  // $id_{txo}$ is unrecognized
            return false
        end if

        $(amount_{txo}, \varphi^{spend}_{txo}) \leftarrow txo$
        if $\varphi^{spend}_{txo}(data^{spend}_{txi}, id_{tx}, UTXO) =$ false then
            return false // $\varphi^{spend}_{txo}$ isn't satisfied
        end if
        Remove $txo$ from $UTXO$
    end for
    Add $outputs_{tx}$ to $UTXO$
    return true
end procedure

function IS-LIVE$^{bitcoin}$($tx$)
    if $tx$'s fee is larger than or equal to $min\_fee$ then
        return true
    else
        return false
    end if
end function
```

Functionality 3.2 (part 1): $\mathcal{F}_{ledger}^{\mathcal{L}_{spend},\mathcal{L}_{lock},\delta,\text{IS-LIVE}}$
(Honest API)

The functionality maintains:

- *mempool*: a set of transactions pending processing
- *UTXO*: maps output IDs to a tuple $(state, amount, \varphi^{lock}, \varphi^{spend})$
- *TxIds*: maps transactions to their IDs

```
procedure GET-TX-IDS($\widehat{tx}$)  // Called upon receiving (Get-Tx-IDs-Request, $\widehat{tx}$)
    $\widehat{tx}_{stripped}$ ← strip $\widehat{tx}$ of locking inputs and input amounts
    Send (Create-Tx-IDs-Request, $\widehat{tx}_{stripped}$) to $\mathcal{A}$
    Await (Create-Tx-IDs-Response, $tx$, $aux$) response  // $tx$ is of the form $(IDs, \widehat{tx}_{stripped})$

    if $tx$'s IDs are missing or already used then
        Replace them with fresh random IDs
    end if
    Add $(tx, aux)$ to the set $TxIds\left[\widehat{tx}_{stripped}\right]$
    return $(tx, aux)$
end procedure

procedure SEND-TX($tx$, $aux$)  // Called upon receiving (Send-Tx, $tx$, $aux$)
    $tx_{stripped}$ ← strip $tx$ of locking inputs and input amounts
    $\widehat{tx}_{stripped}$ ← strip $tx_{stripped}$ of transaction and output IDs
    if $(tx_{stripped}, aux) \in TxIds\left[\widehat{tx}_{stripped}\right]$ then
        Add $tx_{stripped}$ to $mempool$
        Send (Tx-Received, $tx_{stripped}$, $aux$) to $\mathcal{A}$
    end if
end procedure
```

...Continued in functionality 3.2 (part 2)...

Functionality 3.2 (part 2): $\mathcal{F}_{ledger}^{\mathcal{L}_{spend},\mathcal{L}_{lock},\delta,\textsc{is-live}}$
(Adversarial API)

...Continued from functionality 3.2 (part 1)...

```
procedure PROCESS-TX($tx$)
    $tx_{stripped} \leftarrow$ strip $tx$ of locking inputs and input amounts
    $\widehat{tx}_{stripped} \leftarrow$ strip $tx_{stripped}$ of transaction and output IDs
    if $\nexists aux$ such that $(tx_{stripped}, aux) \in TxIds\left[\widehat{tx}_{stripped}\right]$ then
        return
    end if

    $UTXO' \xleftarrow{fresh\ copy} UTXO$
    $is\_success \leftarrow$ VALIDATE-AND-APPLY-TX($tx$)
    if $is\_success$ then
        Remove $tx_{stripped}$ from $mempool$
        Send (Tx-Published, $tx$) to all parties
    else
        $UTXO \leftarrow UTXO'$   // Restore "backup"
    end if
end procedure

procedure RELEASE-TXOS($tx$)
    if $tx$'s outputs are in state pending then
        Set the states of $tx$'s output to unspent in $UTXO$
        Send (Txos-Released, $tx$) to all parties
    else if $tx$'s outputs are in state pending-locked then
        Set the states of $tx$'s output to locked in $UTXO$
        Send (Txos-Released, $tx$) to all parties
    end if
end procedure

procedure REMOVE-TX($tx$)   // Syntactic Sugar
    $\widehat{tx}_{stripped} \leftarrow$ strip $tx$ of locking inputs and input amounts
    if $\widehat{tx}_{stripped} \in mempool$ and $\neg$IS-STRONGLY-VALID($\widehat{tx}_{stripped}$) then
        Remove $\widehat{tx}_{stripped}$ from $mempool$
    end if
end procedure
```

...Continued in functionality 3.2 (part 3)...

Functionality 3.2 (part 3): $\mathcal{F}_{ledger}^{\mathcal{L}_{spend},\mathcal{L}_{lock},\delta,\textsc{is-live}}$
(Adversarial API)

...Continued from functionality 3.2 (part 2)...

```
51: procedure TICK()   // Can only be called by $\mathcal{E}$
52:     for all $tx \in mempool$ do
53:         if
54:             $tx$ was added to $mempool$ $\delta$ ticks ago and
55:             IS-LIVE($tx$)
56:         then
57:             PROCESS-TX($tx$)
58:         end if
59:     end for
60:     for all $tx \in \{$transactions published$\}$ do
61:         if
62:             $tx$ was added to $mempool$ $\delta$ ticks ago and
63:             $tx$'s outputs are pending or pending-locked in $UTXO$
64:         then
65:             RELEASE-TXOS($tx$)
66:         end if
67:     end for
68:     Send (Tick) to all parties
69: end procedure
```

...Continued in functionality 3.2 (part 4)...

Functionality 3.2 (part 4): $\mathcal{F}_{ledger}^{\mathcal{L}_{spend},\mathcal{L}_{lock},\delta,\text{IS-LIVE}}$
(Internals)

...Continued from functionality 3.2 (part 3)...

```
procedure VALIDATE-AND-APPLY-TX($tx$)
    $((id_{tx}, txo\_ids_{tx}), (inputs_{tx}, outputs_{tx})) \xleftarrow{parse} tx$
    // incoming coins ≥ outgoing coins:
    if $\sum_{txo \in outputs_{tx}} amount_{txo} > \sum_{txi \in inputs_{tx}} amount_{txi}$ then
        return false
    end if
    // valid inputs:
    if $\neg$VALIDATE-AND-APPLY-INPUTS($inputs_{tx}, id_{tx}$) then
        return false
    end if
    APPLY-OUTPUTS($outputs_{tx}, txo\_ids_{tx}$)
    return true
end procedure

procedure VALIDATE-AND-APPLY-INPUTS($inputs_{tx}$, $id_{tx}$)
    for all $k \in [1 \dots \text{SIZE}(inputs_{tx})]$ do:
        $txi \leftarrow inputs_{tx}[k]$
        $(id, amount, data^{lock}, data^{spend}) \xleftarrow{parse} txi$
        if $\varphi^{spend} = \perp$ then
            if $\neg$VALIDATE-AND-APPLY-LOCKING-INPUT($txi, id_{tx}$) then
                return false
            end if
        else
            if $\neg$VALIDATE-AND-APPLY-SPENDING-INPUT($txi, id_{tx}$) then
                return false
            end if
        end if
    end for
    return true
end procedure
```

...Continued in functionality 3.2 (part 5)...

Functionality 3.2 (part 5): $\mathcal{F}_{ledger}^{\mathcal{L}_{spend},\mathcal{L}_{lock},\delta,\text{IS-LIVE}}$
(Internals (cont.))

...Continued from functionality 3.2 (part 4)...

```
99:  procedure VALIDATE-AND-APPLY-SPENDING-INPUT($txi$, $id_{tx}$)
100:     $(id_{txo}, amount_{txi}, data_{txi}^{lock}, data_{txi}^{spend}) \xleftarrow{parse} txi$
101:     // referenced output is in a spendable state:
102:     if STATE($UTXO, id_{txo}$) $\in \{\bot, \mathbf{pending}, \mathbf{pending\text{-}locked}\}$ then
103:         return false
104:     end if
105:     $(state_{txo}, amount_{txo}, \varphi_{txo}^{lock}, \varphi_{txo}^{spend}) \leftarrow UTXO[id_{txo}]$
106:     // referenced output has sufficient remaining coins:
107:     if $amount_{txi} > amount_{txo}$ then
108:         return false
109:     end if
110:     // referenced output is already spent:
111:     if $state_{txo} = \mathbf{spent}$ then
112:         SPEND-OUTPUT($id_{txo}$, $amount_{txo}$-$amount_{txi}$ )
113:         return true
114:     end if
115:     // locking condition is satisfied:
116:     if $\varphi_{txo}^{lock}(data_{txi}^{lock}, id_{tx}, UTXO) = \mathbf{false}$ then
117:         return false
118:     end if
119:     // spending condition is satisfied:
120:     if $\varphi_{txo}^{spend}(data_{txi}^{spend}, id_{tx}, UTXO) = \mathbf{false}$ then
121:         return false
122:     end if
123:     SPEND-OUTPUT($id_{txo}$, $amount_{txo}$-$amount_{txi}$ )
124:     return true
125: end procedure

126: procedure SPEND-OUTPUT($id_{txo}$, $amount_r$)
127:     $(state_{txo}, amount_{txo}, \varphi_{txo}^{lock}, \varphi_{txo}^{spend}) \leftarrow UTXO[id_{txo}]$
128:     $UTXO[id_{txo}] \leftarrow (\mathbf{spent}, amount_r, \varphi_{txo}^{lock}, \varphi_{txo}^{spend})$
129: end procedure
```

...Continued in functionality 3.2 (part 6)...

Functionality 3.2 (part 6): $\mathcal{F}_{ledger}^{\mathcal{L}_{spend},\mathcal{L}_{lock},\delta,\text{IS-LIVE}}$
(Internals (cont.))

...Continued from functionality 3.2 (part 5)...

```
procedure VALIDATE-AND-APPLY-LOCKING-INPUT($txi$, $id_{tx}$)
    $(id_{txo}, amount_{txi}, data_{txi}^{lock}, data_{txi}^{spend}) \xleftarrow{parse} txi$ // $data_{txi}^{spend} = \bot$
    // referenced output is in a lockable state:
    if STATE($UTXO, id_{txo}$) $= \bot$ then
        return false
    end if
    $(state_{txo}, amount_{txo}, \varphi_{txo}^{lock}, \varphi_{txo}^{spend}) \leftarrow UTXO[id_{txo}]$
    // locking is done with amount 0:
    if $amount_{txi} \neq 0$ then
        return false
    end if
    // referenced output is already locked:
    if $state_{txo} \in \{$pending-locked, locked$\}$ then
        return true
    end if
    // locking condition is satisfied:
    if $\varphi_{txo}^{lock}(data_{txi}^{lock}, id_{tx}, UTXO) =$ false then
        return false
    end if
    if $txo$'s state in $UTXO$ is pending then
        Set $txo$'s state to pending-locked in $UTXO$
    else if $txo$'s state in $UTXO$ is unspent then
        Set $txo$'s state to locked in $UTXO$
    end if
    return true
end procedure
```

...Continued in functionality 3.2 (part 7)...

Functionality 3.2 (part 7): $\mathcal{F}_{ledger}^{\mathcal{L}_{spend},\mathcal{L}_{lock},\delta,\text{IS-LIVE}}$
(Internals (cont.))

...Continued from functionality 3.2 (part 6)...

```
procedure APPLY-OUTPUTS($outputs$, $txo\_ids$)
    for all $k \in [1 \dots \text{SIZE}(outputs)]$ do
        $txo \leftarrow outputs\,[k]$
        $id \leftarrow txo\_ids\,[k]$
        Add $txo$ to $UTXO$ with ID $id$ and in state pending
    end for
end procedure

procedure IS-STRONGLY-VALID($tx$)
    $UTXO' \xleftarrow{fresh\ copy} UTXO$
    $applied\_successfully \leftarrow \text{VALIDATE-AND-APPLY-TX}(tx)$
    $UTXO \leftarrow UTXO'$   // discard side-effects
    if
        $applied\_successfully$ and
        $\text{IS-LIVE}(tx)$ and
        $tx$ doesn't spend any outputs in state pending-locked/locked/spent
    then
        return true
    end if
    return false
end procedure
```

4 Generically Realizing a Meta-Ledger

4.1 Overview of the Protocol

Our protocol (formally appearing as protocol 4.1) realizes the *meta-ledger*, so a party running it should first and foremost be able to *send meta-transactions.* The protocol realizes the meta-ledger over a base-ledger that does not recognize meta-transactions. Hence its logic can be divided into two main complementary categories:

- Compilation of meta-transactions to base-transactions and sending thereof to the base-ledger and
- Identifying these base-transactions as they are published by the base-ledger and the decompilation back to meta-transactions.

The output of an honest party running the protocol will differ from the output of the base-ledger, as some transactions published by the base-ledger will be modified before being output — the most-notable modification is the replacing of base-transactions with meta-transactions.

Compilation and sending are discussed in section 4.1.2, and monitoring the base-ledger and decompilation are discussed in section 4.1.3. However, we first address in section 4.1.1 a few decompilation complexities that affected the compilation as well.

Abstracting the Compilation/Decompilation of Meta-Transactions

The protocol is parameterized by a compiler/decompiler pair, $(\textsc{comp}, \textsc{decomp})$. Given a meta-transaction tx^m, the current meta-ledger output and the base-ledger output, $\textsc{comp}$ will compile tx^m into a sequence of base-transaction sets (the fragments), whose order indicates the order in which they should be sent (a formal description appears in definition 4.2.1). Given a base transaction, the meta-ledger output and the base-ledger output, $\textsc{decomp}$ will attempt to reconstruct a meta-transaction (a formal description appears in definition 4.2.2).

The protocol works for any $(\textsc{comp}, \textsc{decomp})$ pair satisfying some basic requirements (see section 4.2.4). When instantiated with the compiler/decompiler from chapter 5, $\textsc{comp}^{\text{Circuit}}$ (protocol 5.1) and $\textsc{decomp}^{\text{Circuit}}$ (protocol 5.2), it realizes the meta-ledger with circuit-spending conditions.

In fig. 4.1.1 we give an example of compiling and decompiling two complementary meta-transactions that can be given as input to an honest party running protocol instantiated with the $\textsc{comp}^{\text{Circuit}}$ and $\textsc{decomp}^{\text{Circuit}}$ from chapter 5. The figure is structured as follows:

- Column "Circuit" is dedicated to $tx^m_{circuit}$, whose output $txo^m_{circuit}$ contains circuit condition "$\neg(a \wedge b) = \textbf{true}$".
- Column "Source" is dedicated to tx_{src} whose output txo_{src} is spent by $tx^m_{circuit}$.
- Column "Assignment" is dedicated to tx^m_{assign}, whose input txi^m_{assign} contains assignment data "$a \rightarrow \textbf{true}, b \rightarrow \textbf{false}$".
- Row "Honest Input" shows the transaction as it is given by $\mathcal{E}$ to the honest party to send.
- Row "$\textsc{comp}$ output to Base-Ledger" shows the fragments that are the output of $\textsc{comp}$ and that are sent in the real-world.

- Row “DECOMP output to Meta-Ledger” shows the output of the honest party and DECOMP.

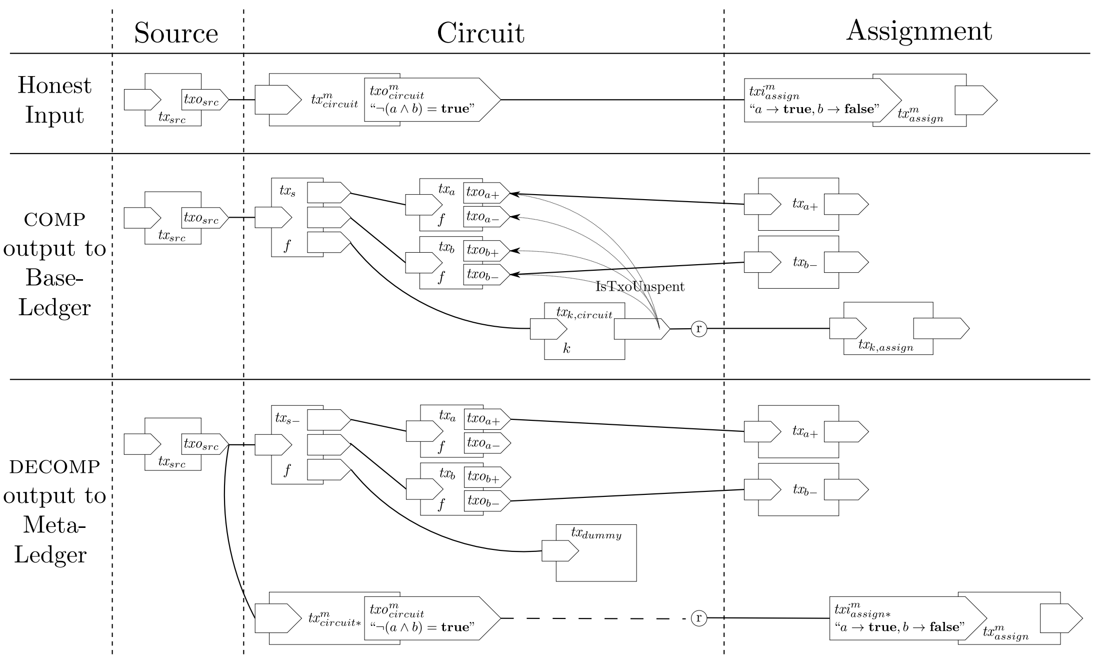


Figure 4.1.1: Compilation and Decompilation

Special Fragments

We identify two special fragments from the output COMP creates for a meta-transaction tx^m. We require of COMP/DECOMP that exactly one of the fragments will share tx^m’s output IDs. We refer to that fragment as the *keystone* (c.f. definition 4.2.9). We also require that exactly one of the fragments will reference the same outputs as tx^m. We refer to that fragment as the *splitter* (c.f. definition 4.2.10).

We say a keystone is *simple* if it is also the splitter, that is, if it also spends the same outputs as tx^m. Such is $tx_{k,assign}$ (fig. 4.1.1, Base-Ledger, Assignment) with respect to tx^m_{assign} (Input, Assignment).

We say a keystone is *non-simple* if the keystone and the splitter are two different fragments. $tx_{k,circuit}$ (fig. 4.1.1, Base-Ledger, Circuit) is a non-simple keystone with respect to $tx^m_{circuit}$ (fig. 4.1.1, Input, Circuit).

Note that both the keystone and the splitter of a decompiled base-transaction are the transaction itself.

High-level Protocol Description

First, the decompiler “queues” all transactions it receives from the base-ledger for a period of time before output. This allows it to modify transactions “retroactively” (up to the liveness delay of the meta-ledger).

For each new base-transaction that is received, the protocol determines which of the following three cases it falls into (this can be done efficiently in the case of circuit meta-transactions by using the keystone and fragment marks, together with pattern-matching on the output conditions):

Case 1: The transaction, together with a subset of the preceding transactions, could have been an output of COMP. In this case, it can also extract the input to COMP, and use that to compute a meta-transaction. (For fig. 4.1.1, this is always the case.)

Case 2: The transaction could not have been part of a COMP output, but an output it spends corresponds to a meta-output in the current meta-ledger.

Case 3: The transaction satisfies neither of the two conditions above.

In the third case, the decompiler can simply ouput the transaction unmodified. In the first two cases, it will output a meta-transaction. There are two main ideas that allow the decompiler to ensure the meta transaction that it outputs is valid in the meta-ledger:

1. If the keystone is *simple* (as is the case in the assignment transaction of fig. 4.1.1), it replaces the keystone with the meta-transaction, keeping the transaction ID and the output IDs unchanged. Thus, it can transfer signatures on the keystone unmodified to use as signatures on corresponding meta-transaction. In this case, the meta-transaction appears in the meta-ledger at exactly the same place at which the keystone appeared, so a valid keystone "automatically" implies that any base-outputs that the meta-transaction spends are also valid (for meta-outputs, validity is guaranteed from the compiler soundness).

2. If the keystone is *not* simple, the handling is a little more complicated. In this case, it can't simply replace the keystone with the meta-transaction—because the *splitter* is the transaction that actually spends the source inputs (e.g., consider tx_s in the Circuit column of fig. 4.1.1). Thus, it needs to transfer signatures from the splitter to the meta-transaction for it to be valid. The solution is to copy the splitter's ID to the meta-transaction. However, we can't just replace the splitter with the meta-transaction—the splitter's outputs are spent by fragment transactions, which should still appear in the meta-ledger. Instead, the protocol gives the splitter a new ID, but inserts the meta-transaction before the splitter in the ledger output (this is one of the retroactive modifications it performs on the queue). The meta-transaction is modified to spend only part of the source inputs, so that the splitter can spend the rest. Because the ledger functionality allows partially spent transactions to be spent by the adversary without satisfying the spending conditions again, the modified splitter is valid in the meta-ledger.

   In order to maintain the invariant that the UTXO state of the meta-ledger remains equivalent to the UTXO state of the base-ledger, the outputs of the meta-transaction aren't released immediately (they enter in a "pending state"), but wait until the corresponding *keystone* outputs are released in the base-ledger (this corresponds to the dashed line in fig. 4.1.1).

Finally, there is one more "trick" used by the decompiler: whenever a transaction spends a fragment of a previously-decompiled transaction that has a meta-output, the decompiler adds a *locking input* to that transaction that locks the corresponding meta-output. This possible because the compiler is required to ensure that all fragment outputs have the same locking condition as the meta-output. The reason the decompiler must do this is because, otherwise an adversary could "wedge" a meta-output by spending some of its fragments (e.g., think of a circuit for which the adversary sets some of the input bits), in which case an honest party that compiles a valid meta-transaction isn't guaranteed to be able to insert the corresponding fragments into the ledger (e.g., if the circuit-input used by the honest party is different from the wedged bits). By locking the meta-output, the meta-ledger prevents this situation from happening.

4.1.1 Decompiling Meta-Transactions

Valid Decompiled Transaction Inputs

For a transaction to be valid, its inputs need to be valid, which among other things, requires that they provide data satisfying the conditions of the outputs they reference.

In fig. 4.1.1, the input data of decompiled $tx^m_{circuit*}$ (Meta-Ledger, Circuit) should satisfy txo_{src}'s (Meta-Ledger, Source) and the input data of decompiled $tx^m_{assign*}$ (Meta-Ledger, Assignment) should satisfy $txo^m_{circuit}$'s (Meta-Ledger, Assignment) conditions.

Output conditions often check whether the signature provided in the input data is valid. Recall that signatures are validated with respect to the ID of the spending transaction.

It is necessary to support meta-transactions that are required to provide signatures. A decompiled, honestly-sent transaction should be funded by the same outputs it originally spent (those that are spent in the base-ledger by the splitter). Additionally, due to the state-transition order preservation requirement, a decompiled transaction should be output by the honest party when its splitter is published in the base-ledger.

The way the protocol settles this is by "giving" the IDs of the splitters to the meta-transactions. In fig. 4.1.1, $tx^m_{circuit*}$'s ID will be that of tx_s (Meta-Ledger, Circuit), and $tx^m_{assign*}$'s ID will be that of $tx_{k,assign}$'s input (Meta-Ledger, Assignment).

Valid Decompiled Inputs of Transactions With Non-Simple Keystones

Ideally, every decompiled meta-transaction would have a simple keystone. If that was the case, the honest party would have been able to simply replace the keystone (which is also the splitter) with the decompiled meta-transaction. However, it is sometimes useful to separate the inputs and the outputs of a meta-transaction to two different fragments, as is done by $\text{COMP}^{\text{Circuit}}$ (see tx_s and $tx_{k,circuit}$ in fig. 4.1.1, Base-Ledger, Circuit, the splitter and the keystone of $tx^m_{circuit*}$ respectively). Hence, not all keystones are simple.

The decompilation of such a meta-transaction is possible only after the keystone transaction is published by the base-ledger. Hence the protocol delays publishing of all transactions if any of those it needs to publish is a potential fragment. Delaying output means that the honest party can output the decompiled meta-transaction before it published the splitter. Delay is implemented using *MsgQueue*— a queue of messages the honest party maintains.

However, if the decompiled meta-transaction simply copies the inputs from the splitter, then between the two, they spend from the referenced outputs twice the amount the original meta-transaction had. Suffice to say for now that the honest party adjusts the input amounts of the two transactions so that added-up they spend the same amount the original meta-transaction had. This is also made possible thanks to the delay the honest party introduces.

In the scenario in fig. 4.1.1, the honest party would not output the splitter tx_s immediately as it is published by the base-ledger. It would would wait, in case the keystone is published. If and when the keystone $tx_{k,circuit}$ is published, $\text{DECOMP}^{\text{Circuit}}$ can be used to decompile $tx^m_{circuit*}$. The honest party would output decompiled meta-transaction $tx^m_{circuit*}$ before tx_s after having adjusted the input amounts of the two transactions.

The Use of Partial Spending

The transfer of IDs causes a different issue though, which is only relevant for meta-transactions with non-simple keystones (tx_s in fig. 4.1.1, Meta-Ledger, Circuit) — on the one hand, we still want the splitter to appear in the meta-ledger and therefore in the output of the honest party. On the other hand, its ID cannot be the same as it was in the base-ledger since the ID is already taken by the meta-transaction, and IDs are unique.

This is where partial spending comes into play (see section 3.5.4) — the honest party first outputs the decompiled meta-transaction. Since the splitter's inputs are valid in the base-ledger with respect to the splitter's ID, the inputs of the decompiled meta-transaction will be valid. Only then does it output the splitter — since its inputs spend already-spent outputs, they are valid.

4.1.2 Compiling and Sending of Transactions

The honest party's API appears in protocol 4.1 (part 1) and includes GET-TX-IDS (line 1) and SEND-TX (line 10).

Both the ledger's SEND-TX and the honest party's SEND-TX accept an already-signed transaction. $\mathcal{E}$ requires a transaction's ID in order to provide a signed transaction to the SEND-TX procedures, since signing a transaction essentially means signing its unique ID. Hence the GET-TX-IDS procedures — both the ledger and the protocol implement it.

When asked for the IDs of transaction *tx*, the protocol's GET-TX-IDS invokes COMP with *tx*. COMP internally uses the ledger's GET-TX-IDS procedure, and all the fragments, apart from the splitter, are returned signed. *tx*'s IDs a returned alongside *aux* auxiliary data required for sending — the entire output of COMP.

The IDs returned by the protocol's GET-TX-IDS are selected in a very specific way, to allow the output of the honest party to be indistinguishable from a meta-ledger as per the considerations discussed in section 4.1.1:

- The transaction ID is always chosen to be that of the splitter. Recall that "later", during decompilation, the ID of the splitter is "moved" to the decompiled meta-transaction.
- The output IDs are chosen to be those of the keystone (in the case of a simple keystone, and therefore also in the case of a base transaction, these are also the output IDs of the splitter).

When asked to send transaction *tx* with auxiliary data *aux* (that is, in fact, the fragments), the honest party sends the fragments instead of *tx*. It "signs" the splitter by planting *tx*'s signature in the splitter's inputs. The honest party also verifies *aux* is a valid and admissible COMP output for *tx*.

The fragments are sent one step at a time — the i^{th} step is sent when all the outputs of step $(i-1)^{th}$ have been released. This logic is implemented in TXOS-RELEASED-BY-$\mathcal{F}_{ledger}$ (line 52), since the function is invoked whenever the ledger releases outputs. SEND-NEXT-STEP (line 77) contains the logic for sending the next step, and it invoked from two places — from SEND-TX for sending the first step, and from TXOS-RELEASED-BY-$\mathcal{F}_{ledger}$ to send all other steps.

This mechanism helps guarantee that if the first step, that contains the splitter, is published by the honest party, then the adversary cannot interrupt the publication of all other steps and fragments (see condition 3 of definition 4.2.18). Consider $tx^m_{circuit}$ (fig. 4.1.1, Input, Circuit) — since tx_a and tx_b spend tx_s's outputs (fig. 4.1.1, Input, Base), they must be sent after tx_s's outputs have been released. Otherwise they are invalid until such time as tx_s's outputs are released, and if the ledger attempts to process them, they will be discarded.

The honest party also verifies when asked to send a transaction that it is live — this is required for proving that the output of the honest party is indistinguishable from the meta-ledger.

4.1.3 Processing Base-ledger Output and Decompilation

An honest party is subscribed to messages output by the base-ledger — when (`Tx-Published`, *tx*) is output, TX-PUBLISHED-BY-$\mathcal{F}_{ledger}(tx)$ is run (line 23 of protocol 4.1 (part 2)).

As the honest party sees transactions being published by the ledger it considers each to see whether they should be replaced with meta-transactions. This is in line 25 done by calling DECOMP with the published transaction *tx*, and based on the base and meta states. The call returns tx^m. If the decompilation succeeds, then tx^m is a meta-transaction and it is returned alongside its fragments; if tx is not a keystone transaction, or if the decompilation fails, then tx^m is identical to tx, and tx is also the only fragment.

The transaction that the honest party outputs is not necessarily identical to tx^m as it was output by DECOMP. It is also not output immediately — instead, it is added somewhere in the Deciding what to output is based (mostly) on whether the keystone is simple.

A Simple Keystone Was Published

SIMPLE-KEYSTONE-PUBLISHED (line 115) is called if the decompiled transaction has a simple keystone, with the keystone tx_k and the meta-transaction tx^m. The honest party pushes a `Tx-Published` message at the end of *MsgQueue* with a transaction made up from tx^m's inputs using tx^m's ID and from tx_k's outputs using tx_k's output IDs, discarding tx^m's outputs.

This applies to fig. 4.1.1's $tx^m_{assign*}$ (Meta-Ledger, Assignment) — it is made up of the inputs of the decompiled meta-transaction and the outputs of $tx_{k,assign}$, using the ID of the decompiled meta-transaction and the output IDs of the keystone.

SIMPLE-KEYSTONE-PUBLISHED can also be called if the decompiled transaction has a non-simple keystone, but its splitter has already been output so it is impossible to reduce its amounts. In which case, the honest party treats the keystone as a standard base-transaction, pushing a `Tx-Published` message at the end of *MsgQueue* with the keystone.

A Non-Simple Keystone Was Published

NON-SIMPLE-KEYSTONE-PUBLISHED (line 120) is called if the decompiled transaction has a non-simple keystone, with the keystone tx_k, the splitter tx_s and the meta-transaction tx^m. The honest party creates three transactions:

1. tx^m_*, from tx_s's inputs using tx_s's ID and tx^m's outputs using tx^m's output IDs (tx^m's inputs are discarded),

2. $tx_{s\text{-}}$, a fresh copy of the splitter, but using a new unique ID,

3. tx_{dummy}, from tx_k's input using tx_k's ID (it is ensured by condition 7 of definition 4.2.13 that there only exists one) and with no outputs.

Denoting $amount_k$ the sum of the output amount of tx_k, the honest party reduces $amount_k$ from tx_{dummy}'s input and from the $tx_{s\text{-}}$'s output it spends. It then fixes the input amounts of $tx_{s\text{-}}$ and of tx^m_* by greedily reducing a total of $amount_k$ from $tx_{s\text{-}}$'s inputs and setting tx^m_*'s amounts to whatever was reduced from $tx_{s\text{-}}$ inputs. Note that now tx^m_*'s inputs specify the same total amount as its outputs ($amount_k$).

Finally three modifications are made to *MsgQueue*:

1. a ($\texttt{Tx-Published}, tx^m_*$) is added before ($\texttt{Tx-Published}, tx_s$),

2. ($\texttt{Tx-Published}, tx_s$) is replaced with ($\texttt{Tx-Published}, tx_{s\text{-}}$) and

3. ($\texttt{Tx-Published}, tx_{dummy}$) is pushed to the end.

This applies to fig. 4.1.1's $tx^m_{circuit*}$ (Meta-Ledger, Circuit) — it is made up of the inputs of $tx_{s\text{-}}$ and the outputs of the decompiled meta-transaction, using the ID of tx_s and the output IDs of the decompiled meta-transaction.

Adding Locking Inputs

Every time TX-PUBLISHED-BY-$\mathcal{F}_{ledger}$ is called, ADD-LOCKING-INPUTS (line 100) is called internally. ADD-LOCKING-INPUTS goes over all `Tx-Published` messages in *MsgQueue* and add locking inputs when required. Adding a locking input to *tx* is required if *tx* spends a locking fragment output that was published by the base-ledger before *tx*. A locking input locks the meta-output corresponding to the fragment output.

4.2 Definitions for Transaction Compiler/Decompiler

4.2.1 Input and Output Behavior

We first give formal definitions for COMP and DECOMP,

Definition 4.2.1 (COMP)**.** On input,

- a base-ledger output out_b,
- a meta-ledger output out_m and
- a meta-transaction tx^m,

COMP outputs $[T_1^{tx^m}, ..., T_n^{tx^m}]$, where $T_i^{tx^m}$ for $i \in [n]$ is a set of tuples of the form: (tx, aux), where tx is transaction with IDs and aux is auxiliary data required for sending tx.

To keep notation simple, we use $(fragments, tx_k) \leftarrow \textsc{comp}(out_b, out_m, tx^m)$ when we only care about distinguishing between the keystone and the rest of the fragments, and treat $fragments$ as a set of transactions when we don't care about the internal order.

Definition 4.2.2 (DECOMP)**.** On input,

- a base-ledger output out_b,
- a meta-ledger output out_m and
- a base-transaction tx^b,

DECOMP outputs a tuple $(tx^m, fragments)$ where tx^m is a meta-ledger transaction and $fragments$ is a set of base-transactions.

4.2.2 Relations between Base and Meta-Ledger States

In order to use a COMP/DECOMP pair, they must satisfy completeness and validity requirements. We first define some terms that will allow us to specify the requirements,.

Definition 4.2.3 (State Order)**.** We define the following complete ordering on transaction output states:

$$\perp < \textbf{pending} < \textbf{pending-locked} < \textbf{unspent} < \textbf{locked} < \textbf{spent} .$$

This ordering satisfies the property that an output's state in the ledger is weakly monotone—it cannot change from a higher to a lower order state.

Definition 4.2.4 (Ledger State Extension)**.** We say a ledger state s^{ext} is an *extension* of a ledger state s if it satisfies for every txo:

- $\textsc{state}(txo, s) \leq \textsc{state}(txo, s^{ext})$

- If $\textsc{state}(txo, s) = \textbf{pending-locked}$ then $\textsc{state}(txo, s^{ext}) \neq \textbf{unspent}$.

Note that a ledger state s^{ext} is an extension of a state s iff there exist ledger outputs out^{ext} and out such that s^{ext} is the state computed from out^{ext}, s is the state computed from out and out^{ext} is an extension of out.

Definition 4.2.5 ($\overset{state}{\underset{txo}{\leftrightsquigarrow}}$ Weak Equivalence)**.** Let s_a and s_b be two ledger states. We say that s_a and s_b are *weakly-equivalent* and denote $s_a \overset{state}{\underset{txo}{\leftrightsquigarrow}} s_b$ if for every transaction output ID id:

- $\textsc{state}(s_a, id) \in \{\perp, \textbf{pending}, \textbf{pending-locked}\} \iff$ $\textsc{state}(s_b, id) \in \{\perp, \textbf{pending}, \textbf{pending-locked}\}$.
- $\textsc{state}(s_a, id) \in \{\textbf{unspent}, \textbf{locked}\} \iff \textsc{state}(s_b, id) \in \{\textbf{unspent}, \textbf{locked}\}$.
- $\textsc{state}(s_a, id) = \textbf{spent} \iff \textsc{state}(s_b, id) = \textbf{spent}$.

The weak equivalence definition captures equivalence with respect to spending and locking conditions—we require that weakly-equivalent states must be "indistinguishable" to conditions that take state into account.

Definition 4.2.6 ($\overset{state}{\underset{txo}{\rightsquigarrow}}$ Consistency)**.** Let s_b and s_m be two ledger states. We say that s_m is consistent with s_b and denote $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ if $s_m \overset{state}{\underset{txo}{\leftrightsquigarrow}} s_b$ and for every transaction output $txo^m \in s_m$:

1. $\textsc{state}(s_b, id_{txo^m}) \leq \textsc{state}(s_m, id_{txo^m})$
2. $\textsc{remaining}(s_m, id_{txo^m}) = \textsc{remaining}(s_b, id_{txo^m})$
3. $\varphi^{lock}_{s_m, id_{txo^m}} = \varphi^{lock}_{s_b, id_{txo^m}}$

This definition captures the property that the adversary has at least as much "control" over outputs in the meta-ledger as it does in the base-ledger (i.e., if a transaction is live in the meta-ledger—that is, if the ledger could "force" the adversary to process, it must also be live in the base-ledger).

Definition 4.2.7 (Explainable TXO)**.** Let out_b and out_m be ledger outputs and txo^m a transaction output of $tx^m \in out_m$. We say txo^m is *explainable* with respect to out_b and out_m if there exists a transaction $tx^b \in out_b$ with output txo^b such that $id_{txo^b} = id_{txo^m}$, and

$$\begin{cases} txo^m = txo^b & \text{if } \varphi^{spend}_{txo^m} \text{ is in } \mathcal{L}_{base} \\ \exists fragments : (tx^m, fragments) = \textsc{decomp}(out^{pre}_b, out^{pre}_m, tx^b) & \text{otherwise} \end{cases}$$

(where out^{pre}_b is the prefix of out_b up to, excluding tx^b and out^{pre}_m is the prefix of out_m up to, excluding tx^m).

When txo^m is explainable w.r.t out_b and out_m, we call the set of fragments returned by $\textsc{decomp}$ the *explanatory fragments* for txo^m (this set is empty if txo^m is a base output).

Definition 4.2.8 ($\underset{tx}{\rightsquigarrow}$ Consistency)**.** Let out_b and out_m be ledger outputs, and s_b and s_m their corresponding ledger states. We say that out_m is consistent with out_b and denote $out_m \underset{tx}{\rightsquigarrow} out_b$ if $out_m = out_b$ or if $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ and for every transaction output $txo^m \in s_m$, txo^m is explainable with respect to out_b and out_m.

This definition captures the property that all outputs of the meta-ledger state "can be explained" as either a valid decompilation of the base-ledger, or an exact copy of a base-ledger output.

We note that $out_m \underset{tx}{\rightsquigarrow} out_b$ implies that a transaction output ID appears in s_m iff it appears in s_b (if it exists in s_b, then item 1 implies it exists in s_m, while explainability implies the other direction).

4.2.3 Special Transactions

In order to concisely define the required behavior of COMP and DECOMP, we give names to base transactions with specific properties with relation to meta-transactions.
Let $(tx^m, fragments) \leftarrow \text{DECOMP}(out_b, out_m, tx)$ be the output of DECOMP.

Definition 4.2.9 (Keystone)**.** If there exists a single base transaction $tx_k \in fragments$ such that tx_k and tx^m have the same output IDs, we call tx_k the *keystone* transaction for $(tx^m, fragments)$.

Definition 4.2.10 (Splitter)**.** If there exists a single base transaction $tx_s \in fragments$ such that tx_s and tx^m reference the same output IDs, we call tx_s the *splitter* transaction for $(tx^m, fragments)$.

Definition 4.2.11 (Simple Keystone)**.** We say a keystone transaction tx_k is a *simple keystone* if it is also the splitter transaction.

A keystone that is not a simple keystone is a *non-simple* keystone.

For $\text{COMP}^{\text{Circuit}}$ and $\text{DECOMP}^{\text{Circuit}}$ (protocols 5.1 and 5.2) that are discussed in sections 5.1 and 5.2 and use the construction explained in chapter 2, the compilation of a meta circuit-condition output yields a non-simple keystone ($tx_{k,circuit}$ in fig. 4.1.1, Base-Ledger, Circuit), and the compilation of meta assignment input yields a simple keystone ($tx_{k,assign}$ in fig. 4.1.1, Base-Ledger, Assignment).

4.2.4 Admissible comp/decomp

Our generic ledger protocol realizes the ledger functionality as long as COMP/DECOMP satisfy a set of requirements. We call such a pair *admissible*. Below, we specify the requirements for admissibility.

(Note that to simplify the formal description of the protocol we also require that when COMP is given a base-transaction that is valid as-is in the base-ledger and does not require compilation, it returns a single set containing that transaction. Similarly, we require that when DECOMP is given a transaction and it doesn't decompile, it outputs the transaction as-is.)

Definition 4.2.12 (COMP/DECOMP Completeness)**.** We say the pair (COMP, DECOMP) is *complete* if they satisfy the following condition:

For every base-ledger output out_b, meta-ledger output out_m such that $out_m \underset{tx}{\rightsquigarrow} out_b$, every meta transaction tx^m that is valid in out_m and every execution $(fragments, tx_k) \leftarrow \text{COMP}(out_b, out_m, tx^m)$, if out_b^{ext} is an extension of out_b that contains all the transactions in $fragments$, then $(tx_*^m, fragments) \leftarrow \text{DECOMP}(out_b^{ext}, out_m^{ext}, tx_k)$, where tx_*^m is identical to tx^m up to input amount changes.

In addition to completeness, we require both COMP and DECOMP to satisfy a set of *validity* requirements separately:

Definition 4.2.13 (DECOMP Validity)**.** We say the decompiler DECOMP is *valid* if, for every base-ledger output out_b, every meta-ledger output out_m such that $out_m \underset{tx}{\rightsquigarrow} out_b$ and every transaction tx such that tx is valid in out_b, $(tx^m, fragments) \leftarrow \text{DECOMP}(out_b, out_m, tx)$ satisfies all of the following conditions:

Condition 1: $fragments \subseteq out_b$

Condition 2: tx is a keystone transaction and there exists a splitter transaction $tx_s \in out_b||tx$ (it is possible that $tx_s = tx$).

Condition 3: For every transaction output txo of tx^m, the output amount of txo (in the meta-ledger) is equal to the amount of the corresponding output of the keystone tx (in the base-ledger).

Condition 4: If tx^m has meta outputs, then there exists a condition $\varphi^{lock} \in \mathcal{L}_{lock}$ such that φ^{lock} is the locking condition for the meta outputs and for all fragment outputs that are not spent by other fragments.

Condition 5: For every base transaction output txo^b of tx^m, the spending and locking conditions of txo^b (in the meta-ledger) are identical to those of the corresponding output of the keystone tx (in the base-ledger).

Condition 6: If tx spends a meta-output in out_m, then tx^m has a simple keystone.

Condition 7: If tx^m has a simple keystone, then tx^m is valid in out_m. (Note that if tx^m has a non-simple keystone, there are no requirements about its validity.)

Condition 8: If tx is not simple, it only spends a single output of the splitter tx_s.

The validity conditions for COMP ensure that compilation generates valid base transactions, and that we can always compute meta-transaction IDs deterministically from base-transaction IDs. The conditions also ensure that a computationally-bounded adversary cannot "block" the transactions generated by COMP from being inserted into the ledger. To this end, we will require some additional definitions.

Definition 4.2.14 (Live Transaction Sequence)**.** Let out_b be a ledger output and $T = (tx_1, \ldots, tx_n)$ a sequence of transactions. We say T is *live* in out_b if for all $i \in \{1, \ldots, n\}$ it holds that tx_i is live in $out_b||tx_1|| \cdots ||tx_{i-1}$.

That is, a sequence is live if an honest party is guaranteed to be able to publish the transactions in the sequence, in order, as long as no other transactions are published in between.

Definition 4.2.15 (Valid Transaction Sequence)**.** Let out_b be a ledger output and $T = (tx_1, \ldots, tx_n)$ a sequence of transactions. We say T is *valid* in out_b if for all $i \in \{1, \ldots, n\}$ it holds that tx_i is valid in $out_b||tx_1|| \cdots ||tx_{i-1}$.

A valid sequence can be appended to the ledger by the adversary, but isn't guaranteed to be live. However, every live sequence is necessarily valid.

Definition 4.2.16 (Blocking Transaction Sequence)**.** Let out_b be a ledger output and $T = (tx_1, \ldots, tx_n)$ a sequence of transactions. We say a transaction sequence X *blocks* T in out_b if the sequence $T' = T \setminus X$ (consisting of all transactions in T that are not in X) is not live in $out_b||X$. (We consider the empty sequence to be live.)

We can now define what it means for a sequence to be "unblockable". Intuitively, we want to capture the property that the transaction fragments output by COMP can either be blocked "early" (in our case this will be before some prefix is published) or not at all. Formally,

Definition 4.2.17 (Unblockable Sequence Distribution)**.** Let out_b be a ledger output and $\mathcal{T}_1, \mathcal{T}_2$ distributions over transaction sequences. We say $\mathcal{T}_2$ is an *unblockable sequence distribution* following $\mathcal{T}_1$ in out_b if for every computationally-bounded adversary $\mathcal{A}$,

$$\Pr\Big[(T_1, T_2) \leftarrow (\mathcal{T}_1, \mathcal{T}_2), X \leftarrow \mathcal{A}(1^\lambda, out_b, T_1, T_2), T_1 \subseteq X, X \text{ is valid and blocks } T_2\Big] < \varepsilon(\lambda)$$

where ε is a negligible function of the security parameter λ, and the probability is over the distribution $\mathcal{T}$ and the coins of the adversary.

To understand the definition, think of T_1, T_2 as a single long sequence output by COMP; the first part might be blockable, but if it's not ($T_1 \subseteq X$), the second part cannot be blocked.

Definition 4.2.18 (COMP Validity)**.** COMP is valid iff it satisfies the following conditions for every base-ledger output out_b, every meta-ledger output out_m such that $out_m \underset{tx}{\rightsquigarrow} out_b$, every meta transaction tx^m that is live in out_m and $(fragments, tx_k) \leftarrow \text{COMP}(out_b, out_m, tx^m)$:

Condition 1: **(Splitter exists)** $(fragments, tx_k)$ contains a splitter for tx^m. (The splitter can be tx_k itself.)

Condition 2: **(Fragments prior to splitter are blockable only by locking relevant outputs)** Let S be the set of *txo*s spent by tx^m. We define the set of *relevant outputs* for S to be:

- Transaction-outputs in S.
- Transaction outputs of explanatory fragments for txos in S (c.f. definition 4.2.7).

For every extension out_b^{ext} of out_b that does not contain the splitter transaction, if $(fragments, tx_k) \setminus out_b^{ext}$ is not live in out_b^{ext}, then either there exists a transaction in out_b^{ext} that locks a relevant output, or for *every* extension out_m^{ext} of out_m such that $out_m^{ext} \underset{tx}{\rightsquigarrow} out_b^{ext}$, it holds that tx^m is not live in out_m^{ext}.

Condition 3: **(Fragments following splitter are unblockable)** Loosely speaking, this condition ensures that if the splitter has been published, no computationally-bounded adversary can prevent the remaining fragments from being published.

Define the distribution $\mathcal{T}^{(out_b, out_m, tx^m)} = fragments$ to consist of the fragments output by $\text{COMP}(out_b, out_m, tx^m)$, $\mathcal{T}_1$ to be the fragments in $\mathcal{T}^{(out_b, out_m, tx^m)}$ up to (including) the the splitter fragment tx_s (ensured by condition 1) and $\mathcal{T}_2$ the remaining fragments.

Then $\mathcal{T}_2$ is an unblockable sequence distribution following $\mathcal{T}_1$ in out_b (c.f., definition 4.2.17).

Definition 4.2.19 (Admissibility)**.** A pair of algorithms (COMP, DECOMP) is admissible if DECOMP satisfies definition 4.2.13, COMP satisfies definition 4.2.18 and the pair satisfies COMP/DECOMP completeness.

The formal description of our generic ledger protocol appears in protocol 4.1 (external APIs are in protocol 4.1 (part 1), handling of ledger events (i.e. methods triggered by output of the base-ledger) is in protocols 4.1 (part 2) and 4.1 (part 3) and internal methods are in protocols 4.1 (part 5) and 4.1 (part 6)).

4.3 Security Analysis of the Generic Meta-Ledger Protocol

We prove the security of the protocol in the UC model. More formally, our main security claim appear in theorem 4.3.1.

Theorem 4.3.1 (Protocol 4.1 realizes $\mathcal{F}_{ledger}^{meta}$)**.** *For all $\mathcal{L}_{base}$, $\mathcal{L}_{lock}$ such that $\mathcal{L}_{base}$ satisfies ...., and every l-valid* $\text{COMP}^{\mathcal{L}_{meta} \to \mathcal{L}_{base}}/\text{DECOMP}^{\mathcal{L}_{base} \to \mathcal{L}_{meta}}$ *pair and all $\delta_{base} > 0$, protocol 4.1 realizes $\mathcal{F}_{ledger}^{\mathcal{L}_{meta} \cup \mathcal{L}_{base}, \mathcal{L}_{lock}, \delta_{meta}, \text{IS-LIVE}^{meta}}$ in the $\mathcal{F}_{ledger}^{\mathcal{L}_{base}, \mathcal{L}_{base}, \delta_{base}, \text{IS-LIVE}^{base}}$-hybrid model,* *for* $\delta_{meta} = f(\delta_{base})$ *and*

$$\begin{aligned}\textsc{is-live}^{meta}(tx) = tx \textit{ is base} \wedge \textsc{is-live}^{base}(tx) \wedge tx \textit{ is not marked as fragment or keystone}\\ \vee\, tx \textit{ is meta} \wedge l(tx)\end{aligned}$$

4.3.1 Proof Overview

Our proof uses a standard hybrid argument. We build four hybrids, H_1-H_4, where H_1 is the real-world and H_4 is the ideal-world with the full simulator (algorithm 4.1).

H_2 and its indistinguishability from H_1 are discussed in section 4.3.2. The main difference between H_2 and H_1 is that in H_2 the ideal-world $\mathcal{S}$ simulates a *single* honest party (denote it as $\mathcal{S}_{\mathcal{P}_h}$) which handles all honest inputs and generates outputs for all honest users, whereas in H_1 there are multiple independent honest parties. (In addition, there is a 'syntactic' difference in that in H_2 the real base-ledger $\mathcal{F}^{base}_{ledger}$, is replaced with a simulated ledger (denote it as $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$); the ideal functionality in H_2 forwards to $\mathcal{S}$ any message it receives and is told by $\mathcal{S}$ what to output (the $\mathcal{S}_{\mathcal{P}_h}$'s output).

$\mathcal{S}$ is able to simulates all honest parties by using a single instance of an honest party since $\mathcal{F}_{ledger}$ is agnostic of the identity of the party that sent it a transaction, and the honest party does not maintain its identity or any other identity-bound property (e.g., signing key or randomness).

Since the clock is shared between all parties, and Π_{ledger} is deterministic (COMP can be random, but it does not accept the identity of the caller) the outputs of all honest parties "monitoring" the base-ledger are identical.

H_3 and its indistinguishability from H_2 are discussed in section 4.3.3. H_3 adds *honest party input extraction* — honest party inputs are now sent to an instance of $\mathcal{F}^{meta}_{ledger}$ and $\mathcal{S}$ learns what input was given to the honest parties from messages it receives from the $\mathcal{F}^{meta}_{ledger}$ instance. Proving indistinguishability is relatively easy, since $\mathcal{F}_{ledger}$ notifies $\mathcal{A}$ when it had received messages, specifying the arguments.

H_4 and its indistinguishability from H_3 are discussed in section 4.3.4. $\mathcal{S}_{\mathcal{P}_h}$'s output is no longer output by the ideal-functionality. Instead the output of the $\mathcal{F}^{meta}_{ledger}$ instance is used as honest party output, and $\mathcal{S}$ translates $\mathcal{S}_{\mathcal{P}_h}$ output to calls to the adversarial API of $\mathcal{F}^{meta}_{ledger}$.

Due to the H_3 leap and since $\mathcal{S}$ forwards messages $\mathcal{E}$ sends $\mathcal{F}^{base}_{ledger}$ to $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$, the inputs to $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ are indistinguishable from those of $\mathcal{F}^{base}_{ledger}$ in real-world (H_1). Hence the messages $\mathcal{S}_{\mathcal{P}_h}$ sees from $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ are indistinguishable from those the honest parties see from $\mathcal{F}^{meta}_{ledger}$ in the real-world.

Since $\mathcal{S}_{\mathcal{P}_h}$ is an instance of Π_{ledger}, it acts like the real-world honest parties. When $\mathcal{S}_{\mathcal{P}_h}$ outputs (`Tx-Published`, *tx*), $\mathcal{S}$ calls $\mathcal{F}^{meta}_{ledger}$'s PROCESS-TX with *tx* and when $\mathcal{S}_{\mathcal{P}_h}$ outputs (`Txos-Released`, *tx*), $\mathcal{S}$ calls $\mathcal{F}^{meta}_{ledger}$'s RELEASE-TXOS with *tx*. In corollary 4.3.26 we show these calls always succeed, and that otherwise the state of $\mathcal{F}^{meta}_{ledger}$ doesn't change.

To prove corollary 4.3.26 we show that $s_{m+q} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$, i.e., that the application of $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue* to $\mathcal{F}^{meta}_{ledger}$ (s_{m+q}) is always consistent with $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$'s state ($s_b$):

- Consistency is defined in definition 4.2.6 and captures the difference between the states of the meta-ledger and the underlying base-ledger (e.g., meta-outputs replace their corresponding base keystone outputs, meta-inputs replace their corresponding base splitter inputs).

- Recall that the honest party maintains a queue *MsgQueue* that causes a delay and a "gap" between $\mathcal{S}_{\mathcal{P}_h}$ and $\mathcal{F}^{meta}_{ledger}$ (messages received from the ledger are not output immediately).

  Applying a *MsgQueue* to a ledger state is defined in definition 4.3.17, and essentially means calling different methods with respect to the ledger state, thereby changing it —

PROCESS-TX for `Tx-Published` messages, RELEASE-TXOS for `Txos-Released` messages and REMOVE-TX for `Tx-Removed` messages.

Hence we prove in lemma 4.3.25 that the combined state of $\mathcal{F}_{ledger}^{meta}$ and $\mathcal{S}_{\mathcal{P}_h}$ is "always" consistent with that of $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$ — to do this we map out all possible modifications to the states of $\mathcal{F}_{ledger}^{meta}$, $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$ and $\mathcal{S}_{\mathcal{P}_h}$, and group them into eight event types (event types 1 to 8). Each event is associated with a list of code segments that change *one or more* states ($\mathcal{F}_{ledger}^{meta}/\mathcal{S}_{\mathcal{F}_{ledger}^{base}}/\mathcal{S}_{\mathcal{P}_h}$). Event type 1's code segments, for example, cover the successful processing of a transaction in $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$ *and* the application of the transaction to $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue*. Event type 4's code segments, cover the popping of a `Tx-Published` message from $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue* and the processing of the transaction in $\mathcal{F}_{ledger}^{meta}$.

In claim 4.3.23 we show that all changes to the three states occur in one of the events, in claim 4.3.22 we show that the code segments of each event run consecutively during simulation and are disjoint from code segments of other events, and in lemma 4.3.24 we show that (most of) the events preserve consistency — that if the three states were consistent, and the event occurred (i.e., all code segments were run), then the resulting states were still consistent. Hence we can prove lemma 4.3.25 ($s_{m+q} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ is always preserved) by induction on events.

To help us prove the each event maintains consistency separately, we show in lemmas 4.3.20 and 4.3.21 that the consistency is preserved "even though" the base-ledger keystone is decompiled into a meta-transaction, and in lemma 4.3.19 we prove the addition of locking inputs to transactions in the meta-ledger (that do not exist in the base-ledger) also preserves consistency.

We prove lemmas 4.3.19 to 4.3.21 by using a number of supporting claims that mainly show consistency is maintained in different settings. In lemma 4.3.16 we show that the application of an identical transaction to two consistent states will yield consistent states. Claim 4.3.15 shows the same holds for an identical output and claim 4.3.14 for an identical list of inputs.

To show that the application of an identical list of inputs preserves consistency (claim 4.3.14), we prove similar arguments for a single identical locking-input (lemma 4.3.13), spending-input (Lemma 4.3.12), and for the spending of an amount from the same output claim 4.3.11.

To prove lemma 4.3.25, it is also required to show that all changes in $\mathcal{F}_{ledger}^{meta}$ correspond are changes in $\mathcal{S}_{\mathcal{P}_h}$ and $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$ — that the $\mathcal{F}_{ledger}^{meta}$ state is not changed due to internal $\mathcal{F}_{ledger}^{meta}$ processes. Particularly, that no event of event types 7 and 8 ever occurs. To prove this, we use claim 4.3.10, that if a (`Send-Tx`, *tx*) is given to any honest party in the ideal-world, then within $2 \cdot s \cdot \delta_{base}$, $\mathcal{S}$ has called either $\mathcal{F}_{ledger}^{meta}$'s PROCESS-TX and RELEASE-TXOS with *tx′* (which is almost identical to *tx*) or $\mathcal{F}_{ledger}^{meta}$'s REMOVE-TX with *tx*.

In order to prove claim 4.3.10, we showed there exist upper bounds on the time it takes an honest party to send a transaction ($s \cdot \delta_{base}$, see claim 4.3.9) and on the time an honest can delay a message after it had been published by the ledger (a further $s \cdot \delta_{base}$, see claim 4.3.8).

4.3.2 H_2 — Single Honest Party

Following are the differences between H_2 and H_1:

1. In H_2 the real base-ledger $\mathcal{F}_{ledger}^{base}$ is replaced with a simulated ledger (denote it as $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$) per definition 4.3.2.

2. In H_2 the ideal-world $\mathcal{S}$ simulates all honest parties by using a single instance (denote it as $\mathcal{S}_{\mathcal{P}_h}$) per definition 4.3.3, whereas in H_1 there are multiple independent honest parties.

3. In H_2 the ideal functionality forwards to $\mathcal{S}$ any message it receives and is told by $\mathcal{S}$ what to output (the $\mathcal{S}_{\mathcal{P}_h}$'s output) — $\mathcal{S}$ is notified when $\mathcal{E}$ sends a message to an honest party; messages output from $\mathcal{S}_{\mathcal{P}_h}$ are passed to $\mathcal{E}$ (simulating the real-world honest parties).

4. When the ideal functionality receives a (`Tick`)message, $\mathcal{S}$ invokes $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$'s TICK.

Definition 4.3.2 ($\mathcal{S}$ simulation of $\mathcal{F}^{base}_{ledger}$)**.** $\mathcal{S}$ simulates $\mathcal{F}^{base}_{ledger}$— the simulated instance is referred to as $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$.

- All messages sent from $\mathcal{E}$ to $\mathcal{F}^{base}_{ledger}$ are sent to $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$.
- All messages output from $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ are passed to $\mathcal{E}$.

Definition 4.3.3 ($\mathcal{S}$ simulation of honest parties)**.** $\mathcal{S}$ simulates an honest party running Π_{ledger} — the simulated instance is referred to as $\mathcal{S}_{\mathcal{P}_h}$.

- When $\mathcal{S}_{\mathcal{P}_h}$ sends a message to $\mathcal{F}^{base}_{ledger}$, $\mathcal{S}$ pauses the $\mathcal{S}_{\mathcal{P}_h}$ simulation, storing its execution state, runs the $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ code that handles the message, and finally restores the $\mathcal{S}_{\mathcal{P}_h}$ execution state and continues simulating it.
- When $\mathcal{S}_{\mathcal{P}_h}$ outputs a message , $\mathcal{S}$ pauses the $\mathcal{S}_{\mathcal{P}_h}$ simulation, storing its execution state, runs the $\mathcal{S}$ code that handles the message, and finally restores the $\mathcal{S}_{\mathcal{P}_h}$ execution state and continues simulating it.
- When $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ outputs a message addressed to honest parties, $\mathcal{S}$ pauses the $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ simulation, storing its execution state, runs the $\mathcal{S}_{\mathcal{P}_h}$ code that handles the message, and finally restores the $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ execution state and continues simulating it.

Lemma 4.3.4. *$H_1 \sim H_2$.*

Proof. $\mathcal{S}$ is able to simulates all honest parties by using a single instance of an honest party since $\mathcal{F}_{ledger}$ is agnostic of the identity of the party that sent it a transaction, and the honest party does not maintain it's identity or any other identity-bound property (e.g., signing key or randomness).

The SEND-TX receives an already-signed transaction (as the transaction or *aux*). Hence it is $\mathcal{E}$'s responsibility to sign transactions.

Since the clock is shared between all parties, and Π_{ledger} is deterministic (COMP can be random, but it does not accept the identity of the caller) the outputs of all honest parties "monitoring" the base-ledger are identical.

Finally, since for every $\mathcal{F}^{meta}_{ledger}$ TICK, $\mathcal{S}$ calls $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$'s TICK the outputs of H_1 and H_2 are indistinguishable. □

4.3.3 H_3 — Honest Party Input Extraction

In H_3, $\mathcal{S}$ extracts $\mathcal{E}$ honest party inputs from $\mathcal{F}^{meta}_{ledger}$ messages it receives, whereas in H_2, $\mathcal{S}$ receives honest party inputs from the ideal functionality.

The extraction is done as follows (see PROCESS-$\mathcal{F}^{meta}_{ledger}$-MESSAGE in line 1 of algorithm 4.1):

- A $\mathcal{F}^{meta}_{ledger}$ (`Tx-Received`, tx) message was triggered by a (`Send-Tx`, tx) input and
- A $\mathcal{F}^{meta}_{ledger}$ (`Create-Tx-IDs-Request`, tx) message was triggered by a (`Get-Tx-IDs-Request`, tx) input.

Lemma 4.3.5. *$H_2 \sim H_3$.*

Proof. By observation of $\mathcal{F}_{ledger}$:

- $\mathcal{F}_{ledger}$ outputs (`Create-Tx-IDs-Request`, $\widehat{tx}$) on each invocation of GET-TX-IDS($\widehat{tx}$) (line 3 of functionality 3.2 (part 1)), and hence on each (`Get-Tx-IDs-Request`, $\widehat{tx}$) honest party input, and

- $\mathcal{F}_{ledger}$ outputs (`Tx-Received`, tx) on each invocation of SEND-TX(tx) (line 17 of functionality 3.2 (part 1)), and hence on each (`Send-Tx`, tx) honest party input.

The outputs are identical since:

- for each (`Create-Tx-IDs-Request`, $\widehat{tx}$) from $\mathcal{F}_{ledger}^{meta}$, the $\mathcal{S}$ simulates $\mathcal{E}$ sending (`Get-Tx-IDs-Request`, $\widehat{tx}$) input to $\mathcal{S}_{\mathcal{P}_h}$ (line 4) and returns a `Create-Tx-IDs-Response` with $\mathcal{S}_{\mathcal{P}_h}$'s output, and
- for each (`Tx-Received`, tx) from $\mathcal{F}_{ledger}^{meta}$, the $\mathcal{S}$ simulates $\mathcal{E}$ sending (`Send-Tx`, tx) input to $\mathcal{S}_{\mathcal{P}_h}$ (line 8).

□

4.3.4 H_4 — Complete Ideal-World Simulation

In H_4 the output of the honest parties is that of the ideal honest parties, i.e., the output of $\mathcal{F}_{ledger}^{meta}$, whereas in H_3 the output of the honest parties is that of $\mathcal{S}_{\mathcal{P}_h}$.

$\mathcal{S}$ "connects" $\mathcal{S}_{\mathcal{P}_h}$ and $\mathcal{F}_{ledger}^{meta}$ in PROCESS-$\mathcal{S}_{\mathcal{P}_h}$-MESSAGE (line 14 of algorithm 4.1):

- when $\mathcal{S}_{\mathcal{P}_h}$ outputs a (`Tx-Published`, tx) message, $\mathcal{S}$ calls $\mathcal{F}_{ledger}^{meta}$'s PROCESS-TX with tx, and
- when $\mathcal{S}_{\mathcal{P}_h}$ outputs a (`Txos-Released`, tx) message, $\mathcal{S}$ calls $\mathcal{F}_{ledger}^{meta}$'s RELEASE-TXOS with tx.

Honest sending of a meta transactions in the idela-world is atomic, however it is not in the real-world where it can be interrupted. To guarantee that a transaction that was sent to $\mathcal{F}_{ledger}^{meta}$ and was interrupted is not published, $\mathcal{S}$ removes the transaction from $\mathcal{F}_{ledger}^{meta}$ when it notices that was interrupted.

The identification part is done in CHECK-$\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$-FRAGMENT-VALIDITY (line 26). The procedure is called after each change to the state of $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$ (a transaction was published or the outputs of a transaction were released), and after $\mathcal{S}_{\mathcal{P}_h}$ has finished applying the change to its internal state. Upon identification, $\mathcal{S}$ pushes a `Tx-Removed` message to $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue*.

The actual removal is done when $\mathcal{S}_{\mathcal{P}_h}$ pops the message from *MsgQueue*. $\mathcal{S}$ "sees" the popping and calls $\mathcal{F}_{ledger}^{meta}$'s REMOVE-TX with the transaction.

Lemma 4.3.6. *$H_3 \sim H_4$.*

Proof. Note that only PROCESS-TX, RELEASE-TXOS calls translate to $\mathcal{F}_{ledger}^{meta}$ output visible to honest parties. By corollary 4.3.26, all $\mathcal{S}$ calls to $\mathcal{F}_{ledger}^{meta}$'s PROCESS-TX and RELEASE-TXOS succeed and these procedures are only called by $\mathcal{S}$.

By observation of $\mathcal{S}$'s PROCESS-$\mathcal{S}_{\mathcal{P}_h}$-MESSAGE (line 14), $\mathcal{S}$ calls these procedures whenever they happen in $\mathcal{S}_{\mathcal{P}_h}$ with the exact same arguments. □

Claim 4.3.7 ($\mathcal{F}_{ledger}$ — Transaction processing delay)**.** *For every transaction tx, if $\mathcal{F}_{ledger}$ (functionality 3.2) is given input (`Send-Tx`, tx) and tx is live (*IS-LIVE(tx) = ***true****), then within δ ticks* PROCESS-TX *is called with tx and then* RELEASE-TXOS *is called with tx.*

Proof. By observation of $\mathcal{F}_{ledger}$'s TICK (line 51, functionality 3.2). □

Claim 4.3.8 (Π_{ledger} — *MsgQueue* publishing delay)**.** *For every message m, if m is added to MsgQueue in time t (ticks), then m is popped from MsgQueue by time $t + s \cdot \delta$.*

Proof. By observation of Π_{ledger}'s FLUSH-MSG-QUEUE (line 84), popping from *MsgQueue* is halted only if the message at the head of the queue is a (`Tx-Published`, tx) message and the following conditions hold:

Algorithm 4.1 (part 1): $\mathcal{S}_{ledger}^{\mathcal{L}_{meta},\text{COMP},\text{DECOMP}}$

- $\mathcal{S}$ simulates $\mathcal{F}_{ledger}^{base}$ per definition 4.3.2 — the simulated instance is referred to as $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$.
- $\mathcal{S}$ simulates all honest parties by using a single instance per definition 4.3.3 — denote it as $\mathcal{S}_{\mathcal{P}_h}$. In fact, is uses a slightly-modified version of Π_{ledger} — *MsgQueue* may now also contain `Tx-Removed` messages.
- When $\mathcal{S}_{\mathcal{P}_h}$ pops a `Tx-Removed` message from *MsgQueue* in line 94 of FLUSH-MSG-QUEUE, and $\mathcal{S}$'s $\mathcal{S}_{\mathcal{P}_h}$-TX-REMOVED-POPPED is called, the message is regarded as an output of $\mathcal{S}_{\mathcal{P}_h}$ in the sense that it pauses the $\mathcal{S}_{\mathcal{P}_h}$ simulation.

```
1: procedure PROCESS-$\mathcal{F}_{ledger}^{meta}$-MESSAGE($m$)
2:     // Called when $\mathcal{F}_{ledger}^{meta}$ sends a message $m$
3:     if $m$ = (Create-Tx-IDs-Request, $\widehat{tx}$) then
4:         Simulate $\mathcal{E}$ sending (Get-Tx-IDs-Request, $\widehat{tx}$) input to $\mathcal{S}_{\mathcal{P}_h}$
5:         Await (Get-Tx-IDs-Response, $tx$, $aux$) response
6:         Send (Create-Tx-IDs-Response, $tx$, $aux$) to $\mathcal{F}_{ledger}^{meta}$
7:     else if $m$ = (Tx-Received, $tx$, $aux$) then
8:         Simulate $\mathcal{E}$ sending (Send-Tx, $tx$, $aux$) input to $\mathcal{S}_{\mathcal{P}_h}$
9:     else if $m$ = (Tick) then
10:        $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$.TICK()
11:    end if
12:    // Tx-Published and Txos-Released messages are discarded
13: end procedure
```

...Continued in algorithm 4.1 (part 2)...

1. *tx* is marked as fragment and
2. *tx*'s outputs are not in *FragmentTxos* and
3. *tx* has been published by $\mathcal{F}_{ledger}$ less than $s \cdot \delta$ ticks ago.

Suppose that messages in *MsgQueue* are sorted by the time they were inserted. In which case the claim holds. Suppose in contradiction the popping is halted due to message m_{halt}, and that *MsgQueue* contains a message $m_{delayed}$ that was added to the queue more than $s \cdot \delta$ ticks ago. Since *MsgQueue* is sorted m_{halt} too was added to the queue more than $s \cdot \delta$ ticks ago and thus should have been popped.

In fact, messages in *MsgQueue* are "almost sorted" — we show that any message that is not added at *MsgQueue*'s end or that is modified in-place, won't cause the popping to halt. We do this by an analysis of all the places where *MsgQueue* is modified:

- In lines 55, 118 and 149 a `Tx-Published` message is pushed at the end of *MsgQueue*.
- In line 148 a message is modified. The modification changes input/output amounts of the transaction in the `Tx-Published` message, and thus does not affect the popping from *MsgQueue*.
- In line 147 a (`Tx-Published`, tx^m) is inserted in the middle of *MsgQueue*. tx^m is the result of a successful DECOMP call and therefore cannot be a fragment and will be output when first considered and will not halt the popping from *MsgQueue*.

□

Claim 4.3.9 (Simulation — `Send-Tx` to `Tx-Published`/`Tx-Removed` delay)**.** *For every transaction tx, if input (`Send-Tx`, tx) is given to $\mathcal{S}_{\mathcal{P}_h}$, then within $s \cdot \delta_{base}$ ticks either:*

- *(`Tx-Published`, tx′) and then (`Txos-Released`, tx′) are added to MsgQueue, where tx′ is identical to tx up to input amount changes and addition of locking inputs or*
- *(`Tx-Removed`, tx) is added to MsgQueue.*

Proof. By observation of Π_{ledger}:

- *tx*'s first step is sent immediately in line 20 of SEND-TX (line 10).
- *tx*'s $(l+1)^{th}$ step is sent when all of the outputs in steps $\leq l$ were released in lines 59 to 61 of TXOS-RELEASED-BY-$\mathcal{F}_{ledger}$ (line 52).
- *tx* is removed from *TxStepQueues* if the last step sent was sent more than δ_{base} ticks ago in line 73 of TICK-BY-$\mathcal{F}_{ledger}$ (line 68).

Hence, if the outputs of each step's transactions are released within δ_{base} (see claim 4.3.7), then by definition 4.2.12, DECOMP returns a transaction tx^m that is identical to *tx* up to input amount changes, and within $s \cdot \delta_{base}$ ticks (`Tx-Published`, tx^m) and (`Txos-Released`, tx^m) are added to *MsgQueue*.

If within $s \cdot \delta_{base}$ such a message was not added to *MsgQueue* then at least one step's outputs were not published, which implies that it was removed from *mempool*, which in turn implies that it was, at some point not-strongly-valid.

By observation of $\mathcal{S}$'s (algorithm 4.1) CHECK-$\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$-FRAGMENT-VALIDITY (line 26), if a fragment of *tx* became not strongly-valid, $\mathcal{S}$ would push a (`Tx-Removed`, *tx*) to $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue*.

□

Claim 4.3.10 (Simulation — `Send-Tx` to $\mathcal{F}^{meta}_{ledger}$ calls delay)**.** *For every transaction tx, if input (`Send-Tx`, tx) is given to any honest party in the idela-world at time t (ticks), then by time $t + 2 \cdot s \cdot \delta_{base}$ $\mathcal{S}$ has called either:*

- *$\mathcal{F}^{meta}_{ledger}$'s* PROCESS-TX *and then* RELEASE-TXOS *with tx′, where tx′ is identical to tx up to input amount changes and addition of locking inputs or*
- *$\mathcal{F}^{meta}_{ledger}$'s* REMOVE-TX *with tx.*

Proof. When $\mathcal{F}^{meta}_{ledger}$'s SEND-TX is called with transaction *tx*, a (`Tx-Received`, *tx*) message is sent to $\mathcal{A}$ (see functionality 3.2 (part 1), line 17). When $\mathcal{S}$ sees the message, it simulates sending a (`Send-Tx`, *tx*) input to $\mathcal{S}_{\mathcal{P}_h}$ (see line 8 of algorithm 4.1).

Suppose in contradiction that by $t + 2 \cdot s \cdot \delta_{base}$ neither happened, that is, neither PROCESS-TX nor REMOVE-TX were called.

By claim 4.3.8 by time $t+s\cdot\delta_{base}$, neither (`Txos-Released`, *tx′*) for any such *tx′* nor (`Tx-Removed`, *tx*) were present in $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue*.

By claim 4.3.9 by time *t*, $\mathcal{S}_{\mathcal{P}_h}$ was not given input (`Send-Tx`, *tx*), contradicting the assumption that it did. □

Claim 4.3.11 ($s_m \underset{txo}{\overset{state}{\rightsquigarrow}} s_b$ is preserved by $\mathcal{F}_{ledger}$'s SPEND-OUTPUT)**.** *Let s_m and s_b be two $\mathcal{F}_{ledger}$ states, and suppose $s_m \underset{txo}{\overset{state}{\rightsquigarrow}} s_b$, and let id_{txo} be an output ID and $amount_r$ be an amount. Define:*

- *s_b' as the state resulting from running* SPEND-OUTPUT*(id_{txo}, $amount_r$) (line 126, functionality 3.2) with respect to s_b and*

- *$s_m^{'}$ as the state resulting from running* SPEND-OUTPUT$(id_{txo}, amount_r)$ *with respect to s_m.*

If STATE$(s_m, id_{txo}) \neq \bot$, *then* $s_m^{'} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b^{'}$.

Proof. Since $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$, STATE$(s_b, id_{txo}) \neq \bot$. Both base and meta states are changed to **spent** and both amounts to $amount_r$. All other $\overset{state}{\underset{txo}{\rightsquigarrow}}$ conditions follow from $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$. □

Lemma 4.3.12 ($s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ is preserved by $\mathcal{F}_{ledger}$'s VALIDATE-AND-APPLY-SPENDING-INPUT). *Let s_m and s_b be two $\mathcal{F}_{ledger}$ states, and suppose $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$, and let txi_b and txi_m be two identical transaction inputs and id_{tx} be a transaction ID and define:*

- *$s_b^{'}$ as the state resulting from running* VALIDATE-AND-APPLY-SPENDING-INPUT(txi_b, id_{tx}) *(line 99, functionality 3.2) with respect to s_b and*
- *$s_m^{'}$ as the state resulting from running* VALIDATE-AND-APPLY-SPENDING-INPUT(txi_m, id_{tx}) *with respect to s_m.*

If $\varphi^{spend}_{s_m, id_{txo}} \neq \bot$ and $\varphi^{spend}_{s_m, id_{txo}} \in \mathcal{L}_{base}$ where id_{txo} is the referenced transaction output ID, then either:

- *both calls return* ***true*** *and $s_m^{'} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b^{'}$ or*
- *both calls return* ***false****.*

Proof. Denote the amount spent as $amount_{txi}$, and the spend and lock data as $data^{spend}$ and $data^{lock}$ respectively.

The proof is by exhaustive case analysis of all lines in which VALIDATE-AND-APPLY-SPENDING-INPUT can return. For line l we show that either both calls return or both don't and that when they do, they return the same boolean value and consistency is preserved.

1. Line 103 (**false** is returned if the referenced output is not in a spendable state): Due to weak equivalence, STATE$(s_a, id) = \{\bot, \textbf{pending}, \textbf{pending-locked}\} \iff$ STATE$(s_b, id) \in \{\bot, \textbf{pending}, \textbf{pending-locked}\}$.

   Hence, either both calls will return **false** in line 103, or neither will and we can assume in the following lines that STATE$(s_m, id_{txo}) \notin \{\bot, \textbf{pending}, \textbf{pending-locked}\}$ and STATE$(s_b, id_{txo}) \notin \{\bot, \textbf{pending}, \textbf{pending-locked}\}$.

2. Line 108 (**false** is returned if the referenced output has insufficient remaining coins): Since $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$, REMAINING$(s_m, id_{txo})$ = REMAINING(s_b, id_{txo}).

   Hence, if $amount_{txi} >$ REMAINING(s_b, id_{txo}) = REMAINING(s_m, id_{txo}), both calls will return **false** in line 108.

   Otherwise, both calls will not return in line 108, and we can assume in the following lines that $amount_{txi} \leq$ REMAINING(s_b, id_{txo}) = REMAINING(s_m, id_{txo}).

3. Line 113 (**true** is returned if the referenced output is **spent**): Since s_b and s_m are weakly-equivalent, STATE$(s_b, id_{txo}) = \textbf{spent} \iff$ STATE$(s_m, id_{txo}) = \textbf{spent}$.

   Hence, if STATE(s_b, id_{txo}) = STATE(s_m, id_{txo}) = **spent**, both calls will return **true** in line 113. Also, claim 4.3.11 applies to the SPEND-OUTPUT call in line 112 so consistency is preserved.

   Otherwise, both calls will not return in line 113, and we can assume in the following lines that STATE$(s_m, id_{txo}) \neq$ **spent** and STATE$(s_b, id_{txo}) \neq$ **spent**.

4. Line 117 (**false** is returned if the locking condition is not satisfied): Since $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$, $\varphi^{lock}_{s_m,id_{txo}} = \varphi^{lock}_{s_b,id_{txo}}$. Since s_m and s_b are weakly equivalent, and because φ^{lock} is agnostic to $\perp$/**pending**/**pending-locked** and **unspent**/**locked** deviations, $\varphi^{lock}_{s_b,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_b}) = \varphi^{lock}_{s_m,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_m})$.

   Hence, if $\varphi^{lock}_{s_b,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_b}) = \varphi^{lock}_{s_m,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_m}) = \textbf{false}$, both calls will return **false** in line 117.

   Otherwise, both calls will not return in line 117, and we can assume in the following lines that $\varphi^{lock}_{s_b,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_b}) = \varphi^{lock}_{s_m,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_m}) = \textbf{true}$.

5. Line 121 (**false** is returned if the spending condition is not satisfied): Since $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ and $\varphi^{spend}_{s_m,id_{txo}} \in \mathcal{L}_{base}$, $\varphi^{spend}_{s_m,id_{txo}} = \varphi^{spend}_{s_b,id_{txo}}$. Since s_m and s_b are weakly equivalent, and because φ^{spend} is agnostic to $\perp$/**pending**/**pending-locked** and **unspent**/**locked** deviations, $\varphi^{spend}_{s_b,id_{txo}}(data^{spend}, id_{tx}, UTXO_{s_b}) = \varphi^{spend}_{s_m,id_{txo}}(data^{spend}, id_{tx}, UTXO_{s_m})$.

   Hence, if $\varphi^{spend}_{s_b,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_b}) = \varphi^{spend}_{s_m,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_m}) = \textbf{false}$, both calls will return **false** in line 121.

   Otherwise, both calls will not return in line 121, and we can assume in the following lines that $\varphi^{spend}_{s_b,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_b}) = \varphi^{spend}_{s_m,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_m}) = \textbf{true}$.

6. Line 124: both calls will return **true** in line 124. Also, claim 4.3.11 applies to the SPEND-OUTPUT call in line 123 so consistency is preserved.

□

Lemma 4.3.13 ($s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ is preserved by $\mathcal{F}_{ledger}$'s VALIDATE-AND-APPLY-LOCKING-INPUT**)**. *Let s_m and s_b be two $\mathcal{F}_{ledger}$ states, and suppose $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$, and let txi_b and txi_m be two identical transaction inputs and id_{tx} be a transaction ID and define:*

- *s_b' as the state resulting from running* VALIDATE-AND-APPLY-LOCKING-INPUT*(txi_b, id_{tx}) (line 130, functionality 3.2) with respect to s_b and*
- *s_m' as the state resulting from running* VALIDATE-AND-APPLY-LOCKING-INPUT*(txi_m, id_{tx}) with respect to s_m.*

If $\varphi^{spend}_{s_m,id_{txo}} = \perp$ and $\varphi^{spend}_{s_m,id_{txo}} \in \mathcal{L}_{base}$ where id_{txo} is the referenced transaction output ID, then either:

- *both calls return **true** and $s_m' \overset{state}{\underset{txo}{\rightsquigarrow}} s_b'$ or*
- *both calls return **false**.*

Proof. Denote the amount spent as $amount_{txi}$ and the lock data as $data^{lock}$.

The proof is by exhaustive case analysis of all lines in which VALIDATE-AND-APPLY-LOCKING-INPUT can return. For line l we show that either both calls return or both don't and that when they do, they return the same boolean value and consistency is preserved.

1. Line 134 (**false** is returned if the referenced output is not in a lockable state): Since $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$, STATE$(s_m, id) \neq \perp \iff$ STATE$(s_b, id) \neq \perp$.

   Hence, either both calls will return **false** in line 134, or neither will and we can assume in the following lines that STATE$(s_m, id_{txo}) \neq \perp$ and STATE$(s_b, id_{txo}) \neq \perp$.

2. Line 139 (**false** is returned if locking is done with amount 0): txi_b and txi_m are identical and both spend $amount_{txi}$.

   Hence, if $amount_{txi} \neq 0$, both calls will return **false** in line 139. Otherwise, both calls will not return in line 139, and we can assume in the following lines that $amount_{txi} = 0$.

3. Line 143 (**true** is returned if the referenced output is **pending-locked/locked**): Since s_b and s_m are weakly-equivalent and $\textsc{state}(s_m, id_{txo}) \neq \perp$ and $\textsc{state}(s_b, id_{txo}) \neq \perp$, $\textsc{state}(s_m, id) \in \{\textbf{pending}, \textbf{pending-locked}\} \iff \textsc{state}(s_b, id) \in \{\textbf{pending}, \textbf{pending-locked}\}$.

   Hence, if $\textsc{state}(s_m, id) \in \{\textbf{pending}, \textbf{pending-locked}\}$ or $\textsc{state}(s_b, id) \in \{\textbf{pending}, \textbf{pending-locked}\}$ both calls will return **true** in line 143. Also, the states remain unchanged so consistency is preserved.

   Otherwise, both calls will not return in line 143, and we can assume in the following lines that $\textsc{state}(s_m, id) \notin \{\textbf{pending}, \textbf{pending-locked}\}$ and $\textsc{state}(s_b, id) \notin \{\textbf{pending}, \textbf{pending-locked}\}$.

4. Line 147 (**false** is returned if the locking condition is not satisfied): Since $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$, $\varphi^{lock}_{s_m,id_{txo}} = \varphi^{lock}_{s_b,id_{txo}}$. Since s_m and s_b are weakly equivalent, and because φ^{lock} is agnostic to $\perp$/**pending**/**pending-locked** and **unspent**/**locked** deviations, $\varphi^{lock}_{s_b,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_b}) = \varphi^{lock}_{s_m,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_m})$.

   Hence, if $\varphi^{lock}_{s_b,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_b}) = \varphi^{lock}_{s_m,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_m}) = \textbf{false}$, both calls will return **false** in line 147.

   Otherwise, both calls will not return in line 147, and we can assume in the following lines that $\varphi^{lock}_{s_b,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_b}) = \varphi^{lock}_{s_m,id_{txo}}(data^{lock}, id_{tx}, UTXO_{s_m}) = \textbf{true}$.

5. Line 154: both calls will return **true** in line 154.

   By observation of lines 149 to 153, only the state of the referenced output is modified, and only **pending** → **pending-locked** and **unspent** → **locked** transitions are possible. Since $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ and because $\textsc{state}(s_m, id_{txo}) \neq \perp$ and $\textsc{state}(s_b, id_{txo}) \neq \perp$, the only base-state/meta-state combinations possible are:

   - such that the two states are the same $\textsc{state}(s_m, id_{txo}) = \textsc{state}(s_b, id_{txo})$,
   - $\textsc{state}(s_b, id_{txo}) = \textbf{pending}$ and $\textsc{state}(s_m, id_{txo}) = \textbf{pending-locked}$,
   - $\textsc{state}(s_b, id_{txo}) = \textbf{unspent}$ and $\textsc{state}(s_m, id_{txo}) = \textbf{locked}$.

   In the first case, both states will be changed to the same state, in the second case $\textsc{state}(s_b, id_{txo})$ will be changed to **pending-locked**, and in the third case $\textsc{state}(s_b, id_{txo})$ will be changed to **locked**. Hence, $s'_m \overset{state}{\underset{txo}{\rightsquigarrow}} s'_b$.

□

Claim 4.3.14 ($s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ is preserved by $\mathcal{F}_{ledger}$'s VALIDATE-AND-APPLY-INPUTS)**.** *Let s_m and s_b be two $\mathcal{F}_{ledger}$ states, and suppose $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$, and let $inputs_b$ and $inputs_m$ be two identical lists of respectively identical transaction inputs and id_{tx} be a transaction ID and define:*

- *s'_b as the state resulting from running* VALIDATE-AND-APPLY-INPUTS$(inputs_b, id_{tx})$ *(line 83, functionality 3.2) with respect to s_b and*
- *s'_m as the state resulting from running* VALIDATE-AND-APPLY-INPUTS$(inputs_m, id_{tx})$ *with respect to s_m.*

If for every txi $\in$ *inputs*$_m$, $\varphi^{spend}_{s_m, id_{txo}} \in \mathcal{L}_{base}$ *where* id_{txo} *is the transaction output ID referenced by txi, then either:*

- *both calls return* ***true*** *and* $s'_m \overset{state}{\underset{txo}{\rightsquigarrow}} s'_b$ *or*
- *both calls return* ***false****.*

Proof. The proof is by induction on the index of the input. Suppose that the induction hypothesis holds for (up to and including) the i^{th} input.

If $\varphi^{spend} = \perp$, then by lemma 4.3.13 either both VALIDATE-AND-APPLY-LOCKING-INPUT calls will return **true** and the consistency is preserved, or both return **false**, in which case both VALIDATE-AND-APPLY-INPUTS calls will return **false** in line 89.

Otherwise, then by lemma 4.3.12 either both VALIDATE-AND-APPLY-SPENDING-INPUT calls will return **true** and the consistency is preserved, or both return **false**, in which case both VALIDATE-AND-APPLY-INPUTS calls will return **false** in line 93. □

Claim 4.3.15 ($s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ is preserved by $\mathcal{F}_{ledger}$'s APPLY-OUTPUTS)**.** *Let* s_m *and* s_b *be two* $\mathcal{F}_{ledger}$ *states, and suppose* $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$*, and let outputs*$_b$ *and outputs*$_m$ *be two identical lists of respectively identical transaction outputs and txo_ids be a list of IDs of the same length. Define:*

- s'_b *as the state resulting from running* APPLY-OUTPUTS(id_{txo}, *txo_ids*) *(line 126, functionality 3.2) with respect to* s_b *and*
- s'_m *as the state resulting from running* APPLY-OUTPUTS(id_{txo}, *txo_ids*) *with respect to* s_m*.*

Then it holds that $s'_m \overset{state}{\underset{txo}{\rightsquigarrow}} s'_b$*.*

Proof. The same IDs are mapped to respectively identical outputs in state **pending**. □

Lemma 4.3.16 ($s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ is preserved by $\mathcal{F}_{ledger}$'s VALIDATE-AND-APPLY-TX)**.** *Let* s_m *and* s_b *be two* $\mathcal{F}_{ledger}$ *states, and suppose* $s_m \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$*, and let* tx_b *and* tx_m *be two identical transactions and define:*

- s'_b *as the state resulting from running* VALIDATE-AND-APPLY-TX(tx_b) *(line 70, functionality 3.2) with respect to* s_b *and*
- s'_m *as the state resulting from running* VALIDATE-AND-APPLY-TX(tx_m) *with respect to* s_m*.*

If $\varphi^{spend}_{s_m, id_{txo}} \in \mathcal{L}_{base}$ *for each output txo spent by* tx_m*, then either:*

- *both calls return* ***true*** *and* $s'_m \overset{state}{\underset{txo}{\rightsquigarrow}} s'_b$ *or*
- *both calls return* ***false****.*

Proof. The proof is by exhaustive case analysis of all lines in which VALIDATE-AND-APPLY-TX can return. For line l we show that either both calls return or both don't and that when they do, they return the same boolean value and it holds that the resulting states are consistent.

1. Line 74 (**false** is returned if incoming coins < outgoing coins): Since tx_b and tx_m are identical:
    - $out_b = \sum_{txo \in outputs_{tx_b}} amount_{txo} = \sum_{txo \in outputs_{tx_m}} amount_{txo} = out_m$
    - $in_b = \sum_{txi \in inputs_b} amount_{txi} = \sum_{txi \in inputs_m} amount_{txi} = in_m$

Thus, $out_b > in_b \iff out_m > in_m$.

Hence, if $out_b > in_b = out_m > in_m = \textbf{true}$, both calls will return **false** in line 74.

Otherwise, both calls will not return in line 74, and we can assume in the following lines that $out_b \leq in_b = out_m \leq in_m = \textbf{true}$.

2. Line 78 (**false** is returned if any of the inputs is invalid): tx_b and tx_m are identical, so their inputs $inputs_b$ and $inputs_m$ are also respectively identical. Hence claim 4.3.14 applies, and therefore either the applications of both $inputs_b$ and $inputs_m$ will fail, or both will succeed and the consistency will be preserved.

3. Line 81: both calls will return **true** in line 81. By claim 4.3.15, $s_m^{'} \underset{txo}{\overset{state}{\rightsquigarrow}} s_b^{'}$ so consistency is preserved.

□

Definition 4.3.17 (Applying *MsgQueue* to $\mathcal{F}_{ledger}$ state)**.** Given an honest party (protocol 4.1) *MsgQueue* q, and a state of $\mathcal{F}_{ledger}$ (functionality 3.2) s, $\textsc{apply-queue}(q, s)$ is defined in algorithm 4.2.

Denote $q_{[:m_x]}$ the prefix of q that ends with message m_x and $q^{'}_{[:m_y]}$ the prefix of $q^{'}$ that ends with message m_y. Let $s_{m_x} = \textsc{apply-queue}(q_{[:m_x]}, s_m)$.

Claim 4.3.18 ($s_m = s_b$ is preserved by $\textsc{apply-queue}$)**.** *Let s_m and s_b be two identical $\mathcal{F}_{ledger}$ states and q a MsgQueue. Define:*

- *$s_b^{'}$ as the state resulting from running $\textsc{apply-queue}(q, s_b)$ (definition 4.3.17 and*
- *$s_m^{'}$ as the state resulting from running $\textsc{apply-queue}(q, s_m)$.*

Then $s_m^{'} = s_b^{'}$.

Proof. Holds since $\textsc{apply-queue}$ is deterministic. □

Lemma 4.3.19 ($s_{m+q} \underset{txo}{\overset{state}{\rightsquigarrow}} s_b$ is preserved by Π_{ledger}'s $\textsc{add-locking-inputs}$)**.** *Let s_m and s_b be two $\mathcal{F}_{ledger}$ states, and q be a state of $\mathcal{S}_{\mathcal{P}_h}$'s MsgQueue. Define:*

- *as $q^{'}$ as q after running $\textsc{add-locking-inputs}$ (protocol 4.1, line 100) with respect to q,*
- *$s_{m+q} = \textsc{apply-queue}(q, s_m)$,*
- *$s_{m+q^{'}} = \textsc{apply-queue}(q^{'}, s_m)$.*

If $s_{m+q} \neq \perp$ and $s_{m+q} \underset{txo}{\overset{state}{\rightsquigarrow}} s_b$, then $s_{m+q^{'}} \neq \perp$ and $s_{m+q^{'}} \underset{txo}{\overset{state}{\rightsquigarrow}} s_b$.

Proof. The proof is by induction on the index of the added locking input. Suppose that the induction hypothesis holds for (up to and including) the i^{th} input.

Let *txi* be the $(i+1)^{th}$ input, and denote its transaction *tx* and the `Tx-Published` message m_{tx}. Also denote as txo_f the fragment output referenced by *txi*, and as txo^m the meta-output that txo_f is mapped to in *LockingFragmentTxos*. Suppose that *tx* doesn't already have an input that locks txo^m.

If q is of the following form: $[m_1, \ldots, m_k, m_{tx}, m_l, \ldots, m_n]$, then $q^{'}$ is of the following form: $[m_1, \ldots, m_k, m_{tx_{lock}}, m_l, \ldots, m_n]$, where $m_{tx_{lock}} = (\texttt{Tx-Published}, tx_{lock})$ and tx_{lock} is identical to *tx* with the addition of the locking input txi_{lock}.

We show that each $m \in q^{'}$ is applied successfully in turn when evaluating $\textsc{apply-queue}(q^{'}, s_m)$:

1. $[m_1, \ldots, m_k]$ — by claim 4.3.18, $s_{m_k} = s'_{m_k}$, and since $s_{m_k} \neq \perp$, then $s'_{m_k} \neq \perp$.

2. $m_{tx_{lock}}$ — since $s_{m_k} = s'_{m_k}$, the two states are consistent. Recall that tx_{lock} differs from tx only in the addition of txi_{lock}. We show that the application of tx_{lock} is successful.

   First note that by construction, txi_{lock} spends amount 0, hence the "incoming coins $\geq$ outgoing coins" condition (line 74) still holds for tx_{lock}.

   Since tx's existing inputs were not modified, the states after the application of these inputs s_{m_k+} and s'_{m_k+}, are still identical.

   We prove that the call VALIDATE-AND-APPLY-LOCKING-INPUT(txi_{lock}) with respect to s'_{m_k+} will return **true** in line 154 or in line 143 and that consistency is preserved by showing that it cannot return **false** in any other line:

   a) Line 134 (**false** is returned if the referenced output is not in a lockable state): Since *LockingFragmentTxos* maps txo_f to txo^m, txo^m can't be in state $\perp$. So the call will not return in line 134.

   b) Line 139 (**false** is returned if locking is done with amount 0): By construction, txi_{lock} spends amount 0, so the call will not return in line 139.

   c) Line 147 (**false** is returned if the locking condition is not satisfied): Since:

      - by condition 4 of definition 4.2.13, txo_f and txo^m share the locking condition,
      - by construction, txi_{lock}'s locking data is identical to that of txi,
      - s_{m_k+} and s'_{m_k+} are identical,
      - txi and txi_{lock} are inputs of the same transaction tx.

      Then: $\varphi^{lock}_{txo^m}(data^{lock}_{txi_{lock}}, id_{tx}, s'_{m_k+}) = \varphi^{lock}_{txo_f}(data^{lock}_{txi}, id_{tx}, s_{m_k+}) =$ **true**. So the call will not return in line 147.

   Since tx's outputs were not modified, $s'_{m_{tx_{lock}}}$ and $s_{m_{tx}}$ differ only in txo^m's state:

   - STATE$(s_{m_{tx}}, txo^m) =$ **pending** $\Rightarrow$ STATE$(s'_{m_{tx_{lock}}}, txo^m) =$ **pending-locked**
   - STATE$(s_{m_{tx}}, txo^m) =$ **unspent** $\Rightarrow$ STATE$(s'_{m_{tx_{lock}}}, txo^m) =$ **locked**
   - STATE$(s_{m_{tx}}, txo^m) \notin \{$**pending**, **unspent**$\} \Rightarrow$ STATE$(s'_{m_{tx_{lock}}}, txo^m) =$ STATE$(s_{m_{tx}}, txo^m)$

   Hence all $\overset{state}{\underset{txo}{\rightsquigarrow}}$ conditions still hold.

3. $[m_l, \ldots, m_n]$ — Suppose $s'_{m_{j-1}}$ and $s_{m_{j-1}}$ differ only in txo^m's state as specified above — we prove the same applies for s'_{m_j} and s_{m_j}, after applying m_j.

   a) If $m_j = (\texttt{Tx-Published}, tx_j)$ and tx_j's input txi_j references txo^m, then it is implied that txo^m has been released in both states, and either:

      - the two states are equal, in which case the application of tx_j will have the same effect, or
      - STATE$(s_{m_{j-1}}, txo^m) =$ **unspent** and STATE$(s'_{m_{j-1}}, txo^m) =$ **locked**. Locking would not affect the states, and spending is agnostic to whether the output that is being spent is in state **unspent** or in state **locked**— the state would transition to **spent** either way.

   b) Otherwise if $m_j = (\texttt{Txos-Released}, tx_j)$ and $tx_j = tx^m$ then STATE$(s_{m_{j-1}}, txo^m) \in \{$**pending**, **pending-locked**$\}$ and STATE$(s'_{m_{j-1}}, txo^m) =$ **pending-locked**:

      - If STATE$(s_{m_{j-1}}, txo^m) =$ **pending**, then STATE$(s_{m_j}, txo^m) =$ **unspent**, STATE$(s'_{m_j}, txo^m) =$ **locked**.

- Otherwise $\textsc{state}(s_{m_{j-1}}, txo^m) = \textbf{pending-locked}$, then $\textsc{state}(s_{m_j}, txo^m) = \textbf{locked}, \textsc{state}(s'_{m_j}, txo^m) = \textbf{locked}$.

c) Otherwise $m_j = (\texttt{Tx-Removed}, tx_j)$, then tx_j wasn't strongly-valid with respect to $s_{m_{j-1}}$ (see line 163 of functionality 3.2) and one of the following must hold:

- $\textsc{validate-and-apply-tx}(tx_j)$ returned **false** with respect to $s_{m_{j-1}}$. Since the conditions hold at $(s'_{m_{j-1}}, s_{m_{j-1}})$, the only relevant difference between the two states is the state of txo^m, which was locked. Hence it is impossible that $\textsc{validate-and-apply-tx}(tx_j) = \textbf{true}$ with respect to $s'_{m_{j-1}}$.
- $\textsc{is-live}(tx_j)$ returned **false**. Since $\textsc{is-live}$ is a function of the transaction (not of the ledger state), $\textsc{is-live}$ will still return **false** in s'_{m_j}.
- tx_j spent an output in state **locked/spent** with respect to $s_{m_{j-1}}$.
  - $\textsc{state}(s_{m_{j-1}}, txo^m) = \textbf{locked} \Rightarrow \textsc{state}(s'_{m_{j-1}}, txo^m) = \textbf{locked}$
  - $\textsc{state}(s_{m_{j-1}}, txo^m) = \textbf{spent} \Rightarrow \textsc{state}(s'_{m_{j-1}}, txo^m) = \textbf{spent}$

  By the induction hypothesis, $\textsc{state}(s_{m_j}, txo) = \textsc{state}(s'_{m_j}, txo)$, so the corresponding output is in the same state in $\textsc{state}(s'_{m_j}, txo)$.

Hence s'_{m_n} and s_{m_n} are identical up to locking of txo^m (**pending** $\rightarrow$ **pending-locked** / **unspent** $\rightarrow$ **locked**), so $s_{m+q'} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$. □

Lemma 4.3.20 ($s_{m+q} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ is preserved by Π_{ledger}'s $\textsc{simple-keystone-published}$). *Denote the states of $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ and $\mathcal{F}^{meta}_{ledger}$ as s_b and s_m respectively and $\mathcal{S}_{\mathcal{P}_h}$'s MsgQueue as q. Define $s_{m+q} = \textsc{apply-queue}(q, s_m)$ and suppose $s_{m+q} \neq \bot$ and $s_{m+q} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$.*

Let tx be a transaction and suppose tx is successfully-applied to s_b, and denote the resulting state as s'_b. If as a result, $\mathcal{S}_{\mathcal{P}_h}$ calls $\textsc{simple-keystone-published}$ *(protocol 4.1, line 115) with respect to q, then $s_{m+q'} \neq \bot$ and $s_{m+q'} \overset{state}{\underset{txo}{\rightsquigarrow}} s'_b$, where q' is q after the* $\textsc{simple-keystone-published}$ *call and $s_{m+q'} = \textsc{apply-queue}(q', s_m)$.*

Proof. If q is of the following form: $[m_1, \ldots, m_n]$, then q' is of the following form: $[m_1, \ldots, m_n, m_{tx^m}]$, where $m_{tx^m} = (\texttt{Tx-Published}, tx^m)$ and tx^m is the transaction output by $\textsc{decomp}$. By claim 4.3.18, $s'_{m_n} = s_{m_n} = s_{m+q}$, and since $s_{m+q} \neq \bot$, then $s'_{m_n} \neq \bot$.

By observation of $\textsc{tx-published-by-}\mathcal{F}_{ledger}$ (line 23 of protocol 4.1) and of $\textsc{simple-keystone-published}$:

- tx^m's ID and outputs are taken from the keystone *tx*.
- tx^m's inputs are taken from:
  - the keystone *tx* if it is non-simple and the splitter is not present in *MsgQueue* or
  - the transaction output by the $\textsc{decomp}$ call in line 25 otherwise.

Due to the lemma condition, $s_{m+q} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$. If tx^m's inputs are taken from the keystone *tx*, tx^m is identical to *tx*, and we can apply lemma 4.3.16 to conclude that tx^m can be successfully-applied to s_{m+q}, and that the consistency is preserved.

Otherwise, tx^m's inputs are taken from the the transaction output by the $\textsc{decomp}$ call, and we can apply condition 7 in definition 4.2.13 to conclude that tx^m can be successfully-applied to s_{m+q} and that the consistency is preserved. □

Lemma 4.3.21 ($s_{m+q} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ is preserved by Π_{ledger}'s NON-SIMPLE-KEYSTONE-PUBLISHED). *Denote the states of $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ and $\mathcal{F}^{meta}_{ledger}$ as s_b and s_m respectively and $\mathcal{S}_{\mathcal{P}_h}$'s MsgQueue as q. Define $s_{m+q} =$ APPLY-QUEUE(q, s_m) and suppose $s_{m+q} \neq \perp$ and $s_{m+q} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$.*

Let tx be a transaction and suppose tx is successfully-applied to s_b, and denote the resulting state as $s^{'}_b$. If as a result, $\mathcal{S}_{\mathcal{P}_h}$ calls NON-SIMPLE-KEYSTONE-PUBLISHED *(protocol 4.1, line 120) with respect to q, then $s_{m+q^{'}} \neq \perp$ and $s_{m+q^{'}} \overset{state}{\underset{txo}{\rightsquigarrow}} s^{'}_b$, where $q^{'}$ is q after the* NON-SIMPLE-KEYSTONE-PUBLISHED *call and $s_{m+q^{'}} =$* APPLY-QUEUE$(q^{'}, s_m)$.

Proof. If q is of the following form: $[m_1, \ldots, m_k, m_{tx_s}, m_l, \ldots, m_n]$, then $q^{'}$ is of the following form: $[m_1, \ldots, m_k, m_{tx^m}, m_{tx_{s}\text{-}}, m_l, \ldots, m_n, m_{tx_{dummy}}]$, where:

- $m_{tx_s} =$ (`Tx-Published`, tx_s) and tx_s is the *MsgQueue*-version of splitter,
- $m_{tx^m} =$ (`Tx-Published`, tx^m) and tx^m is initialized in line 126 with tx_s's inputs and the outputs of the meta-transaction returned from the call to DECOMP in line 25,
- $m_{tx_{s}\text{-}} =$ (`Tx-Published`, tx_s-) and tx_s- initialized to tx_s in line 128,
- $m_{tx_{dummy}} =$ (`Tx-Published`, tx_{dummy}) and tx_{dummy} is initialized in line 129 with tx_k's single input and no outputs.

Note that $s_{m_n} = s_{m+q}$ and $s^{'}_{m_y} =$ APPLY-QUEUE$(q'_{[:m_y]}, s_m)$.

We show that each $m \in q^{'}$ is applied successfully in turn when evaluating APPLY-QUEUE$(q^{'}, s_m)$:

1. $[m_1, \ldots, m_k]$ — by claim 4.3.18, $s^{'}_{m_k} = s_{m_k} = s_{m+q}$, and since $s_{m+q} \neq \perp$, then $s^{'}_{m_k} \neq \perp$.
2. $m_{tx^m}, m_{tx_{s}\text{-}}$ — Following is an exhaustive list of lines in which VALIDATE-AND-APPLY-TX can return **false**— for each we show simultaneously for tx^m and for tx_s- that the calls don't return there.
   a) Line 74 (**false** is returned if incoming coins < outgoing coins):

      By condition 3 in definition 4.2.13, tx^m's output amounts are identical to those of tx_k; the same total amount ($amount_k$) is set to tx^m inputs in line 142 of protocol 4.1.

      The same amount reduced from tx_s-'s inputs ($amount_k$) in line 135 is reduced from its outputs in line 141. Since tx_s is valid in this regard, then so is tx_s-.

      Hence the two calls will not return (**false**) in line 74.

   b) Line 78 (**false** is returned if any of the inputs is invalid):

      tx^m's inputs are initialized as a fresh copy of tx_s inputs, tx_s- is initialized as a fresh copy of tx_s. tx^m also receives tx_s's ID.

      Following is an exhaustive list of lines in which VALIDATE-AND-APPLY-SPENDING-INPUT can return **false**— for each we show simultaneously for corresponding spending inputs of tx^m and tx_s- that the calls don't return there.

      - Line 103 (**false** is returned if the referenced output is not in a spendable state): Since the referenced outputs are shared by tx^m and tx_s, then because $s^{'}_{m_k} = s_{m_k}$ and tx_s's inputs are valid in this regard, then so are tx^m's inputs.

        Since the referenced outputs are shared by tx_s- and tx^m, then because the outputs have already been spent by tx^m, tx_s-'s inputs are valid in this regard.

        Hence the two calls will not return (**false**) in line 103.

- Line 108 (**false** is returned if the referenced output has insufficient remaining coins): The (vector) addition tx^m's input amounts and tx_{s^-}'s input amounts sums up to tx_s's input amounts (see lines 141 and 142). Then because $s^{'}_{m_k} = s_{m_k}$ and tx_s's inputs are valid in this regard, then so are tx^m's and tx_{s^-}'s inputs.

  Hence the two calls will not return (**false**) in line 108.
- Line 113 (**true** is returned if the referenced output is **spent**): Since the referenced outputs are shared by tx^m and tx_s, then because $s^{'}_{m_k} = s_{m_k}$ a VALIDATE-AND-APPLY-SPEND call for an tx^m input will return **true** in line 113 iff the call for the corresponding tx_s input will.

  Hence the tx^m call will return **true** in line 113 if the call for the corresponding tx_s input did.

  Since the referenced outputs are shared by tx_{s^-} and tx^m, then because the outputs have already been spent by tx^m, all VALIDATE-AND-APPLY-SPENDING-INPUT calls for tx_{s^-}'s inputs will return **true** in line 113.
- Line 117 (**false** is returned if the locking condition is not satisfied): Since tx^m's inputs are copied from tx_s and only the amounts are modified, then because $s^{'}_{m_k} = s_{m_k}$ and tx_s's inputs are valid in this regard, then so are tx^m's inputs.

  Hence the call will not return (**false**) in line 117.
- Line 121 (**false** is returned if the spending condition is not satisfied): Since tx^m's inputs are copied from tx_s and only the amounts are modified, then because $s^{'}_{m_k} = s_{m_k}$ a VALIDATE-AND-APPLY-SPENDING-INPUT call for an tx^m input will return in line 121 iff the call for the corresponding tx_s input will.

  Hence the call will not return (**false**) in line 121.

Following is an exhaustive list of lines in which VALIDATE-AND-APPLY-LOCKING-INPUT can return **false**— for each we show simultaneously for corresponding locking inputs of tx^m and tx_{s^-} that the calls don't return there.

i. Line 134 (**false** is returned if the referenced output is not in a lockable state): Since the referenced outputs are shared by tx^m, tx_{s^-} and tx_s, then because $s^{'}_{m_k} = s_{m_k}$ and tx_s's inputs are valid in this regard, then so are tx^m's inputs.

   Hence the two calls will not return (**false**) in line 134.

ii. Line 139 (**false** is returned if locking is done with amount 0): Since the tx^m and tx_{s^-} are instantiated with tx_s's inputs, then because tx_s's inputs are valid in this regard, then so are tx^m's and tx_{s^-}'s inputs.

   Hence the two calls will not return (**false**) in line 139.

iii. Line 143 (**true** is returned if the referenced output is **pending-locked/locked**): Since the referenced outputs are shared by tx^m and tx_s, then because $s^{'}_{m_k} = s_{m_k}$ a VALIDATE-AND-APPLY-LOCKING-INPUT call for an tx^m input will return **true** in line 143 iff the call for the corresponding tx_s input will.

   Hence the tx^m call will return **true** in line 143 if the call for the corresponding tx_s input did.

   Since the referenced outputs are shared by tx_{s^-} and tx^m, then because the outputs have already been locked by tx^m, all VALIDATE-AND-APPLY-LOCKING-INPUT calls for tx_{s^-}'s inputs will return **true** in line 143.

iv. Line 147 (**false** is returned if the locking condition is not satisfied): Since tx^m's inputs are copied from tx_s and only the amounts are modified, then because $s^{'}_{m_k} = s_{m_k}$ and tx_s's inputs are valid in this regard, then so are tx^m's inputs.

Hence the call will not return (**false**) in line 147.

Note that the outputs spent by tx_s were spent in the same manner by tx^m and tx_s-. Also note that tx_s- shares tx_s's outputs and output IDs with one difference — the output spent by the tx_k and by tx_{dummy} — in $s^{'}_{m_{tx_s\text{-}}}$ its amount is $amount_k$ coins short. Hence the following conditions hold for $s^{'} = s^{'}_{m_{tx_s\text{-}}}$ and $s = s_{m_{tx_s}}$:

a) $s^{'} \neq \bot$

b) For all output ID $id \in s$:
  - $\textsc{state}(s^{'}, id) = \textsc{state}(s, id)$ and
  - $\varphi^{lock}_{s^{'},id} = \varphi^{lock}_{s,id}$ and
  - $\varphi^{spend}_{s^{'},id} = \varphi^{spend}_{s,id}$ and
  - If id is the ID of the output of the splitter tx_s that is spent by the keystone tx_k,

$$\textsc{remaining}(s^{'}, id) = \textsc{remaining}(s, id) - amount_k$$

where $amount_k$ is the total output amount of the keystone. Otherwise,

$$\textsc{remaining}(s^{'}, id) = \textsc{remaining}(s, id)$$

c) For all output ID $id \in s^{'} \setminus s$, $s^{'}[id]$ is the state of an output of tx^m.

3. $[m_l, \ldots, m_n]$. We prove by induction that the above conditions hold for all $(s^{'}, s) \in [(s^{'}_{m_{tx_s\text{-}}}, s_{m_{tx_s}}), (s^{'}_{m_l}, s_{m_l}), \ldots, (s^{'}_{m_n}, s_{m_n})]$. For the base of the induction, we've already shown the conditions hold for $(s^{'}, s) = (s^{'}_{m_{tx_s\text{-}}}, s_{m_{tx_s}})$.

For the induction step, note that $s \neq \bot$ for all $s \in [s_{m_{tx_s}}, s_{m_l}, \ldots, s_{m_n}]$.

Since the keystone tx_k has not yet been applied in s_{m_n} (nor has its output been released), the transactions published in $[m_l, \ldots, m_n]$ do not reference its outputs. This implies that tx^m's outputs are not referenced by the transactions published in $[m_l, \ldots, m_n]$. Hence they remain in **pending** for all $s^{'} \in [s^{'}_{m_l}, \ldots, s^{'}_{m_n}]$. **pending** outputs are effectively treated as missing by conditions, hence the existence of tx^m's output in $s^{'}_{m_l}, \ldots, s^{'}_{m_n}$ does not affect the evaluation of any condition.

Suppose the conditions hold at $(s^{'}_{m_{j-1}}, s_{m_{j-1}})$ — we prove they hold for $(s^{'}_{m_j}, s_{m_j})$, after applying m_j.

If $m_j = (\texttt{Tx-Published}, tx_j)$, the only way for VALIDATE-AND-APPLY-TX to fail with respect to $s^{'}_{m_{j-1}}$ is if an input of tx_j attempts to spend from the reduced-remaining-amount output more than it has left. However, in this case the remaining amount in s_{m_j} (*after* tx_j was applied) must be less than the input amount of the keystone tx_k, hence tx_k could not have been applied successfully (contradicting our assumption that the application of tx_k to s_b is successful).

Otherwise suppose $m_j = (\texttt{Txos-Released}, tx_j)$. $tx_j \neq tx^m$ since tx_k has not yet been published in $s_{m_{j-1}}$, hence the tx_j's outputs are in the same states in $s^{'}_{m_{j-1}}$ and in $s_{m_{j-1}}$.

Otherwise if $m_j = (\texttt{Tx-Removed}, tx_j)$, then tx_j wasn't strongly-valid with respect to $s_{m_{j-1}}$ (see line 163 of functionality 3.2) and one of the following must hold:

- VALIDATE-AND-APPLY-TX(tx_j) returned **false** with respect to $s_{m_{j-1}}$. Since the conditions hold at $(s^{'}_{m_{j-1}}, s_{m_{j-1}})$, the only relevant difference between the two states is the lower amount of the splitter output. Hence it is impossible that VALIDATE-AND-APPLY-TX$(tx_j) =$ **true** with respect to $s^{'}_{m_{j-1}}$.

- $\textsc{is-live}(tx_j)$ returned **false**. Since $\textsc{is-live}$ is a function of the transaction (not of the ledger state), $\textsc{is-live}$ will still return **false** in $s^{'}_{m_j}$.
- tx_j spent an output in state **locked/spent** with respect to $s_{m_{j-1}}$. By the induction hypothesis, $\textsc{state}(s_{m_j}, txo) = \textsc{state}(s^{'}_{m_j}, txo)$, so the corresponding output is in the same state in $\textsc{state}(s^{'}_{m_j}, txo)$.

Hence the conditions holds for $(s^{'}_{m_n}, s_{m_n})$, so $s^{'}_{m_n}$ identical s_{m_n} but for the $amount_k$-reduced amount of the output spent by tx_k, and the "extra" IDs added by tx^m.

4. $m_{tx_{dummy}}$ — tx_{dummy} is constructed in line 129. To show that $\textsc{validate-and-apply-tx}(tx_{s}\text{-})$ (line 70) with respect to $s^{'}_{m_n}$ returns **true** in line 81, we show that it doesn't return (**false**) in lines 74 and 78.
   a) Line 74 (**false** is returned if incoming coins < outgoing coins):

   tx_{dummy} has no outputs, so the "incoming coins < outgoing coins" condition doesn't hold, hence tx_{dummy} is valid in this regard.

   b) Line 78 (**false** is returned if any of the inputs is invalid):

   Recall that $s_{m_n} = s_{m+q}$ and therefore $s_{m_n} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$. Hence $s^{'}_{m_n} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ but for the $amount_k$-reduced amount of txo_s the output spent by tx_k, and the "extra" IDs added by tx^m. Also recall that tx_k is valid with respect to s_b.

   tx_{dummy} is initialized with tx_k's only input and no outputs.

   The amount of $txo_{s\text{-}}$, the output spent by tx_{dummy}, is later reduced and is lower in $s^{'}_{m_n}$ than that of txo_s, the output spent by tx_k, in s_{m_n}. However, the amount of tx_{dummy}'s input is also reduced by $amount_k$.

   $txo_{s\text{-}}$ and txo_s share the same conditions since they're base, so they are satisfied with tx_{dummy} and tx_k data respectively.

   Hence, $s_{m+q'} \neq \bot$. Since tx^m and tx_k share output IDs and amounts $s_{m+q'} \overset{state}{\underset{txo}{\leftrightsquigarrow}} s^{'}_b$.
   Finally, since tx^m was output by $\textsc{decomp}$, $s_{m+q'} \overset{state}{\underset{txo}{\rightsquigarrow}} s^{'}_b$.

□

Define the following simulation events. Each event is specified by a line number in $\mathcal{S}/\mathcal{S}_{\mathcal{F}^{base}_{ledger}}/\mathcal{S}_{\mathcal{P}_h}/\mathcal{F}^{meta}_{ledger}$. The event triggers after the line was executed. For each event we specify an ordered list of code segments that we later prove is run sequentially prior to the event:

1. Line 44 of $\mathcal{S}_{\mathcal{P}_h}$'s $\textsc{tx-published-by-}\mathcal{F}_{ledger}$.

   Code segments:
   - $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$'s $\textsc{process-tx}$ (line 20): up to line 31 — tx was applied to the state of $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$,
   - $\mathcal{S}_{\mathcal{P}_h}$'s $\textsc{tx-published-by-}\mathcal{F}_{ledger}$ (line 23): up to line 44 — tx was applied to $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue*.

2. Line 55 of $\mathcal{S}_{\mathcal{P}_h}$'s $\textsc{txos-released-by-}\mathcal{F}_{ledger}$.

   Code segments:
   - $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$'s $\textsc{release-txos}$ (line 36): up to line 39 or line 42 — tx's outputs were released in $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$,
   - $\mathcal{S}_{\mathcal{P}_h}$'s $\textsc{txos-released-by-}\mathcal{F}_{ledger}$ (line 52): up to line 55 — a `Txos-Released` for tx message was added to $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue*.

3. Line 34 of $\mathcal{S}$'s CHECK-$\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$-FRAGMENT-VALIDITY.

   Code segments:

   - $\mathcal{S}$'s CHECK-$\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$-FRAGMENT-VALIDITY (line 26): line 34 — $\mathcal{S}$ has added a `Tx-Removed` message for *tx* to $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue*.

4. Line 17 of $\mathcal{S}$'s PROCESS-$\mathcal{S}_{\mathcal{P}_h}$-MESSAGE.

   Code segments:

   - $\mathcal{S}_{\mathcal{P}_h}$'s FLUSH-MSG-QUEUE (line 84): line 96 — (`Tx-Published`, *tx*) was popped from $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue*,
   - $\mathcal{S}$'s PROCESS-$\mathcal{S}_{\mathcal{P}_h}$-MESSAGE (line 14): up to line 17 — $\mathcal{S}$ called $\mathcal{F}^{meta}_{ledger}$'s PROCESS-TX with *tx*,
   - $\mathcal{F}^{meta}_{ledger}$'s PROCESS-TX (line 20).

5. Line 19 of $\mathcal{S}$'s PROCESS-$\mathcal{S}_{\mathcal{P}_h}$-MESSAGE.

   Code segments:

   - $\mathcal{S}_{\mathcal{P}_h}$'s FLUSH-MSG-QUEUE (line 84): line 96 — (`Txos-Released`, *tx*) was popped from $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue*,
   - $\mathcal{S}$'s PROCESS-$\mathcal{S}_{\mathcal{P}_h}$-MESSAGE (line 14): up to line 19 — $\mathcal{S}$ called $\mathcal{F}^{meta}_{ledger}$'s RELEASE-TXOS with *tx*,
   - $\mathcal{F}^{meta}_{ledger}$'s RELEASE-TXOS (line 36).

6. Line 24 of $\mathcal{S}$'s $\mathcal{S}_{\mathcal{P}_h}$-TX-REMOVED-POPPED.

   Code segments:

   - $\mathcal{S}_{\mathcal{P}_h}$'s FLUSH-MSG-QUEUE (line 84): line 96 — (`Tx-Removed`, *tx*) was popped from $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue*,
   - $\mathcal{S}$'s $\mathcal{S}_{\mathcal{P}_h}$-TX-REMOVED-POPPED (line 22): up to line 24 — $\mathcal{S}$ called $\mathcal{F}^{meta}_{ledger}$'s REMOVE-TX with *tx*,
   - $\mathcal{F}^{meta}_{ledger}$'s REMOVE-TX (line 45).

7. Line 44 of $\mathcal{F}^{meta}_{ledger}$'s TX-PUBLISHED-BY-$\mathcal{F}_{ledger}$.

   Code segments:

   - $\mathcal{F}^{meta}_{ledger}$'s TICK (line 57) — PROCESS-TX was called with *tx*,
   - $\mathcal{F}^{meta}_{ledger}$'s PROCESS-TX (line 20): up to line 31 — *tx* was applied to the state of $\mathcal{F}^{meta}_{ledger}$.

8. Line 55 of $\mathcal{F}^{meta}_{ledger}$'s TXOS-RELEASED-BY-$\mathcal{F}_{ledger}$.

   Code segments:

   - $\mathcal{F}^{meta}_{ledger}$'s TICK (line 65) — RELEASE-TXOS was called with *tx*,
   - $\mathcal{F}^{meta}_{ledger}$'s RELEASE-TXOS (line 36): up to line 39 or line 42 — *tx*'s outputs were released in $\mathcal{F}^{meta}_{ledger}$.

Claim 4.3.22 (Consecutive Execution of Event Code Segments)**.** *For each event of event types 1 to 6 it holds for the code segments of the event that:*

- *they are run consecutively prior to the event, and*
- *they are disjoint from code segments of other events.*

Proof. By observation of the code of functionality 3.2, protocol 4.1, and algorithm 4.1, the code segment are disjoint, but this is true because each event handles a different process during simulation:

- Event types 1 and 2 handle state changes in $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ and the $\mathcal{S}_{\mathcal{P}_h}$ changes they cause, processing transactions and releasing of outputs, respectively.
- Event type 3 deals with the addition of `Tx-Removed` messages to $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue* when a fragment has become not-strongly-valid.
- Event types 4 to 6 handle with popping of different messages from $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue* (`Tx-Published`,`Txos-Released`,`Tx-Removed`), and their effect on the state of $\mathcal{F}^{meta}_{ledger}$.
- Event types 7 and 8 deal with processes internally-triggered in $\mathcal{F}^{meta}_{ledger}$, processing of transactions and releasing of outputs respectively.

We prove by analysis of each event type that the code segments happen consecutively:

- Event types 1 and 2:

  When $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ outputs a message addressed to honest parties, $\mathcal{S}$ pauses the $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ simulation and immediately runs the $\mathcal{S}_{\mathcal{P}_h}$ code that handles the message:

  – When a `Tx-Published` message is output from $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ (line 31), $\mathcal{S}_{\mathcal{P}_h}$'s TX-PUBLISHED-BY-$\mathcal{F}_{ledger}$ (line 23) is called (where $\mathcal{S}_{\mathcal{P}_h}$ applies *tx* to *MsgQueue*),

  – When a `Txos-Released` message is output from $\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$ (line 39 or line 42), $\mathcal{S}_{\mathcal{P}_h}$'s TXOS-RELEASED-BY-$\mathcal{F}_{ledger}$ (line 52) is called (where $\mathcal{S}_{\mathcal{P}_h}$ adds a `Txos-Released` message for *tx* to *MsgQueue*).

  Hence the respective second code segments of event types 1 and 2 **immediately** follow the respective first code segments during simulation.

  By observation of protocol 4.1:

  – `Tx-Published` messages are only added to *MsgQueue* in TX-PUBLISHED-BY-$\mathcal{F}_{ledger}$. Specifically, in SIMPLE-KEYSTONE-PUBLISHED (line 115) and NON-SIMPLE-KEYSTONE-PUBLISHED (line 120).

  – `Txos-Released` messages are only added to *MsgQueue* in TXOS-RELEASED-BY-$\mathcal{F}_{ledger}$.

  Hence the respective second code segments of event types 1 and 2 **only** follow the respective first code segments during simulation.

- Event type 3: CHECK-$\mathcal{S}_{\mathcal{F}^{base}_{ledger}}$-FRAGMENT-VALIDITY is called after $\mathcal{S}_{\mathcal{P}_h}$ runs either:

  – line 44 of TX-PUBLISHED-BY-$\mathcal{F}_{ledger}$ or

  – line 55 of TXOS-RELEASED-BY-$\mathcal{F}_{ledger}$,

  that is, **immediately** after event types 1 and 2.

- Event types 4 to 6:

  When $\mathcal{S}_{\mathcal{P}_h}$ outputs a message or pops a (`Tx-Removed`, *tx*) message from *MsgQueue* in line 94 of FLUSH-MSG-QUEUE, $\mathcal{S}$ pauses the $\mathcal{S}_{\mathcal{P}_h}$ simulation and immediately runs the $\mathcal{S}$ code that handles the message:

  – when $\mathcal{S}_{\mathcal{P}_h}$ outputs a `Tx-Published` message (line 96), $\mathcal{F}^{meta}_{ledger}$'s PROCESS-TX is called (line 17),

  – when $\mathcal{S}_{\mathcal{P}_h}$ outputs a `Txos-Released` message (line 96), $\mathcal{F}^{meta}_{ledger}$'s RELEASE-TXOS is called (line 19),

- when $\mathcal{S}_{\mathcal{P}_h}$ outputs a `Tx-Removed` message (line 94), $\mathcal{F}_{ledger}^{meta}$'s REMOVE-TX is called (line 24).

Hence the respective second code segments of event types 4 to 6 **immediately** follow the respective first code segments during simulation.

By observation of algorithm 4.1:

- Line 17 is the only place $\mathcal{S}$ calls $\mathcal{F}_{ledger}^{meta}$'s PROCESS-TX.
- Line 19 is the only place $\mathcal{S}$ calls $\mathcal{F}_{ledger}^{meta}$'s RELEASE-TXOS.
- Line 24 is the only place $\mathcal{S}$ calls $\mathcal{F}_{ledger}^{meta}$'s REMOVE-TX.

Hence the respective last two code segments of event types 1 to 8 **only** follow the respective first code segments during simulation.

- Event types 7 and 8: The code segments are internal to $\mathcal{F}_{ledger}^{meta}$, hence they happen consecutively.

□

We say a state change is attributed to an event e if it happens due to the execution of e's code segments.

Claim 4.3.23 (State Change Attribution Coverage)**.** *All changes to the states of $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$, $\mathcal{S}_{\mathcal{P}_h}$ and $\mathcal{F}_{ledger}^{meta}$ during simulation can be attributed to an event of event types 1 to 6.*

Proof. By observation of functionality 3.2, the state of $\mathcal{F}_{ledger}$ can only be modified when processing a transaction in PROCESS-TX (line 20) and when releasing the outputs of a transaction in RELEASE-TXOS (line 36). The two procedures can be called adversarially, or otherwise internally from TICK (line 51). The four combinations are covered in event types 1, 2, 7 and 8.

By observation of protocol 4.1, Π_{ledger}'s *MsgQueue* contains `Tx-Published` and `Txos-Released` messages — their additions to *MsgQueue* are covered in event types 1 to 8 respectively and the poppings in event types 4 and 5 respectively.

By observation of algorithm 4.1, $\mathcal{S}$ also adds `Tx-Removed` messages to $\mathcal{S}_{\mathcal{P}_h}$'s *MsgQueue*. Their additions are covered in event type 3 and the poppings in event type 6. □

Lemma 4.3.24 ($s_{m+q} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$ is preserved by event types 1 to 6)**.** *Denote the states of $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$ and $\mathcal{F}_{ledger}^{meta}$ as s_b and s_m respectively and $\mathcal{S}_{\mathcal{P}_h}$'s MsgQueue as q. Define s_{m+q} = APPLY-QUEUE(q, s_m) and suppose $s_{m+q} \neq \perp$ and $s_{m+q} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b$.*

Let e be an event of event types 1 to 6, and denote s_b, s_m and q after e as $s_b^{'}$, $s_m^{'}$ and $q^{'}$, and define $s_{m^{'}+q^{'}}$ = APPLY-QUEUE$(q^{'}, s_m^{'})$. Then $s_{m^{'}+q^{'}} \neq \perp$ and $s_{m^{'}+q^{'}} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b^{'}$.

Proof. We do a case analysis by the type of the event.

- **Event type 1:** (`Tx-Published`, tx) was published by $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$, hence tx was applied to s_b. $\mathcal{S}_{\mathcal{P}_h}$'s TX-PUBLISHED-BY-$\mathcal{F}_{ledger}$ (line 23 of protocol 4.1) is called with tx.

  If SIMPLE-KEYSTONE-PUBLISHED is called in line 39, then lemma 4.3.20 applies. Otherwise, NON-SIMPLE-KEYSTONE-PUBLISHED is called in line 42, then lemma 4.3.21 applies. Regardless, $s_{m^{'}+q^{'}} \neq \perp$ and $s_{m^{'}+q^{'}} \overset{state}{\underset{txo}{\rightsquigarrow}} s_b^{'}$.

- **Event type 2:** First note that $\mathcal{S}_{\mathcal{P}_h}$ pushes (`Txos-Released`, tx^m) to *MsgQueue*, where tx^m is the meta version of tx^m— the one that shares it's outputs.

Since $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$ releases *tx*'s outputs they must be **pending** or **pending-locked** in s_b. Due to consistency, if the outputs are **pending-locked** in s_b, they must be **pending-locked** in s_m too, hence releasing will succeed s_m and will have the same effect as in s_b. If the outputs are **pending** in s_b, they can be either **pending** or **pending-locked** in s_m, so releasing will succeed in s_m. The outputs will be **unspent** in $s_b^{'}$ and either **unspent** or **locked** in $s_m^{'}$ so consistency is maintained.

- **Event type 3:** $\mathcal{S}$ pushes (`Tx-Removed`, *tx*) when $\mathcal{S}_{\mathcal{P}_h}$ reaches line 73 of TICK-BY-$\mathcal{F}_{ledger}$ (line 68).

  This implies a transaction tx_f in COMP(tx) was sent to $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$ but was not published within δ_{base} tick. By condition 3 of definition 4.2.18 tx_f cannot appear after the splitter (since this would mean the environment found a blocking sequence for COMP(tx) that includes the splitter). Thus, by condition 2 *tx* is removable in s_m.

- **Event types 4 to 6 ($\mathcal{F}_{ledger}^{meta}$ calls):** By the lemma hypothesis, $s_{m+q} \neq \bot$, hence:

$$\begin{aligned}\bot \neq s_{m+q} &= \text{APPLY-QUEUE}(q, s_m) && //msg \leftarrow q.\text{POP}()\\ &= \text{APPLY-QUEUE}(q^{'}, \text{APPLY-MESSAGE}(msg, s_m))\\ &= \text{APPLY-QUEUE}(q^{'}, s_m^{'})\\ &= s_{m'+q'}\end{aligned}$$

  The APPLY-MESSAGE call succeeds because $s_{m+q} \neq \bot$. So $s_{m'+q'} \neq \bot$. Also, since this is a Pop event, nothing was changed in the base-ledger, hence:

$$s_{m'+q'} = s_{m+q} \underset{txo}{\overset{state}{\rightsquigarrow}} s_b = s_b^{'}$$

□

Lemma 4.3.25 ($\underset{txo}{\overset{state}{\rightsquigarrow}}$ is always preserved)**.** *Denote s_b and s_m the states of $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$ and $\mathcal{F}_{ledger}^{meta}$ respectively and q $\mathcal{S}_{\mathcal{P}_h}$'s MsgQueue. Then it holds that:*

- *no event of event types 7 and 8 ever occurs, and*
- *after every event of event types 1 to 6 it holds that $s_{m+q} \neq \bot$ and $s_{m+q} \underset{txo}{\overset{state}{\rightsquigarrow}} s_b$, where $s_{m+q} =$ APPLY-QUEUE(q, s_m).*

Proof. The proof is by induction on the index of the event (event types 1 to 6). Suppose the induction hypothesis holds for (up to and including) the i^{th} event.

By claim 4.3.22 the code segments leading to an event are executed consecutively and are not interrupted.

By claim 4.3.23 the code segments cover all changes to the state, hence no state changes can be made after one event and before the first segment of the following event.

To show that no event of event types 7 and 8 ever occurs, suppose in contradiction that the event is of one of these event types. Hence the transaction *tx* that was processed or whose outputs were released, was live (IS-LIVE$^{meta}(tx)$ = **true**) and has been in $\mathcal{F}_{ledger}^{meta}$'s *mempool* more than δ_{meta} ticks. Thus it was added to $\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$'s *mempool* more than δ_{meta} ticks ago. By claim 4.3.10 within $(2 \cdot s \cdot \delta_{base}) \leq \delta_{meta}$ $\mathcal{S}$ has called either:

- $\mathcal{F}_{ledger}^{meta}$'s PROCESS-TX and then RELEASE-TXOS with *tx'*, where *tx'* is identical to *tx* up to input amount changes or
- $\mathcal{F}_{ledger}^{meta}$'s REMOVE-TX with *tx*.

If the first case holds, due to the induction hypothesis, tx' was successfully applied to $\mathcal{F}_{ledger}^{meta}$'s state and its outputs were released. Since $\mathcal{F}_{ledger}$'s PROCESS-TX strips input amounts and locking inputs, tx is successfully removed from its *mempool*.

If the second case holds, REMOVE-TX(tx) was called and by the induction hypothesis was successful, thereby removing tx from *mempool*.

If the event is of event type 8 then the transaction tx whose outputs were released has been in

If the event is of event types 1 to 6 then by lemma 4.3.24 $s_{m+q} \neq \perp$ and $s_{m+q} \overset{state}{\underset{txo}{\leadsto}} s_b$ after the $(i+1)^{th}$ event. □

Corollary 4.3.26 ($\mathcal{S}$ calls to $\mathcal{F}_{ledger}^{meta}$'s adversarial APIs always succeed)**.** *Every $\mathcal{S}$ call to $\mathcal{F}_{ledger}^{meta}$'s* PROCESS-TX *(line 20 of functionality 3.2 (part 2)) and* RELEASE-TXOS *(line 36) succeeds and the two procedures are never called internally from* TICK *(line 51).*

Proof. Every PROCESS-TX call is in the code segments of event type 4 and every RELEASE-TXOS call is in the code segments of event type 5. By lemma 4.3.25 these events always succeed and no event of event types 7 and 8 ever occurs. □

Protocol 4.1 (part 1): $\Pi_{ledger}^{\mathcal{L}_{meta},\mathcal{L}_{base},\text{COMP},\text{DECOMP},\text{IS-LIVE}}$
(API)

The party maintains:

- *TxStepQueues*— maps transactions to their steps
- *LockingFragmentTxos*— maps locking fragment outputs to explicitly-meta outputs
- *FragmentTxos*— all fragment outputs
- *MsgQueue*— a queue of messages to be output

```
procedure GET-TX-IDS($\widehat{tx}$)
    $[T_1^{tx^m}, ..., T_n^{tx^m}] \leftarrow$ COMP$(out_b, out_m, \widehat{tx})$
    $tx_k \leftarrow$ the keystone  // $(tx_k, aux_k) \in \bigcup_{i=1}^{n} T_i^{tx^m}$, see definition 4.2.9
    $tx_s \leftarrow$ the splitter  // $(tx_s, aux_s) \in \bigcup_{i=1}^{n} T_i^{tx^m}$, see definition 4.2.10
    $((id_k, txo\_ids_k), \widehat{tx}_k) \xleftarrow{parse} tx_k$
    $((id_s, txo\_ids_s), \widehat{tx}_s) \xleftarrow{parse} tx_s$
    $aux \leftarrow$ entire output of COMP // $[T_1^{tx^m}, ..., T_n^{tx^m}]$
    return $(((id_s, txo\_ids_k), \widehat{tx}), aux)$
end procedure

procedure SEND-TX($tx, aux$)
    if
        IS-LIVE$^{meta}(tx)$ and
        $aux$ is a valid and admissible COMP output for $tx$
    then
        $[T_1^{tx^m}, ..., T_n^{tx^m}] \xleftarrow{parse} aux$  // $aux$ is output of COMP
        Copy signatures from $tx$ to the splitter's inputs
        $pending\_steps_{tx} \leftarrow [T_1^{tx^m}, ..., T_n^{tx^m}]$
        $sent\_steps_{tx} \leftarrow$ empty list
        $TxStepQueues[tx] \leftarrow (sent\_steps_{tx}, pending\_steps_{tx})$
        SEND-NEXT-STEP($tx$)
    end if
end procedure
```

...Continued in protocol 4.1 (part 2)...

Protocol 4.1 (part 2): $\Pi_{ledger}^{\mathcal{L}_{meta},\mathcal{L}_{base},\textsc{comp},\textsc{decomp},\textsc{is-live}}$
(Ledger Event)

...Continued from protocol 4.1 (part 1)...

```
23: procedure TX-PUBLISHED-BY-$\mathcal{F}_{ledger}(tx)$
24:     // Called when $\mathcal{F}_{ledger}$ outputs $(\texttt{Tx-Published}, tx)$
25:     $(tx^m_{\textsc{decomp}}, fragments) \leftarrow \textsc{decomp}(out_b, out_m, tx)$
26:     $tx_k \leftarrow$ the keystone  // $tx_k = tx$, see definition 4.2.9
27:     $tx_s \leftarrow$ the splitter  // $tx_s \in fragments$, see definition 4.2.10
28:     // Existence of $tx_k$ and $tx_s$ is ensured by condition 2 in definition 4.2.13
29:     if
30:         $tx_k$ is a non-simple keystone with respect to $tx^m_{\textsc{decomp}}$ and
31:         // i.e., $tx_k \neq tx_s$, see definition 4.2.11
32:         $tx_s \notin MsgQueue$
33:     then
34:         $tx^m \leftarrow tx_k$
35:     else
36:         $tx^m \leftarrow tx^m_{\textsc{decomp}}$
37:     end if
38:     if $tx_k$ is a simple keystone with respect to $tx^m$ then
39:         SIMPLE-KEYSTONE-PUBLISHED$(tx_k, tx^m)$
40:     else
41:         $tx'_s \leftarrow tx_s$'s version in $MsgQueue$
42:         NON-SIMPLE-KEYSTONE-PUBLISHED$(tx_k, tx'_s, tx^m)$
43:     end if
44:     ADD-LOCKING-INPUTS()  // End of $MsgQueue$ modifications
45:
46:     for all meta output $txo^m \in tx^m$ do
47:         Add all fragment outputs in $fragments$ to $FragmentTxos$
48:         Map all locking fragment outputs in $fragments$ to $txo^m$ in $LockingFragmentTxos$
49:     end for
50:     FLUSH-MSG-QUEUE()
51: end procedure
```

...Continued in protocol 4.1 (part 3)...

Protocol 4.1 (part 3): $\Pi_{ledger}^{\mathcal{L}_{meta},\mathcal{L}_{base},\text{COMP},\text{DECOMP},\text{IS-LIVE}}$
(Ledger Event)

...Continued from protocol 4.1 (part 2)...

```
52: procedure TXOS-RELEASED-BY-$\mathcal{F}_{ledger}$($tx$)
53:     // Called when $\mathcal{F}_{ledger}$ outputs (Txos-Released, $tx$)
54:     $tx^m \leftarrow$ the transaction that was already published and shares $tx$'s outputs
55:     $MsgQueue$.PUSH((Txos-Released, $tx^m$))  // End of $MsgQueue$ modifications
56:
57:     for all $tx \in TxStepQueues$ do
58:         $(sent\_steps_{tx}, next\_steps_{tx}) \xleftarrow{parse} TxStepQueues[tx]$
59:         if all of $sent\_steps_{tx}$'s outputs were released by $\mathcal{F}_{ledger}$ then
60:             if $next\_steps_{tx}$ contains more steps then
61:                 SEND-NEXT-STEP($tx$)
62:             else
63:                 Remove $tx$ from $TxStepQueues$
64:             end if
65:         end if
66:     end for
67: end procedure

68: procedure TICK-BY-$\mathcal{F}_{ledger}$()
69:     // Called when $\mathcal{F}_{ledger}$ outputs (Tick)
70:     for all $tx \in TxStepQueues$ do
71:         $T_{last\_sent} \leftarrow$ last sent step in $TxStepQueues[tx]$
72:         if $T_{last\_sent}$ was sent to $\mathcal{F}_{ledger}$ more than $\delta_{base}$ ticks ago then
73:             Remove $tx$ from $TxStepQueues$
74:         end if
75:     end for
76: end procedure
```

...Continued in protocol 4.1 (part 4)...

Protocol 4.1 (part 4): $\Pi_{ledger}^{\mathcal{L}_{meta},\mathcal{L}_{base},\textsc{comp},\textsc{decomp},\textsc{is-live}}$ (Internals)

...Continued from protocol 4.1 (part 3)...

```
77: procedure SEND-NEXT-STEP($tx$)
78:     $([T_1^{tx}, ..., T_{m-1}^{tx}], [T_m^{tx}, ..., T_n^{tx}]) \xleftarrow{parse} TxStepQueues[tx]$
79:     for all $(tx_f, aux_f) \in T_m^{tx}$ do
80:         $\mathcal{F}_{ledger}$.SEND-TX($tx_f, aux_f$)
81:     end for
82:     $TxStepQueues[tx] \leftarrow ([T_1^{tx}, ..., T_m^{tx}], [T_{m+1}^{tx}, ..., T_n^{tx}])$
83: end procedure

84: procedure FLUSH-MSG-QUEUE()
85:     while $MsgQueue$ is not empty do
86:         if
87:             $MsgQueue$.PEEK() is (Tx-Published, $tx$) and
88:             $tx$ is marked as fragment and
89:             $tx$'s outputs are not in $FragmentTxos$ and
90:             $tx$ has been published by $\mathcal{F}_{ledger}$ less than $s \cdot \delta_{base}$ ticks ago
91:         then
92:             break
93:         end if
94:         $msg \leftarrow MsgQueue$.POP()
95:         if $msg$ is Tx-Published or Txos-Released then
96:             Output $msg$
97:         end if
98:     end while
99: end procedure
```

...Continued in protocol 4.1 (part 5)...

Protocol 4.1 (part 5): $\Pi_{ledger}^{\mathcal{L}_{meta},\mathcal{L}_{base},\text{COMP},\text{DECOMP},\text{IS-LIVE}}$
(Decompilation)

...Continued from protocol 4.1 (part 4)...

```
100: procedure ADD-LOCKING-INPUTS()
101:     for all $m \in MsgQueue$ if $m = (\texttt{Tx-Published}, tx)$ do
102:         $inputs_{temp} \xleftarrow{fresh\ copy} tx$'s inputs
103:         for all $txi \in inputs_{temp}$ do
104:             if $txi$ references a fragment output $txo_f$ in $LockingFragmentTxos$ then
105:                 $(id_{txo_f}, amount, data^{lock}, data^{spend}) \xleftarrow{parse} txi$
106:                 $txo^m \leftarrow LockingFragmentTxos[txo_f]$
107:                 $txi_{lock} \leftarrow (id_{txo^m}, 0, data^{lock}, \perp)$
108:                 if $tx$ doesn't already have an input identical to $txi_{lock}$ then
109:                     Append $txi_{lock}$ to $tx$'s inputs in $m$ in $MsgQueue$
110:                 end if
111:             end if
112:         end for
113:     end for
114: end procedure

115: procedure SIMPLE-KEYSTONE-PUBLISHED($tx_k$, $tx^m$)
116:     $((id_k, txo\_ids_k), (inputs_k, outputs_k)) \xleftarrow{parse} tx_k$
117:     $((id_m, txo\_ids_m), (inputs_m, outputs_m)) \xleftarrow{parse} tx^m$
118:     $MsgQueue.\text{PUSH}((\texttt{Tx-Published}, ((id_m, txo\_ids_k), (inputs_m, outputs_k))))$
119: end procedure
```

...Continued in protocol 4.1 (part 6)...

Protocol 4.1 (part 6): $\Pi_{ledger}^{\mathcal{L}_{meta},\mathcal{L}_{base},\text{COMP},\text{DECOMP},\text{IS-LIVE}}$
(Decompilation)

...Continued from protocol 4.1 (part 5)...

```
120: procedure NON-SIMPLE-KEYSTONE-PUBLISHED($tx_k$, $tx_s$, $tx^m$)
121:     $((id_k, txo\_ids_k), ([txi_k], outputs_k)) \xleftarrow{parse} tx_k$
122:     $((id_s, txo\_ids_s), (inputs_s, outputs_s)) \xleftarrow{parse} tx_s$
123:     $((id_m, txo\_ids_m), (inputs_m, outputs_m)) \xleftarrow{parse} tx^m$  // $txo\_ids_m = txo\_ids_k$
124:
125:     // Initializations, fresh copies:
126:     $tx^m_* \leftarrow ((id_s, txo\_ids_m), (inputs_s, outputs_m))$  // $inputs_m$ are discarded
127:     $id_{s\text{-}} \leftarrow$ new ID in the "meta namespace"
128:     $tx_{s\text{-}} \leftarrow ((id_{s\text{-}}, txo\_ids_s), (inputs_s, outputs_s))$
129:     $tx_{dummy} \leftarrow ((id_k, [\,]), ([txi_k], [\,]))$  // no outputs
130:
131:     $amount_k \leftarrow$ the sum of $outputs_k$'s amounts
132:     $txo_s \leftarrow tx_s$'s output spent by $txi_k$
133:     $txo_{s\text{-}} \leftarrow tx_{s\text{-}}$'s output that corresponds to $txo_s$
134:
135:     Subtract $amount_k$ from $txo_{s\text{-}}$'s output amount
136:     Subtract $amount_k$ from the amount of $tx_{dummy}$'s input
137:     $i \leftarrow 1$
138:     while $i \leq tx_{s\text{-}}$'s input count do
139:         $txi_{s\text{-},i} \leftarrow tx_{s\text{-}}$'s $i^{th}$ input
140:         $amount_{k,i} \leftarrow min(amount_{txi_{s\text{-},i}}, amount_k)$
141:         Subtract $amount_{k,i}$ from $txi_{s\text{-},i}$'s input amount
142:         Set the input amount of $tx^m_*$'s $i^{th}$ input to $amount_{k,i}$
143:         $amount_k \leftarrow amount_k - amount_{k,i}$
144:         $i \leftarrow i + 1$
145:     end while
146:
147:     Insert (Tx-Published, $tx^m_*$) in $MsgQueue$ before (Tx-Published, $tx_s$)
148:     Replace (Tx-Published, $tx_s$) with (Tx-Published, $tx_{s\text{-}}$) in $MsgQueue$
149:     $MsgQueue$.PUSH((Tx-Published, $tx_{dummy}$))
150: end procedure
```

Algorithm 4.1 (part 2): $\mathcal{S}_{ledger}^{\mathcal{L}_{meta},\text{COMP},\text{DECOMP}}$

...Continued from algorithm 4.1 (part 1)...

```
14: procedure PROCESS-$\mathcal{S}_{\mathcal{P}_h}$-MESSAGE($m$)
15:     // Called when $\mathcal{S}_{\mathcal{P}_h}$ outputs a message $m$
16:     if $m =$ (Tx-Published, $tx$) then
17:         $\mathcal{F}_{ledger}^{meta}$.PROCESS-TX($tx$)
18:     else if $m =$ (Txos-Released, $tx$) then
19:         $\mathcal{F}_{ledger}^{meta}$.RELEASE-TXOS($tx$)
20:     end if
21: end procedure

22: procedure $\mathcal{S}_{\mathcal{P}_h}$-TX-REMOVED-POPPED($tx$)
23:     // Called when $\mathcal{S}_{\mathcal{P}_h}$ pops a (Tx-Removed, $tx$) message from $MsgQueue$ in line 94 of
    FLUSH-MSG-QUEUE.
24:     $\mathcal{F}_{ledger}^{meta}$.REMOVE-TX($tx$)
25: end procedure

26: procedure CHECK-$\mathcal{S}_{\mathcal{F}_{ledger}^{base}}$-FRAGMENT-VALIDITY()
27:     // Called after $\mathcal{S}_{\mathcal{P}_h}$ runs either:
        • line 44 of TX-PUBLISHED-BY-$\mathcal{F}_{ledger}$ or
        • line 55 of TXOS-RELEASED-BY-$\mathcal{F}_{ledger}$,
28:     Pause the $\mathcal{S}_{\mathcal{P}_h}$ simulation, store its execution state
29:     for all $tx \in \mathcal{S}_{\mathcal{F}_{ledger}^{base}}$'s $mempool$ do
30:         if $tx$ was sent by $\mathcal{S}_{\mathcal{P}_h}$ and $\neg$IS-STRONGLY-VALID($tx$) then
31:             Let $tx_{origin}$ be the transaction that
32:                 $\mathcal{S}_{\mathcal{P}_h}$ was asked by $\mathcal{E}$ to send originally and
33:                 triggered the sending of $tx$.
34:             $\mathcal{S}_{\mathcal{P}_h}$.$MsgQueue$.PUSH((Tx-Removed, $tx_{origin}$))
35:         end if
36:     end for
37:     Restore the $\mathcal{S}_{\mathcal{P}_h}$ execution state, continue its simulation
38: end procedure
```

Algorithm 4.2: APPLY-QUEUE

```
procedure APPLY-QUEUE($q$, $s$)
    $s' \xleftarrow{fresh\ copy} s$; $q' \xleftarrow{fresh\ copy} q$
    while $q'$ is not empty do
        $msg \leftarrow q'$.POP()
        if $s' \leftarrow$ APPLY-MESSAGE($msg, s'$) $= \perp$ then
            return $\perp$
        end if
    end while
    return $s'$
end procedure

procedure APPLY-MESSAGE(msg, $s$)
    $s' \xleftarrow{fresh\ copy} s$
    if $msg$ is (Tx-Published, $tx$) then
        Call $\mathcal{F}_{ledger}$.PROCESS-TX($tx$) with respect to $s'$ (see functionality 3.2, line 20)
        if the internal call to VALIDATE-AND-APPLY-TX returned false then
            return $\perp$
        end if
    else if $msg$ is (Txos-Released, $tx$) then
        Call $\mathcal{F}_{ledger}$.RELEASE-TXOS($tx$) with respect to $s'$ (see functionality 3.2, line 36)
        if $tx$'s outputs were not released then
            return $\perp$
        end if
    else // $msg$ is (Tx-Removed, $tx$)
        Call $\mathcal{F}_{ledger}$.REMOVE-TX($tx$) with respect to $s'$ (see functionality 3.2, line 45)
        if $tx$ was not removed from $s'$'s mempool in line 48 then
            return $\perp$
        end if
    end if
    return $s'$
end procedure
```

5 A Meta-Ledger With Circuit Support

In this chapter we describe the protocols for COMP and DECOMP—the remaining pieces that, together with our generic ledger protocol complete the construction of a meta-ledger with support for boolean circuit spending conditions.

5.1 Circuit Compiler

The compiler for circuit transactions appears in protocol 5.1. This compiler uses the construction explained in chapter 2. For brevity, we don't repeat the details of the compilation process in pseudocode. However, it is helpful to make the following points explicit:

- Our compiler does not use the current base-ledger and meta-ledger states.
- When we say "Construct a transaction" in our pseudocode, this means generating a transaction tuple that can be sent as input to the base-ledger's SEND-TX method. In particular, it includes querying the base-ledger for the transaction's ID and auxiliary data (using the ledger's GET-TX-IDS method).
- All the fragments generated by the compiler are also signed by the compiler—except for the splitter transaction, which is returned unsigned. (The splitter cannot be signed by the compiler, since these signatures are verified by the outputs it spends, with respect to the verification keys specified in these outputs—unlike internal fragments, whose outputs are generated by the compiler, including their signature key pairs). To compute the splitter signatures, the generic protocol converts the honest party's signature on the meta-transaction to a signature on the splitter base transaction.
- Fragment transactions are specially marked in a way that is allowed in the base-ledger but not in the meta-ledger (c.f. section 3.5.5).

5.1.1 Validating Compilation

The output of the compiler is easily identifiable, using the fragment marks and transaction graph. Given a potential keystone transaction tx_k and current base-ledger state, the following algorithm will verify if it could indeed have been generated by the compiler (and if so, reconstruct the input to the compiler).

Case 1: If tx_k satisfies all of the following:

- tx_k is marked as a keystone
- tx_k has a single input that spends a fragment-marked transaction tx_s
- The other outputs of tx_s are all spent by fragment-marked transactions that match the format of input-bit transactions or circuit-gate transactions (cf. section 2.1.1) and all have the same locking condition.

  Input-bit and gate transactions can be recognized and told apart by the content of their output conditions (see section 2.1). Both input-bit and gate transactions have two outputs each; while an input-bit output condition doesn't reference other

Protocol 5.1: Circuit Compiler

```
1: procedure COMP$^{\text{Circuit}}(out_b, out_m, tx^m)$
2:     if $tx^m$ has a circuit-output $C$ then
3:         $[T_1^{splitter}, ..., T_i^{splitter}] \leftarrow$ Construct splitter transactions as described in section 2.1.4
4:         $T_1^{input-bit} \leftarrow$ Construct input-bit transactions as described in section 2.1.1
5:         $\left[T_1^{gate}, \{tx^b_{C-keystone}\}\right] \leftarrow$ Construct gate transactions as described in section 2.1.1
6:         return $\left[T_1^{splitter}, \ldots, T_i^{splitter}, T_1^{input-bit} \cup T_1^{gate}, \{tx^b_{C-keystone}\}\right]$
7:     else if $tx^m$ has a circuit-input assignment then
8:         $T_1^{input-bit-label} \leftarrow$ Construct input-bit-labeling transactions as described in section 2.2.1
9:         $[T_1^{gate-label}, ..., T_i^{gate-label}] \leftarrow$ Construct gate-labeling transactions as described in section 2.2.1
10:        return $\left[T_1^{input-bit-label}, T_1^{gate-label}, \ldots, T_i^{gate-label}, \{tx^b_{A-keystone}\}\right]$
11:    else // $tx^m$ is a base transaction
12:        return $[\{tx^m\}]$
13:    end if
14: end procedure
```

transaction outputs, each gate transaction output condition will reference two "previous" input-bit and gate transactions using `OP_IS_TXO_UNSPENT`. Because the output conditions have a fixed format, with the only difference between them being the id of the referenced transaction outputs, simple pattern matching can be used to identify them.

Then tx_k is the keystone of a circuit-output transaction. The circuit can be recovered by parsing the `OP_IS_TXO_UNSPENT` references between the gate and input bit transactions. The fragments consist of the splitter transaction tx_s and all the input-bit and gate transactions that spend outputs of tx_s.

Case 2: If tx_k satisfies all of the following:

- tx_k is marked as a keystone
- tx_k has a single input that spends a keystone-marked transaction $tx_{k'}$
- $tx_{k'}$ is recognized as the keystone of a circuit-output transaction tx^m.
- All transactions that spend fragments of tx^m are themselves fragment-marked.

Then tx_k is the keystone of a circuit-assigment transaction. The assignment itself can be recovered using the EXTRACT-ASSIGNMENT procedure (cf. section 2.3). (By observation, the result of this procedure on an honestly-compiled circuit-assignment will return the same assignment). The corresponding fragments are the transactions that spend fragments of tx^m.

Case 3: Otherwise, tx_k is not a keystone that could have been output by COMP$^{\text{Circuit}}$.

5.2 Circuit Decompiler

The circuit decompiler, given the outputs of the base and meta-ledgers, and keystone base transaction (i.e., one that is, potentially, the keystone of a meta-transaction), can distinguish the following cases:

Case 1: The keystone is validated as an output of $\textsc{comp}^{\text{Circuit}}$ on transaction tx^m, and tx^m is valid in out_m. In this case, the decompiler returns tx^m.

Case 2: The keystone is *not* validated as an output of $\textsc{comp}^{\text{Circuit}}$, but spends an output that is a meta-output in the meta-ledger. In this case, the decompiler uses the extraction algorithm guaranteed by section 2.3 to create a valid meta-transaction with a meta-input, whose outputs are identical to the keystone outputs (i.e., they are base outputs in this case).

Case 3: Otherwise, the decompilation "fails". In this case, the decompiler returns the keystone transaction as-is (i.e., treated as a base transaction).

Protocol 5.2: Circuit Decompiler

```
1: procedure $\textsc{decomp}^{\textsc{Circuit}}(out_b, out_m, tx_k)$
2:     if $(fragments, tx_k)$ is a valid output of $\textsc{comp}^{\textsc{Circuit}}(\cdot, \cdot, tx^m)$ then // Case 1
3:         return $(fragments, tx^m)$
4:     else if $tx_k$ spends an output that is a meta-output in $out_m$ then // Case 2
5:         for every meta-output $txo^m$ spent by $tx_k$ do
6:             Compute the assignment $A_{txo^m}$ by calling EXTRACT-ASSIGNMENT with $out_b$ and the corresponding input.
7:             create the meta-input $txi^m$ using this assignment
8:         end for
9:         return a new $tx^m$ that is identical to $tx_k$, except that all inputs that spend meta-outputs are replaced by the corresponding meta-inputs.
10:    else // Case 3
11:        return $tx_k$
12:    end if
13: end procedure
```

5.3 Security Analysis: Circuit Compiler and Decompiler are Admissible

We note that the decompilation cannot fail if a meta-transaction was compiled honestly (since in that situation case 1 will hold; thus, by construction:

Lemma 5.3.1 ($\textsc{comp}^{\text{Circuit}}$/$\textsc{decomp}^{\text{Circuit}}$ are complete)**.** *$\textsc{comp}^{Circuit}$ and $\textsc{decomp}^{Circuit}$ satisfy definition 4.2.12.*

On the other hand, if the decompiler does fail, case 2 cannot hold, so all of the keystone's inputs are base outputs in out_m. This implies that the unmodified keystone transaction is valid in out_m, because it is valid in out_b and $out_m \overset{state}{\underset{txo}{\rightsquigarrow}} out_b$.

In order to plug our compiler/decompiler pair into the generic ledger protocol, we must prove they are admissible (c.f. definition 4.2.19). We do this in lemmas 5.3.2 to 5.3.1:

Lemma 5.3.2 ($\textsc{comp}^{\text{Circuit}}$ is valid)**.** *$\textsc{comp}^{Circuit}$ (cf. protocol 5.1) satisfies definition 4.2.18.*

Proof Sketch. We will consider separately each validity condition:

Condition 1: The output of $\textsc{comp}^{\text{Circuit}}$ always contains a splitter—i.e., a base transaction whose inputs are identical to the meta transaction. This can be seen by observation: if tx^m has a meta-input, the splitter is the keystone, otherwise it is explicitly constructed (c.f. section 2.1.4).

Condition 2: If tx^m has a meta-output, the splitter is the first fragment output by $\textsc{comp}$. Let out_b^{ext} be an extension of out_b that does not contain the splitter. Then if the splitter is not live in out_b^{ext}, one of its inputs must not be valid. Since, by definition, the inputs of the splitter are identical to the inputs of tx^m, for every ledger output $out_m^{ext} \underset{tx}{\rightsquigarrow} out_b^{ext}$ it must hold that tx^m is not live in out_m^{ext}.

If tx^m has a meta-input, the splitter transaction is the keystone. All the inputs of all other fragments spend explanatory fragments for the meta-output spent by tx^m. These are fragments that could have been output by $\textsc{comp}^{\text{Circuit}}$, so their locking conditions must all be identical to each other and to the corresponding meta-output's locking condition, while their spending conditions only consider the state of the other explanatory fragments for the same meta-output.

Thus, in order to block a fragment (or the splitter itslef), one of the explanatory fragments' outputs must have been locked.

Condition 3: This condition is trivial for the simple keystone case (if tx^m has a meta-input, or tx^m is a base transaction), since in this case the splitter is always the last fragment in $\mathcal{T}^{(out_b, out_m, tx^m)}$.

When tx^m has a meta-output, the splitter is always the *first* fragment of $\mathcal{T}^{(out_b, out_m, tx^m)}$. The keystone and all fragments after the splitter spend only the splitter's outputs. The spending (and locking) conditions for these outputs only require a signature under a key that is freshly generated by the compiler. Hence, in order to block the fragment transactions (which are correctly signed), the adversary must be able to lock the splitter outputs before the fragments are processed. However, given an adversary that can do this, we can use it as a black box to forge signatures. Thus, no computationally-bounded adversary can do so (by the security of the signature scheme).

□

Lemma 5.3.3 ($\textsc{decomp}^{\text{Circuit}}$ is valid). $\textsc{decomp}^{\textit{Circuit}}$ *(cf. protocol 5.2) satisfies definition 4.2.13.*

Proof Sketch. We will consider separately each validity condition:

Condition 1: By observation, the fragments returned by $\textsc{decomp}^{\text{Circuit}}$ always exist in the base-ledger output.

Condition 2: By observation, $\textsc{decomp}^{\text{Circuit}}$ always returns the input base transaction as the keystone, and if it is not the splitter itself, the first fragment returned is the splitter (since this means we are in case 1, and $\textsc{comp}^{\text{Circuit}}$ outputs a splitter as the first fragment).

Conditions 3 to 5: If tx^m has a meta-output, we are in case 1 therefore the keystone was output by $\textsc{comp}^{\text{Circuit}}$, which always preserves the output amount and sets the locking condition of all unspent-output fragments (i.e., the input bits and gate outputs) to be identical to the locking condition of tx^m. Otherwise (cases 2 and 3), the meta-outputs generated by $\textsc{decomp}^{\text{Circuit}}$ are identical to the keystone outputs (including in amount).

Condition 6: If tx^m is output due to case 1, this condition holds because $\textsc{comp}^{\text{Circuit}}$ will only accept a meta-transaction with a meta-input if it does not also have a meta-output—in which case it outputs a simple keystone. If we're in case 3, $\textsc{decomp}^{\text{Circuit}}$ returns the keystone itself as tx^m, hence it is simple. Otherwise (we're in case 2), $\textsc{decomp}^{\text{Circuit}}$ outputs a meta-transaction that spends the meta-output corresponding to the base output spend by tx_k, and has the same outputs as tx_k—hence, by definition, tx_k is a simple keystone for tx^m.

Condition 7: If we are in case 1, tx^m is by definition valid. If we are in case 3, tx^m is simply the keystone, which does not spend a meta input (otherwise we would be in case 2). Since the keystone is valid in out_b, and the outputs it spends are identical in out_m, due to $out_m \overset{state}{\underset{txo}{\rightsquigarrow}} out_b$, it will also be valid in out_m. If we are in case 2, the keystone does spend a meta-output, which by consistency must be explainable as a valid output $\textsc{comp}^{\text{Circuit}}$. Thus, we can use compiler-soundness (theorem 2.3.1) to conclude that the extracted assignment must be valid in out_m.

Condition 8: The only case that could return a non-simple keystone is case 1. However, in case 1 the decompiler returns tx^m only if the fragments could have been generated by an honest execution of $\textsc{comp}^{\text{Circuit}}$. Since $\textsc{comp}^{\text{Circuit}}$ generates a simple keystone when tx^m has a meta-input, the condition is satisfied.

□

Taken together, lemmas 5.3.1 to 5.3.3 immediately imply the following:

Corollary 5.3.4 (Admissibility). $\textsc{comp}^{Circuit}$ *and* $\textsc{decomp}^{Circuit}$ *are admissible (satisfy definition 4.2.19).*

6 Discussion and Open Questions

We have shown how to construct a meta-ledger that supports arbtrary circuit conditions using a slightly enhanced version of Bitcoin. Our ledger functionality is composable and “stackable”—given a suitable compiler/decompiler pair, our protocol can construct a meta-meta-ledger.

Many interesting open questions remain.

- Can we construct an admissible compiler/decompiler pair for arbitrary *script* conditions'? As a first approximation, it may be useful to consider the current Bitcoin scripting language, but remove the length limitation (i.e., allow longer scripts by using multiple transactions)
- Our requirements from comp/decomp may appear unduly restrictive. While some requirements appear to be inherent, it is an interesting open question to design meta-ledger protocols that support a wider variety of spending conditions. In particular, our protocols do not support spending conditions that can depend on the *contents* of a spending transaction (rather than just its ID), such as the extensions proposed by Bitcoin covenants.

Bibliography

[1] C. Badertscher, U. Maurer, D. Tschudi, and V. Zikas. Bitcoin as a transaction ledger: A composable treatment. In J. Katz and H. Shacham, editors, *Advances in Cryptology - CRYPTO 2017 - 37th Annual International Cryptology Conference, Santa Barbara, CA, USA, August 20-24, 2017, Proceedings, Part I*, volume 10401 of *Lecture Notes in Computer Science*, pages 324–356. Springer, 2017.

[2] R. Canetti, Y. Dodis, R. Pass, and S. Walfish. Universally composable security with global setup. In S. P. Vadhan, editor, *Theory of Cryptography, 4th Theory of Cryptography Conference, TCC 2007, Amsterdam, The Netherlands, February 21-24, 2007, Proceedings*, volume 4392 of *Lecture Notes in Computer Science*, pages 61–85. Springer, 2007.

[3] M. Chakravarty, J. Chapman, K. MacKenzie, O. Melkonian, M. P. Jones, and P. Wadler. The extended utxo model. In *Workshop on Trusted Smart Contracts (Financial Cryptography 2020)*, Jan. 2020.

[4] J. Garay, A. Kiayias, and N. Leonardos. The bitcoin backbone protocol: Analysis and applications. In *Eurocrypt*, 2015.

[5] J. Katz, U. Maurer, B. Tackmann, and V. Zikas. Universally composable synchronous computation. In A. Sahai, editor, *Theory of Cryptography - 10th Theory of Cryptography Conference, TCC 2013, Tokyo, Japan, March 3-6, 2013. Proceedings*, volume 7785 of *Lecture Notes in Computer Science*, pages 477–498. Springer, 2013.

[6] A. Kiayias, H. Zhou, and V. Zikas. Fair and robust multi-party computation using a global transaction ledger. In M. Fischlin and J. Coron, editors, *Advances in Cryptology - EUROCRYPT 2016 - 35th Annual International Conference on the Theory and Applications of Cryptographic Techniques, Vienna, Austria, May 8-12, 2016, Proceedings, Part II*, volume 9666 of *Lecture Notes in Computer Science*, pages 705–734. Springer, 2016.

[7] M. Möser, I. Eyal, and E. G. Sirer. Bitcoin covenants. In J. Clark, S. Meiklejohn, P. Y. A. Ryan, D. S. Wallach, M. Brenner, and K. Rohloff, editors, *Financial Cryptography and Data Security - FC 2016 International Workshops, BITCOIN, VOTING, and WAHC, Christ Church, Barbados, February 26, 2016, Revised Selected Papers*, volume 9604 of *Lecture Notes in Computer Science*, pages 126–141. Springer, 2016.

[8] S. Nakamoto. Bitcoin: A peer-to-peer electronic cash system, 2008. `http://bitcoin.org/bitcoin.pdf`.

[9] R. O'Connor and M. Piekarska. Enhancing bitcoin transactions with covenants. In M. Brenner, K. Rohloff, J. Bonneau, A. Miller, P. Y. A. Ryan, V. Teague, A. Bracciali, M. Sala, F. Pintore, and M. Jakobsson, editors, *Financial Cryptography and Data Security - FC 2017 International Workshops, WAHC, BITCOIN, VOTING, WTSC, and TA, Sliema, Malta, April 7, 2017, Revised Selected Papers*, volume 10323 of *Lecture Notes in Computer Science*, pages 191–198. Springer, 2017.

[10] R. Pass, L. Seeman, and abhi shelat. Analysis of the blockchain protocol in asynchronous networks. In *Eurocrypt 2017*, 2017.

[11] J. Rubin. Checktemplateverify. `https://github.com/bitcoin/bips/blob/master/bip-0119.mediawiki`.

[12] J. Swambo, S. Hommel, B. McElrath, and B. Bishop. Bitcoin covenants: Three ways to control the future, 2020.

[13] J. Zahnentferner. An abstract model of utxo-based cryptocurrencies with scripts. *IACR Cryptol. ePrint Arch.*, 2018:469, 2018.

[14] J. Zahnentferner. Chimeric ledgers: Translating and unifying utxo-based and account-based cryptocurrencies. *IACR Cryptol. ePrint Arch.*, 2018:262, 2018.

bitcoin-plus

תקציר

השימוש הבסיסי במטבע מבוזר (cryptocurrency) הוא העברת ערך כספי בין זהויות, אך רבים מהשימושים המעניינים יותר דורשים שימוש בחוזים, למשל – "העברה של X מטבעות מישות S לישות R תתבצע רק אם תנאים A ו-B יתקיימו". ביטקוין (ומטבעות מבוזרים דומים) מציבים גבולות נוקשים על השפה בה מנוסחים אותם תנאים. בפרט, התנאים מוגבלים באורכם ולא יכולים להכיל לולאות.

בעבודה זו אנו מראים כיצד ניתן להרחיב את שפת התסריט (scripting language) של ביטקוין עם פקודה (opcode) יחידה, שהוספתה לא מזיקה, והיא מאפשרת ליצור "מטא-תנאים" בעלי כוח הבעתי רב יותר (בצורת מעגליים בוליאניים בגודל שרירותי).

אנחנו בונים פרוטוקול על-מנת להדר (compile) מטא-תנאים לסדרה של עסקאות ביטקוין (transactions) בסיסיות שכוללות את הפקודה החדשה. אנחנו מראים כיצד להשתמש במהדר זה כדי לממש פונקציונאליות meta-ledger, ואף מוכיחים שהוא בטוח במודל של Universal Composability.

עבודה זו בוצעה בהדרכתו של דר' טל מורן מבי"ס אפי ארזי למדעי המחשב, המרכז הבינתחומי, הרצליה.

המרכז הבינתחומי בהרצליה
בית-ספר אפי ארזי למדעי המחשב
התכנית לתואר שני (M.Sc.) – מסלול מחקרי

Bitcoin+: תמיכה זולה בחוזים חכמים מורכבים ב-UTXO Ledger

מאת
ירון קנר

עבודת תזה המוגשת כחלק מהדרישות לשם קבלת תואר מוסמך M.Sc.
במסלול המחקרי בבית ספר אפי ארזי למדעי המחשב, המרכז הבינתחומי הרצליה

ספטמבר 2020