The Optimal Earned Income Tax Credit

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Abstract

In this paper we characterize the optimal linear and piecewise linear EITC schedule. In the linear framework we demonstrate that in the presence of unemployment, an increase of social inequality aversion and a decrease in labor aversion both derive in a lower optimal EITC. For the piecewise linear schedule, we show that in most cases the optimal schedule is a triangle, which is at odds with actual policy, that is based on a trapezoid. According to our simulation, the use of a trapezoid instead of a triangle implies a substantial loss in terms of Social Welfare. We show that a trapezoid is optimal only when the wage distribution among the working poor is even, with a discrete jump for higher wage groups. After mimicking the wage distribution in different countries, we show that changes in the share of the "very rich" have a lower impact on the optimal EITC compared to changes in the wage variance. Finally, we show that the main impact of an increased minimum wage on the optimal EITC schedule is a more pronounced phasing out.

Key Words: Optimal EITC, trapezoid, piecewise linear schedule

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1. Introduction

The earned income tax credit has been adopted in the U.S. in mid-seventies and has become the most prominent social program with a participation of 27.5 million individuals, who get substantial wage subsidies from the federal government. The program has been adopted also in many countries like UK and New Zealand and is gradually being extended to new ones , like Israel, that implemented the program in 2008.

So far the theoretical analysis on the optimal EITC schedule is relatively scarce. The most prominent paper was written by Saez (2002) who analyzed the optimality of the EITC vis-à-vis a basic transfer to the unemployed, and found that that the result depends on extensive and intensive margin elasticities at the labor market. Note, however that in Saez's framework elasticities are exogenous which is a drawback since lower wage individuals entering the labor market are crucially affected by the government transfer (income effect) – and thus it is important to study first the direct implications of this fact on the extensive margin reaction of the working poor. In this paper we build a simple model to study the optimal linear EITC at first[2], and then we move forward to analyze the optimal piecewise linear EITC according to Saez's framework.

The paper is organized as follows. In the first section we study the optimal linear EITC in a situation where individuals decide whether to enter the labor market according to their tastes toward consumption and leisure, given the demogrant received from the government. Understanding first the optimal linear framework has been the way transited in optimal income taxation, as a first step for enhancing the optimality analysis. Once we get the insights into this exercise, we move forward to study the more realistic case of an optimal piecewise linear EITC. For this purpose we perform an analysis with exogenous elasticities along the lines of Saez (2002) in which we ask the following question: is the optimal EITC schedule a trapezoid, as we see in most implemented systems

[2] In the linear model the income effect plays a role. For a more sophisticated framework that combines the income effect with an optimal piecewise optimal schedule see Regev and Strawczynski (2015).

at different countries? Finally, we ask two additional questions: what is the optimal EITC for a given wage distribution and how it is affected by the existence of a minimum wage?

2. The optimal linear EITC

In this section we characterize the optimal linear EITC using a simple stylized model. While the relationship between different variables at the optimum is analyzed in next section using a less restrictive framework, we still get value added in this section by using a simple model with a linear EITC, in which the results can be proved – as a first step for understanding the optimal EITC schedule. In this model the government chooses the optimal EITC given the maximization by individuals.

Individuals

Assume that individuals obtain utility (U) from consumption (c) and leisure (1-l):

$$(1)\quad U_i = \ln(c_i) + \delta_i \ln(1 - l_i)$$

Where δ represents the labor aversion, which is usually higher for low income individuals. For simplicity we will assume that there are three types of individuals in the economy, differentiated by the wage they can achieve at the labor market:

$$(2)\quad w_0 < w_1 < w_2$$

Individual 0 is the unemployed (as a consequence of his/her low w), individual 1 is the working poor since he/she gets a relatively low salary, and individual 2 is the high-income type, who pays the income tax and finance the unemployment benefits and the EITC. The budget constraints of these three individuals are:

$$(3)\quad c_0 = T_0 + A$$

$$c_1 = T_0 + (1 + e)w_1 l_1$$

$$c_2 = T_0 + (1 - t)w_2 l_2$$

Where T_0 represents a demogrant, A is the unemployment benefit, e is the linear EITC and t is the linear income tax which we will assume is given.[3] While individual 0 has no choice, individuals 1 and 2 will maximize equation 1 subject to budget constraints 3. Let us start with the solution of individual 3:

$$(4)\quad \text{F.O.C.:}\ \frac{w_2(1-t)}{w_2(1-t)l_2} = \frac{\delta_2}{1-l_2}$$

And it is easy to show that his labor supply is:

$$(5)\quad l_2 = \frac{1}{1+\delta_2}$$

It is well-known empirically that the labor aversion parameter for high-income individuals is very low. For simplicity we shall assume that $\delta_2 = 0$ and consequently labor supply of the rich individuals is inelastic: $l_2 = 1$. By a similar technique we obtain the labor supply for individual 1:

$$(6)\quad l_1 = \frac{1}{1+\delta_1}\left[1 - \frac{\delta_1 T_0}{w_1(1+e)}\right]$$

Note that in order to participate at the labor market individual 1 needs to obtain a wage that is higher than his threshold wage (including the EITC) which can be derived using the last term of equation 6:

$$(7)\quad w_1(1+e) > \delta_1 T_{0;}$$

When δ_1 is low, the working poor participates at the labor market even if his hourly wage is low and e=0. This means that theoretically the working poor could have an income that is lower than the one obtained by the unemployed (who gets A in addition). As discussed by Saez (2002), this would be the case of a deserving working poor, who would be favored by a conservative government and clearly would receive an EITC. Note, however, that by not participating at the labor market he/she would be able to receive a higher transfer from

[3] The optimal linear and non-linear income tax has been extensively characterized. For a survey see Diamond and Saez (2011). The implicit assumption is that there is a tax threshold, and consequently the working poor does not pay income tax since his/her wage is below the threshold.

the government $(T_0 + A)$, and consequently it would be better for him/her to remain unemployed. Thus, the benchmark case shall be based on a representative working poor with a higher δ_1 which implies by equation (7) that he has a higher wage.[4] Summarizing, the benchmark (and realistic) assumption for this case requires that the income of the working poor without an EITC is equal to the one of the unemployed – in order to resemble a situation of indifference.[5]

The Government

Government redistributes income in order to obtain a maximal social welfare. The government budget constraint is:

$$(8)\quad \mathrm{T} = \mathrm{tw}_2 = 3\mathrm{T}_0 + \mathrm{A} + \mathrm{ew}_1 \mathrm{l}_1$$

While a more general optimization would avoid assuming that l_2 is given, we will solve this case in the next sections. In this section, given that labor supply of high-income individuals is fixed, we assume for simplicity that tax revenues T are also given. Note that since t and labor supply of the third individual are given, he/she will not take part at the government maximization. While this assumption is clearly arbitrary, it allows concentrating the government dilemma on re-distribution among the poor: the government decision is to transfer money from the high-income individual either to the unemployed or to the working poor. We assume that the government chooses e optimally for a given A. The maximization problem is:

$$(9)\quad \mathrm{MAX}_{\mathrm{e}} \quad \mathrm{W} = \sum_{\mathrm{i}=0}^{1} \frac{\mathrm{U}_{\mathrm{i}}^{1-\mathrm{v}}}{1-\mathrm{v}}$$

In equation 9, W is the social welfare function and v represents the inequality aversion parameter. For simplicity, we will assume that since the government does not see the realization of labor, U will be based

[4] As discussed by Saez, in the general model if an EITC is optimal for this individual, it would clearly be optimal also in the case of a lower wage, because his/her social weight would be even higher.
[5] In the simulation we use a case in which the working poor income is slightly higher.

only on consumption. This assumption is consistent with governments' discourse, which stresses poverty alleviation as measured by income or consumption (i.e., governments do not put a weight on leisure).[6] By substituting equation 8 in the individuals' solution and taking the F.O.C. that results from the maximization problem stated in 9, we can calculate the optimal e.

F.O.C.:

$$\left\{\ln\left[\mathrm{w}_1\left(\frac{1}{1+\delta_1}\left(1-\frac{\delta_1 T_0}{w_1(1+e)}\right)\right)(1+\mathrm{e})+\frac{\mathrm{T}-\mathrm{A}-\mathrm{e}w_1 l_1}{3}\right]\right\}^{-\mathrm{v}}\frac{w_1/(1+\delta_1)-w_1 l_1/3}{\left[\mathrm{w}_1 l_1(1+\mathrm{e})+\frac{\mathrm{T}-\mathrm{A}-\mathrm{e}w_1 l_1}{3}\right]}=$$

$$\left\{\ln\left[\frac{\mathrm{T}-\mathrm{A}-\mathrm{e}w_1 l_1}{3}+A\right]\right\}^{-\mathrm{v}}\frac{3}{T-A-ew_1 l_1+3A}\frac{w_1 l_1}{3}$$

Note that the first term in the LHS represents social marginal utility of the working poor, multiplied by the marginal effect of the EITC divided by his income. In the terminology used by Saez (2002), the nominator includes both a behavioral effect (enhancement of labor supply) and a mechanical effect (loss of demogrant). In the RHS we only see, beyond the social marginal utility of the unemployed divided by his income, the mechanical effect - that is related to the reduction of the demogrant. The difference compared to Saez's framework is that in our case we get straightforward the final result - which does not depend on elasticities. We can re-write this condition in the following way:

(10)

$$\left\{\ln\left[\mathrm{w}_1 l_1(1+\mathrm{e})+\frac{\mathrm{T}-\mathrm{A}-\mathrm{e}w_1 l_1}{3}\right]\right\}^{\mathrm{v}}$$

$$=\left\{\ln\left[\frac{T-ew_1 l_1-A}{3}+A\right]\right\}^{\mathrm{v}}\frac{T-A-ew_1 l_1+3A}{\left(\mathrm{w}_1 l_1(1+\mathrm{e})+\frac{\mathrm{T}-\mathrm{A}-\mathrm{e}w_1 l_1}{3}\right)}\left[\frac{w_1}{(1+\delta_1)l_1}-\frac{1}{3}\right]$$

[6] Note also that this assumption implies a departure from Mirlees's (1971) framework, where social welfare accounts also for leisure.

The advantage of this analysis is that we get a tractable F.O.C. that allows checking whether the EITC is optimal. Since the incomes of the working poor and the unemployed are equal, the LHS equals the first term of the RHS, and the second term of the RHS equals 1. Thus, the optimality of the EITC depends on the third term – which must be higher than 1. The intuition of the third term is the following: an extra dollar allocated to the EITC increases income by the positive term, while at the same time the working poor loses a third of a dollar through the demogrant. Thus, when the behavioral reaction is strong enough, an EITC is optimal. Note that the first component of this term is positive and higher than 1 (since the nominator is higher than 1 and the denominator is lower than 1). Note also that for plausible parameters, this term is higher than 1 and consequently the RHS>LHS for e=0.[7] The single way to restore the equality is by imposing an EITC (e>0).

We use equations 6, 8 and 10 to prove four important results that will be characterized in the next section using simulations in a more realistic framework:

Result 1: If an EITC is optimal, increasing inequality aversion results in a lower EITC.

To obtain this result note that if the EITC is optimal the left hand side (LHS) of equation 9 equals the right hand side (RHS). When v goes up the LHS becomes lower than the RHS. The way to restore the equality is by reducing e.

Result 2: If an EITC is optimal, a reduction in labor aversion results in a lower optimal EITC.

As explained by Regev and Strawczynski (2015), a reduction in labor aversion may result as a consequence of a successful "From welfare to work" government policy, which makes this question particularly interesting. While in that paper the result was inconclusive (EITC can be higher or lower depending on different scenarios), note that in the present context the result is defined and it implies a reduction of the

[7] The denominator equals $\left[1-\frac{\delta_1 T_0}{w_1(1+e)}\right]$ which is lower than 1. For the parameters shown in the simulation this term equals 1.47. If in addition w_1 is higher than 1 as assumed in the simulation, the last term of the RHS in 10 is significantly higher than 1.

EITC. This result is implied by the fact that the working poor works more, which implies an increase in his income. Thus, it is clear that after this change LHS<RHS. Restoring the equality requires a lower e.

Result 3: If an EITC is optimal, increasing the resources available to the government results on a higher EITC.

Also here an optimal EITC implies that the departure is when LHS equals the RHS. For the same labor aversion parameters, when we allocate all the money to the demogrant, the RHS>LHS. This means that we shall reallocate some money to the EITC, in order to restore the equality.

Result 4: If an EITC is optimal, increasing the unemployment benefits results on a higher EITC.

Note that an increase of unemployment benefits causes that the LHS<RHS. In order to restore the equality the EITC must go up.

In order to characterize these results we perform a simulation and calculate the optimal linear EITC. Results are shown in Table 1.

Table 1

Scenario	$\boldsymbol{l_1}$	$\boldsymbol{T_0}$	**e (%)**
Benchmark: $w_1 = 2.2; T = 3;\ \delta_1 = 0.4; A = 1.6; v = 2$	0.68	0.351	0.350
Higher inequality aversion: $w_1 = 2.2; T = 3;\ \delta_1 = 0.4; A = 1.6; \boldsymbol{v = 3}$	0.68	0.378	0.266
Lower labor aversion: $w_1 = 2.2; T = 3;\ \boldsymbol{\delta_1 = 0.2}; A = 1.6; v = 2$	0.74	0.380	0.260
Higher Government Budget Resources: $w_1 = 2.2; \boldsymbol{T = 3.2};\ \delta_1 = 0.4; A = 1.6; v = 2$	0.68	0.403	0.392

Higher Unemployment Benefits: $w_1 = 2.2; T = 3;\ \delta_1 = 0.4; \boldsymbol{A} = \mathbf{1.8}; v = 2$	0.69	0.248	0.455

The results of the simulation show that the optimal linear EITC is 35 percent, while the optimal transfer to the unemployed is 0.351 which represents 16 percent of the income of the working poor. The results of the sensitivity analysis are in line with the results shown above: higher inequality aversion and lower labor aversion derive in a lower optimal linear EITC and a higher demogrant, while higher resources and higher unemployment benefits imply both a higher EITC (the former imply a higher demogrant while the latter a lower one).

3. The optimal piecewise linear EITC

Previous papers that characterized the optimal EITC are Liebman (2002) and Lehman et al. (2011) . Liebman (2002) found that social inequality aversion is a crucial parameter for justifying the implementation of an EITC. Lehman et al. (2011) found that the inexistence of unemployment increases the optimality of an EITC, which would be optimal also with a Rawlsian social planer. Note that these papers did not analyze the optimality of particular types of schedule, as used in reality. In particular, in this paper we ask whether the optimal schedule is a triangle or a trapezoid as implemented in real life by most countries (U.S.A., U.K. and Israel).

3.1 Simulations Methodology

We further develop Saez's general equilibrium formula, by allowing at the optimum that the size of the bottom 4 wage groups depends on the tax rate. For this purpose we assume that the extensive margin elasticities in the bottom 4 wage groups are larger than zero, which is a necessary (but insufficient) condition for obtaining a wage subsidy in the bottom 3 wage groups. This framework allows for an optimal EITC trapezoid, as explained later. A detailed description of our methodology can be found in Appendix 1. It is important to note that once our iterative approach is adopted, the resulting EITC levels that are obtained in equilibrium are substantially smaller than those reported by Saez (2002). The intuition for this result is simple – as redistribution occurs, consumption levels (and the resulting social weights) become more equal, which makes redistribution less appealing for the central planner.

3.2 The optimal piecewise EITC schedule

We proceed with simulations for the case where the extensive margin elasticities in the bottom 4 wage groups are larger than zero. We initially use, for all relevant parameters (i.e. the extensive and intensive margin elasticities, and the government's redistributive tastes) the same range of values used in Saez (2002). In that respect, a key result of the Saez's (2002) model is that an EITC subsidy is optimal when the extensive margin elasticity is high. Thus, since our interest is in the optimal shape of the EITC schedule, we shall focus on the cases where $\eta = 0.5$ and $\eta = 1$. Furthermore, as EITC subsidies are obtained in the presence of relatively low intensive margin elasticities, which correspond to the ones that are accepted as realistic, we shall focus in our simulations mainly on those cases (e.g. $\zeta = 0.25$, $\zeta = 0.05$).[8] Thus, under these conditions, which favor an EITC subsidy, we shall examine how the shape of the optimal EITC varies with different values of parameters of interest, such as the central planner's redistributive tastes (v), the intensive margin elasticity and the distribution of wages among the working poor.

An important aspect that must be taken into account is that the initial share of unemployed individuals and the distribution of wages of the working poor can vary substantially between countries, and the optimal tax schedule for country A (all else being equal) is not necessarily suitable for country B. Thus, for the generality of the analysis we run the same simulations over a range of wage distributions, with different levels of income inequality among the working poor, and different levels of (pre-tax) unemployment. We will re-visit this characteristic in the next section by estimating the wage distribution using empirical data.

[8] Gruber and Saez (2002) show that real life intensive margin elasticities for low wage groups are particularly low.

If our target population is, say, the bottom 60%[9] of the working age population (the unemployed and the working poor), we can divide the working poor into 5 groups with a predetermined pre-tax size. For example, if prior to redistribution, the unemployed accounts for 10% of the population then, prior to redistribution the bottom four wage groups each account for 12.5% of the working age population. Or, if prior to redistribution the unemployed accounts for 20% of the working age population, then the bottom 4 wage groups would each (initially) account for 10% of the working age population[10]. Specifically, we explore in the benchmark case the cases where the initial share of the unemployed, is either 5%, 10%, or 20%[11].

3.2.1 Benchmark case

We start from a benchmark wage distribution - the actual distribution of non-student working age (25-64) population in Israel in 2014. The non-employed account for about 22% of this population - and by construction the lowest wage group accounts for 8% of this population, and groups 2-4 account for 10% each. The wage distribution ratio in the four bottom wage groups with respect to the bottom wage group is 1, 2.18, 2.90, and 3.55, respectively. Figure 1 shows for different levels of inequality aversion (v), the cumulative subsidy/tax that is obtained in each wage group, as a share of the bottom group's pre-subsidy wages; the simulations are performed using an extensive margin elasticity of $\eta = 1$, and an intensive margin elasticity of $\zeta = 0.25$. For the simplicity of the benchmark analysis, we initially assume that each of the 4 lowest wage groups has the same extensive margin elasticity $\eta_{1-4} = 1$, and the same intensive margin elasticity $\zeta_{1-4} = 0.25$.[12]

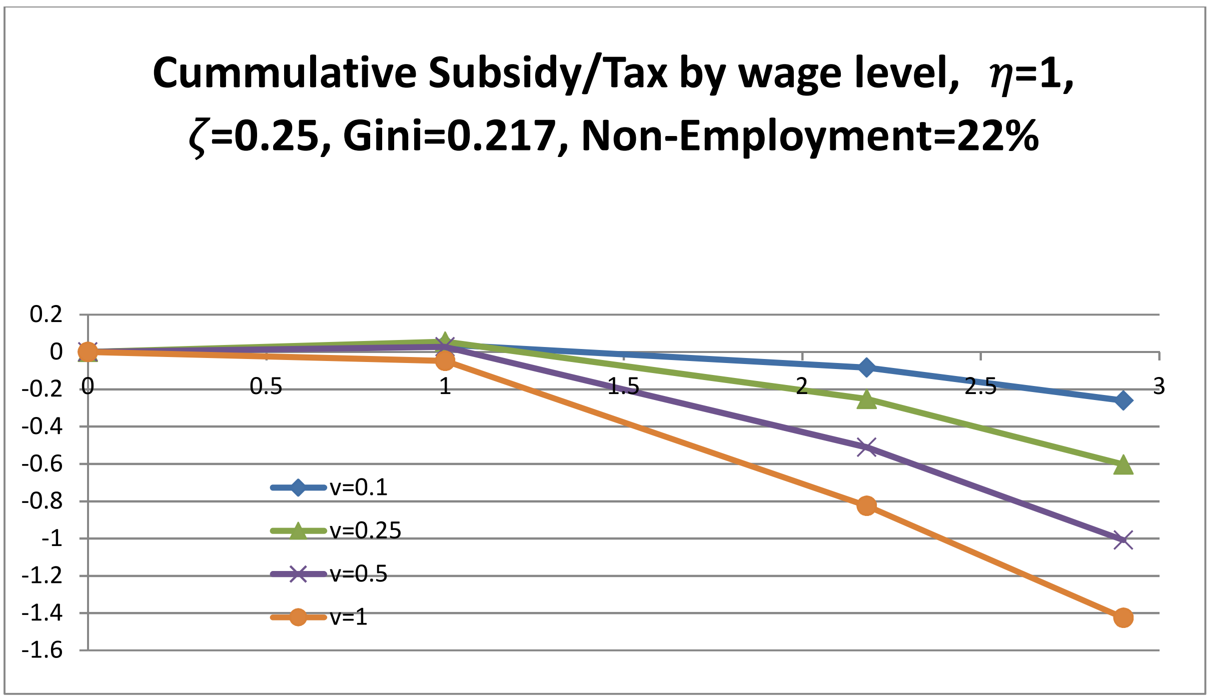
As shown in figure 1, in all levels of inequality aversion, wage groups 2-4 do not receive a wage subsidy but rather pay a tax; i.e., under the conditions specified above, the subsidy turns into a tax (in absolute terms) from the

[9] In Israel for example, about 60% of the non-student working age population, are either not employed or earn less than 8000 NIS – which is currently the income level in which EITC eligibility is canceled out.
[10] It is important to recall that these initial group sizes will later change, as a reaction to the adopted tax schedule.

[11] Simulations for cases where the initial share of the unemployed is larger than 20% are less instructive, because the social weight of the unemployed is quite high and thus the EITC becomes a less appealing tool for the central planner.
[12] We shall later explore more complex cases where the intensive margin elasticity differs between wage groups.

second wage group on. The phasing out of the second group is steep – which means that the optimal schedule looks like a triangle and not as a trapezoid. Note however, that since the actual wage distribution is continuous (rather than discrete) there would always be a wage range in which the (gradual) phasing-out of the subsidy occurs. Nonetheless, an important insight that can be drawn from this discrete analysis is that when the wage level of the second lowest group is substantially higher than that of the bottom wage group – then a wage subsidy for the second wage group (and on) is usually not an optimal policy. In the Israeli case, the wage ratio between wage groups 1 and 2 is 2.18, and thus the 2nd group's wages are not subsidized.

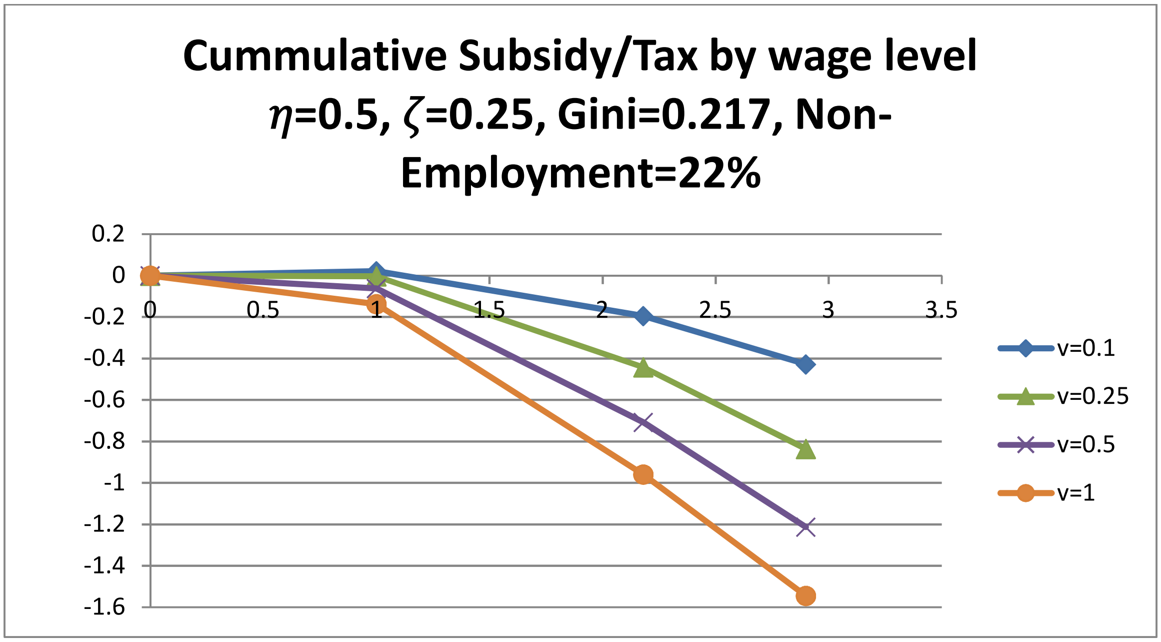
Figure 1



3.2.2 Sensitivity to the extensive margin

In our benchmark case, very low EITC subsidies were obtained for the lowest wage group, and from the 2nd group on, there was no entitlement to a subsidy and a tax was payed. This result was obtained under the benchmark assumption of an exogenous extensive margin elasticity $\eta = 1$. Recalling Saez's (2002) analysis, we know that an EITC subsidy is more likely to be obtained when the extensive margin elasticity is high (e.g. $\eta = 1$). Thus, it is not surprising to see (figure 2) that given a lower extensive margin elasticity $\eta = 0.5$, there is practically no EITC subsidy for neither wage group – even when inequality aversion (v) is very low.

Figure 2



This case ($\eta = 0.5$) is less interesting in the context of characterizing the shape of the optimal EITC trapezoid, and we shall therefore proceed with $\eta = 1$ in all other simulations. This assumption seems plausible also from an empirical point of view.[13]

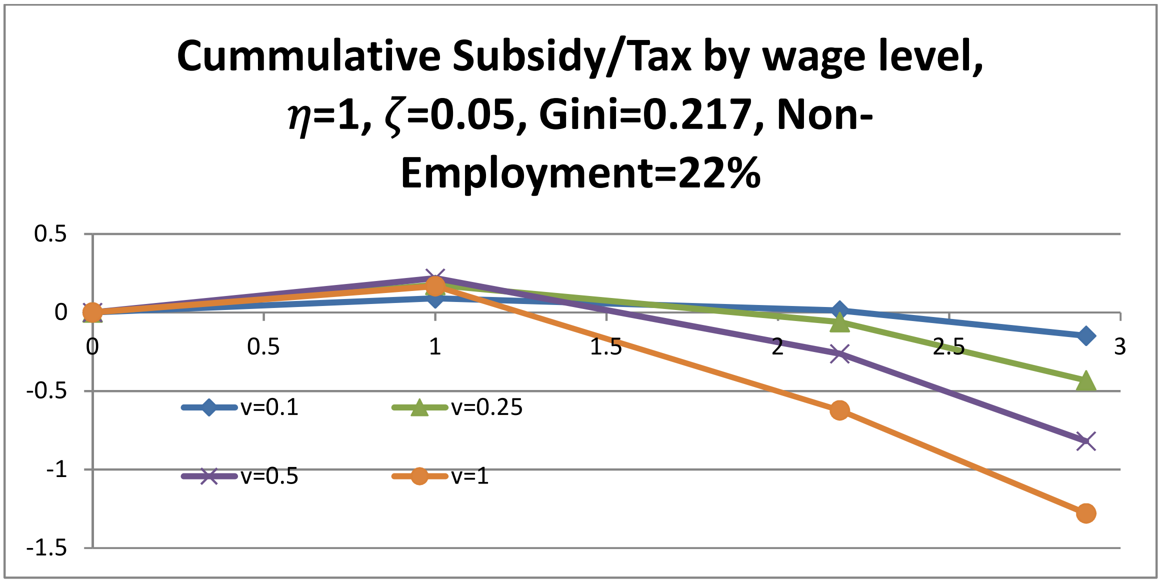
3.2.3 The intensive margin

Despite the results shown above, there are specific conditions under which the second wage group is not taxed, despite the presence of high wage gaps between groups 1 and 2. Figure 3 shows the optimal cumulative subsidy/tax in each wage group, when all conditions are similar to those described in figure 1, except for the intensive margin elasticity, which is significantly lower: $\zeta_{1-4} = 0.05$. As evident in figure 2, when both the

[13] Brender and Strawczynski (2006) show that in Israel extensive margin elasticities are higher for particular groups like Arab women.

intensive margin elasticity and the level of inequality aversion (v) are very low, than the second lowest wage group is not taxed, despite the large wage gaps relative to the bottom wage group.

Figure 3



In general the optimal tax schedule is flatter when inequality aversion is low. This is not surprising, given that under lower inequality aversion less redistribution is required and tax rates are therefore relatively low. However, as evident in figures 1 and 2, the EITC subsidy for the bottom wage group is highest when the level of inequality aversion is 0.5. A higher level of inequality aversion (e.g. v=1) or a lower level (e.g. v=0.25) yields a smaller EITC subsidy for the bottom wage groups. The intuition here is that when inequality aversion is very low, there is no need for much redistribution and consequently the EITC subsidy is small; and when inequality aversion is very high, the social weights of the non-employed are high relative to the working poor and thus a large lump-sum transfer is preferred over an EITC subsidy.

3.2.4 The effect of non-employment rates on the shape of the optimal EITC trapezoid

Another important aspect that affects its shape is the share of non-employed individuals in the population. As intuitively implied from Saez's (2002) analysis, the lower is non-employment the higher the attractiveness of the EITC for the central planner. In figure 4 we employ conditions that favor more an EITC trapezoid: Non employment is reduced to 10%, and the wage distribution ratio in the four bottom wage groups with respect to the bottom wage group is 1, 1.21, 1.4, and 4 – yielding (within those 4 groups) a Gini coefficient of 0.302. As evident in figure 4, this setup – a relatively even wage distribution for the bottom 3 groups, with a discrete jump in the wages of the fourth group – yields a flat and wide EITC trapezoid (which differs from the triangle shaped EITC schedules that obtained in the previous examples).

Figure 4

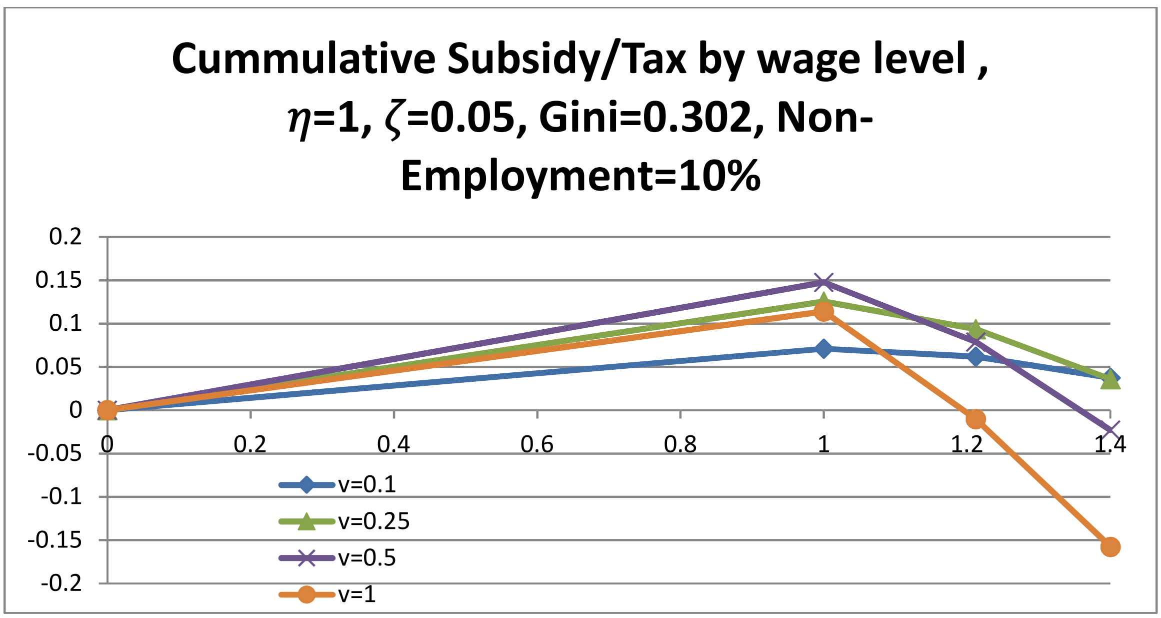
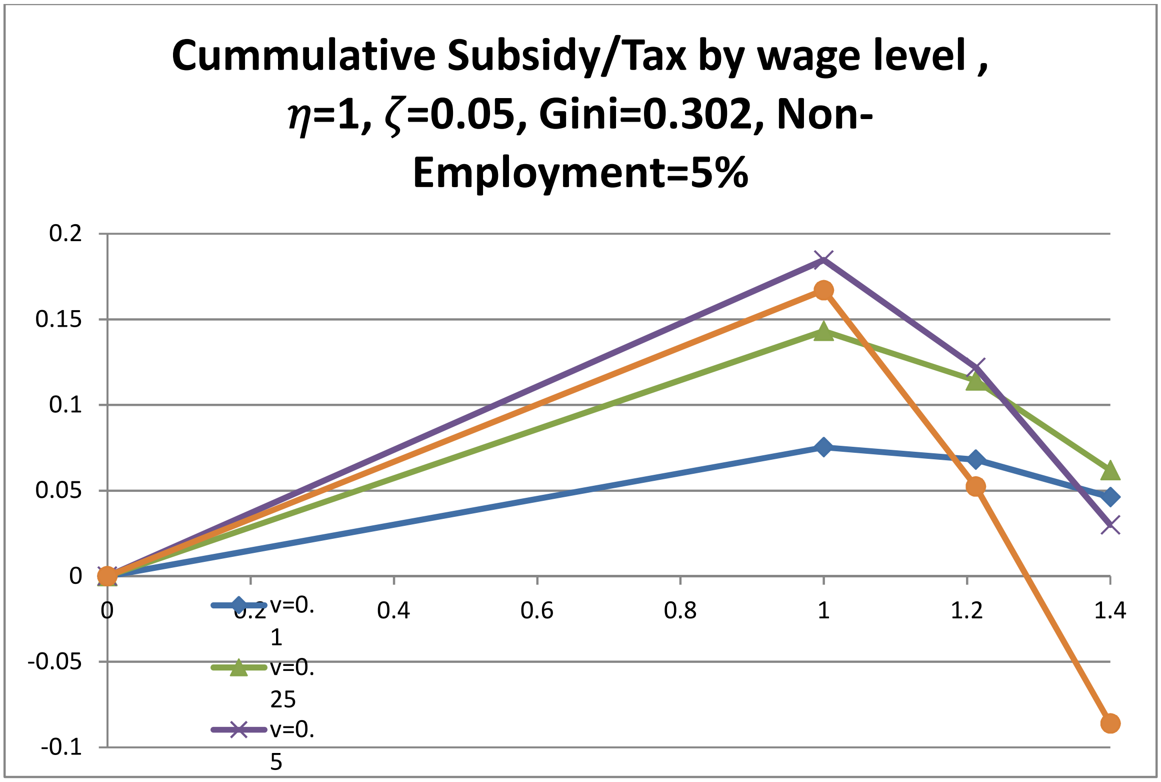


Figure 5 shows the optimal EITC schedules for similar conditions of those discussed in figure 4, with the exception of initial share of non-employed individuals, which was changed from 10% to 5%. Accordingly, the extra 5% of working individuals were equally distributed in the bottom 4 wage groups.

Compared to figure 4, the EITC triangles/ trapezoids in figure 5 are taller and cover a wider income range; i.e., when non-employment is lower, the EITC subsidy is higher, and a larger share of the working poor receives it.

When visually comparing figures 4 and 5, it is easy to see that in figure 5 (lower non-employment) all of the tax/subsidy schedules were simply shifted upwards.

Figure 5

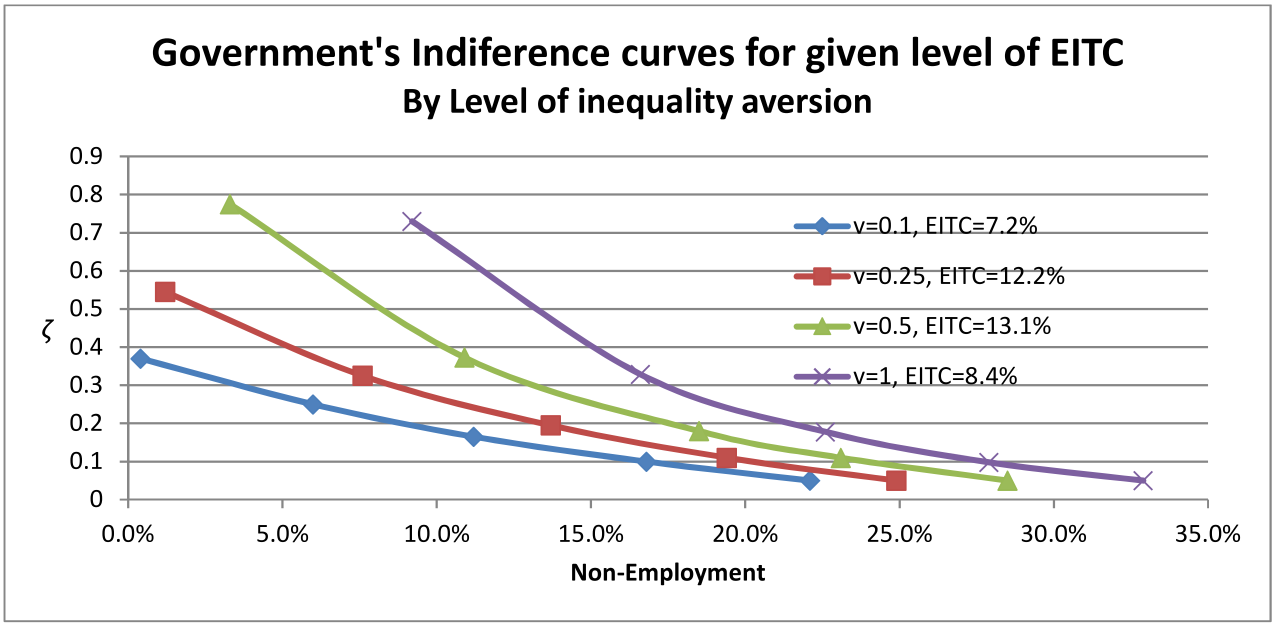


Note that in all these cases the optimal schedule resembles a triangle. We turn now to the characterization of an optimal trapezoid.

3.2.5 The tradeoff between the non-employed and the working poor

As we have shown, the intensive margin elasticity of the working poor ζ_{low} , and the non-employment rate h_0, are two key elements that influence the magnitude of the EITC subsidy - when they are low the subsidy is high, and when they are high the subsidy is low. From the policymaker's point of view, it can be instructive to examine, which of these two factors is more dominant (with respect to the EITC subsidy), when non-employment is high and the intensive elasticity is low (or vice-versa). Figure # shows the government's indifference curves which consist of different combinations of ζ_{low} and h_0 that yield, in optimum, the same given level of EITC subsidy. Each curve represents a different level of inequality aversion, v, and a different (fixed) level of EITC subsidy. As can be seen, the higher the level of inequality aversion, the higher is the concavity of the indifference curves. This means when inequality aversion is high, there is a stronger tradeoff (regarding the EITC subsidy) between the non-employment rate and the intensive elasticity. When v=1, for instance, low-non employment rates make an EITC subsidy optimal, even if the working poor's intensive margin elasticity is very high. In contrast, when inequality aversion is low (v=0.1), this tradeoff is much weaker, and an EITC subsidy is optimal only when the intensive margin elasticity is relatively low.

Figure 6



3.3 Is the optimal EITC a trapezoid?

In actual EITC systems the usual case is a trapezoid. However, we have seen in the previous sub-section that in most cases the optimal schedule is a triangle. One interesting question is: under which condition the trapezoid used in actual systems is optimal?

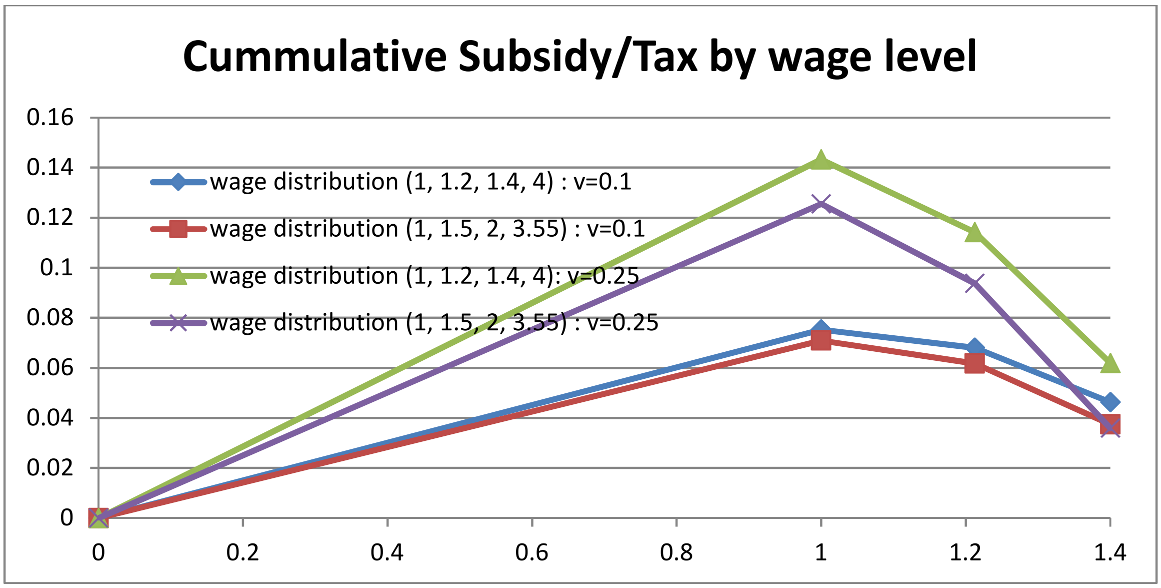
In order to obtain a tax schedule which, in the lower wage range, takes the shape of a trapezoid, the following conditions must be met: the Marginal tax rate must be negative for the lowest wage group, Zero (or very close to Zero) for the 2nd lowest wage group, and positive for the 3rd lowest wage group. Thus the existence of an EITC subsidy is a necessary but insufficient condition for a trapezoid shaped tax schedule. Given a negative marginal tax rate for the lowest wage group, the marginal tax rate for the 2nd lowest wage group can be either positive, zero (or very close to zero), or in some extreme cases even negative.

As we have shown in figures 1 and 2, in the presence of high wage gaps between the two bottom wage groups, the 2nd lowest wage group would (in most cases) pay a tax and not receive an EITC subsidy. Thus, when one's aim is to characterize the optimal shape of the EITC trapezoid, such a case is somewhat less instructive. It is therefore useful to examine cases in which the pre-tax wage gaps between the 1st, 2nd and 3rd lowest wage groups are lower, and an EITC trapezoid is more likely to be obtained.

Figure 6 shows the optimal schedules for two different levels of inequality aversion, in two different wage distributions of the bottom four groups. In one case the wage distribution ratio in the four bottom wage groups with respect to the bottom wage group is 1, 1.5, 2, and 3.55, respectively. And in the other case the wage distribution ratio in the four bottom wage groups with respect to the bottom wage group is 1, 1.2, 1.4, and 4, respectively. In both cases the non-employed account for 10% of this population – and the four bottom wage groups account for 12.5% each; the extensive margin elasticity is $\eta = 1$, and the intensive margin elasticity is $\zeta = 0.05$. As can be seen, when the wage ratio between the two bottom wage groups is smaller, an EITC subsidy for the 2nd wage group is more likely. Furthermore, for low levels of inequality aversion (e.g. v=0.1), the EITC schedule takes the familiar shape of a (flat) trapezoid - with a negative marginal tax for the bottom wage group; approximately zero marginal tax for the 2nd lowest wage group; and a positive marginal tax for the 3rd lowest wage group, which phases-out the subsidy to (approximately) zero.

The comparison between the two cases shows that in the latter - , where wage gaps between the bottom 3 groups are very small, but the wage gap between the fourth group and the third group is substantially larger - positive EITC subsidies are obtained not only for the two lowest wage groups but also for the 3rd group (when inequality aversion is low). This subsidy is completely phased out and turns into a high tax from the 4th group on. This result is driven by the large wage gap between the 4th group and the 3rd group. Since wage gaps between the bottom 3 groups are small, the phasing-stage is "shifted" towards the 4th group - that (due to its relatively high wages) can be taxed to allow for the redistribution of resources to the bottom 3 groups.

Figure 6



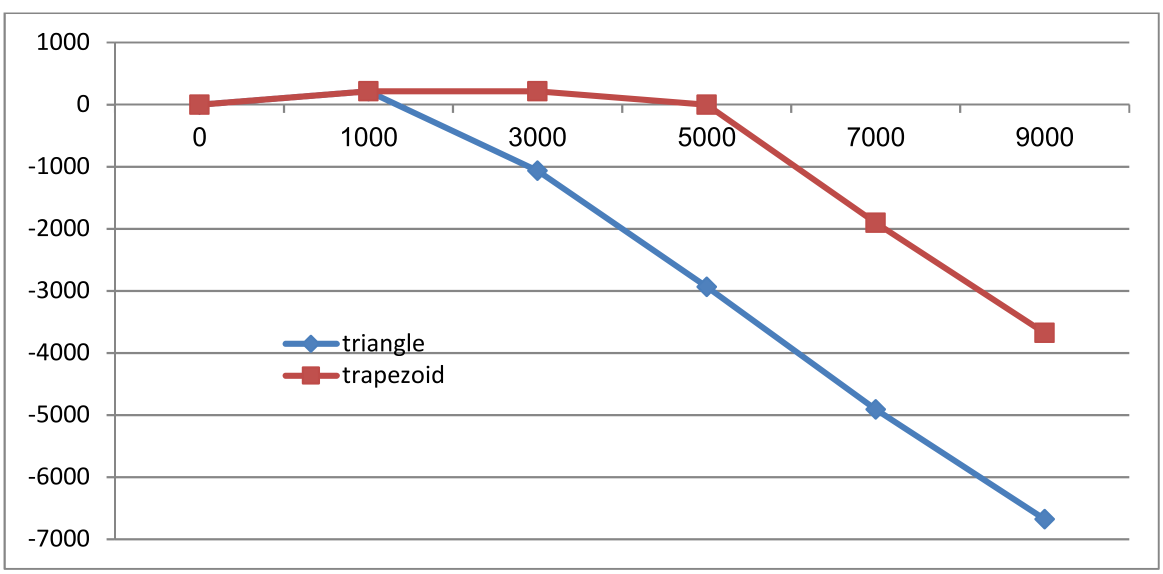
Note that when wage gaps in the bottom groups are small the EITC trapezoid is flatter and wider.

Social welfare loss: Triangle vs. Trapezoid

What is the magnitude of social welfare loss that can be attributed to the use of an EITC trapezoid instead of a triangle? While the answer to this question depends on the choice of parameters, an exercise that compares the aggregate social welfare that is obtained in each case – can yield some insights. Table A.2 (Appendix 2) presents such a comparison, in which the benchmark case is the EITC triangle that is obtained at optimum given moderate inequality aversion (v=1) and the same benchmark parameters that were used in table 5. As shown in the table, such conditions yield a marginal wage subsidy of 21% for the bottom wage group, followed by a 35% marginal tax rate for the 2nd wage group. Now suppose that an EITC trapezoid is employed instead. This would mean a marginal tax rate of 0% for the 2nd group, and an average tax rate of 0% for the 3rd group. By forcing these exact conditions

– we are able to give a close estimate of the welfare loss associated with the (suboptimal) use of an EITC trapezoid. Note that since this exercise altered (mechanically) the tax rates for groups 2 and 3, the average tax rates of the higher wage groups (and group 4 in particular) must also be adjusted in order to avoid a situation where the resulting marginal tax rate for group 4 is higher than 1 - which is of course irrational, since it would create a disincentive to work. Thus, in the trapezoid case the average tax rates of groups 4-8 were lowered (as shown in figure 7) to insure marginal tax rates that are lower than 1. Note that in the trapezoid case the lower tax rates for groups 2 and 3 yield higher participation rates and a lower level of non-employment. In other words, in the trapezoid case, more individuals work and more money is earned, but fewer taxes are being collected. Thus the level of guaranteed income (and the consumption level of the non-employed) is lower. As in Saez (2002), we assume that apart from redistributive transfers the government must also finance, through taxes, a public good. Since the population size is normalized to 1, this means that (in the triangle case) out of a total of 6,042 NIS of collected taxes, 2300 NIS go to the public good and the remaining 3742 NIS go to redistribution as guaranteed income (T_0). In the trapezoid case however, only 4095 NIS are collected, and thus only 1795 NIS go to redistribution.

Figure 7: Triangle vs. Trapezoid



The tradeoff in these two cases is between more redistribution (triangle), and more employment and earnings (trapezoid). To compare the aggregate social welfare of these two cases we compute the respective sum of products of the social weights (g_i), the consumption levels (c_i), and groups' share in the population (h_i). This yields, for the triangle case, an aggregate social welfare of 5158, compared to 4754 in the trapezoid case. In other words, given plausible parameters, the aggregate social welfare is 8.5% higher in the triangle case. Given a high enough level of inequality aversion (v=1), the "price" of a lower guaranteed income (in social welfare terms), can be higher than the gains of increased employment. Note however, that this result depends heavily on the assumption that there is a fixed budget constraint for redistributive purposes and the EITC subsidy comes at the expense of the guaranteed income; i.e., the trapezoid might still be preferable over the triangle if the loss of tax revenue is not deducted from the funds allocated to guaranteed income transfers. For example, if the increased employment levels and additional earnings of the working poor would result in higher levels of consumption – then the increased amount of indirect taxes collected might offset the lower revenues from direct taxes.

We have so far seen how differences in lower-end wage distributions and in non-employment rates can significantly affect the shape of the optimal EITC. It might therefore be a useful exercise, to examine to what extent these factors vary between countries, and within countries over time.

4. An empirical approximation to the optimal EITC

4.1 Estimating the Income distribution

The Pareto log-normal (PLN) and the double Pareto log-normal (DPLN) distributions have been shown to be good approximations for income distributions (Reed and Wu 2008). The pdf and the cdf for the PLN distribution are given by Colombi (1990) and Griffiths and Gholamreza (2012).

$$11)\ f^{PLN}(y; m, \sigma, \alpha) = \frac{\alpha}{y}\phi\left(\frac{\ln y - m}{\sigma}\right)R(x_1)$$

$$12)\ F^{PLN}(y; m, \sigma, \alpha) = \Phi\left(\frac{\ln y - m}{\sigma}\right) - \phi\left(\frac{\ln y - m}{\sigma}\right) R(x_1)$$

And the pdf and cdf for the DPLN distribution, which was developed by Reed (2003) and Reed and Jorgensen (2004), are also given in Griffiths and Gholamreza (2012):

$$13)\ f^{dPLN}(y; m, \sigma, \alpha) = \frac{\alpha\beta}{(\alpha+\beta)y} \phi\left(\frac{\ln y - m}{\sigma}\right) \{R(x_1) + R(x_2)\}$$

$$14)\ F^{dPLN}(y; m, \sigma, \alpha) = \Phi\left(\frac{\ln y - m}{\sigma}\right) - \phi\left(\frac{\ln y - m}{\sigma}\right) \left\{\frac{\beta R(x_1) - \alpha R(x_2)}{\alpha + \beta}\right\}$$

Where $R(t) = [1 - \Phi(t)]/\phi(t)$ is a Mill's ratio, $\phi(.)$ and $\Phi(.)$ are, respectively, the pdf and cdf for a standard normal random variable,

$$x_1 = \alpha\sigma - \frac{\ln y - m}{\sigma} \text{ and } x_2 = \beta\sigma + \frac{\ln y - m}{\sigma}$$

<u>Data and Empirical analysis</u>

Using these specifications and sample based estimates for m and for σ we derive approximations for income distributions for 11 countries and compare them to the actual sampled distributions. These countries are Israel, US, GBR, Germany, Spain, Brazil, Colombia, Uruguay, Russia, Denmark and Finland. For all countries, data was extracted from the Luxemburg Income Survey (LIS) database – which allows for a relatively harmonious comparison. For each country the latest available LIS data was used (which in most cases corresponds to 2013).

<u>Deriving estimates for actual income distributions</u>

Estimates for the actual income distributions in each country were derived as follows: the annual incomes of the bottom 97% wage earners were arbitrarily divided into 49 equal wage intervals and, where the bottom interval ranges from zero to $\frac{1}{50} \cdot W_{97\text{th percntile}}$ and the top interval is unbounded and ranges from $\frac{49}{50} \cdot W_{97\text{th percntile}}$ to infinity (in theory). Where $W_{97\text{th percntile}}$ is the income of the top earner in the 97th percentile. This specification was required in order to obtain wage intervals of reasonable size. Wages are bounded at the bottom but not at the top and thus an unbounded specification would have yielded wage intervals that are too large to provide any useful insight.

Deriving PLN and DPLN approximations for income distributions

In order to derive PLN and DPLN approximations for the income distribution of each country, sample estimates were obtained for m and σ, the first and the second moment of the (log-normalized) income distribution. Note that these parameters were computed for the entire sample of wage earners (not omitting the top 3 percent), in order to account for the Pareto behavior at right tail of the distribution. Once m and σ are obtained the distributions' approximation is quite straightforward.

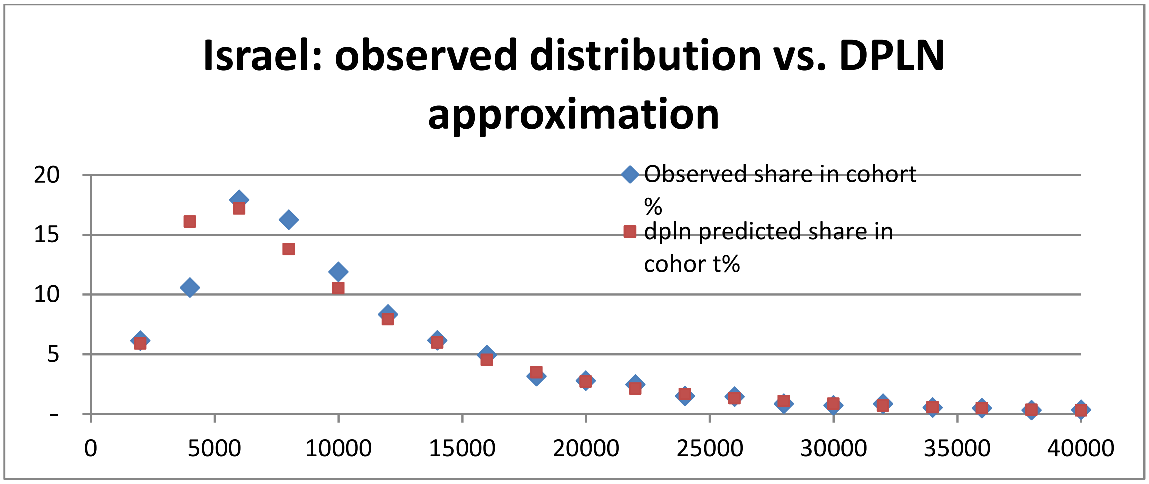
Using (12) and (14), the cumulative share of wage earners in each of the 50 wage brackets was approximated – which allowed (by simple subtraction) the computation of the relative share of wage earners in each of the 50 cohorts. Since α and β are initially unknown, they are computed via an iterative algorithm which minimizes the root mean of squared errors between the observed and approximated relative shares (c_i and $\hat{c}_i$). I.e. between the (binned) sampled distribution and the PLN/dPLN approximated distribution. Note that by minimizing the sum of square errors of the relative share of wage earners, (rather than the sum of square errors of the relative income shares s_i and $\hat{s}_i$), we are aiming for the best fit relative to the distribution of wage earners (and not relative to the distribution of income). It is of course possible to minimize the sum of errors relative to the distribution of income but such a design would give excess weight to discrepancies in higher income levels and little weight to discrepancies at low income levels – which is undesirable for our purposes given that the focus population in this paper is the working poor.

Since the observed income distributions are computed from sample surveys, with sample sizes varying between countries (but generally in the range of 10K-20K observations[14]), the number of bins chosen to represent the observed income distributions can significantly affect the goodness of fit of the PLN and DPLN approximations. If too many bins are used than the observed sample data is too "noisy" and inaccurate to resemble a smooth distribution. This occurs (among other reasons) because sample data (which usually relies on self-reporting) tends to be bunched at round values. E.g., a survey participant who earns, say, 14,917 Dollars a year is likely to report that he earns 15,000[15]. This phenomenon, which is very visible in the data of each country, mandates that the number of bins would be small enough to allow for a relatively smooth (binned) observed distribution. On the other hand, if the number of bins is too small then much information is lost and thus the exercise is less instructive. The optimal balance can vary between countries, and depends on the size of the sample and the quality of the data. As noted, in our analysis, we used the same number of bins for each country, i.e. 50 bins. However, for Israel, for which we had data of slightly better quality[16], we also experimented with different numbers of bins, and obtained a better fit with 20 bins.

Figure 8 compares between the observed (binned) income distribution in Israel 2014, to the approximated DPLN distribution that was computed using the (log-normalized) sample mean and standard deviation. As can be seen, in most bins (cohorts) the observed frequency and the approximated frequency are very similar. An exception is the second cohort where the observed frequency is about 11% while the DPLN approximated frequency is about 16%. This Discrepancy is mainly due to bunching of the actual income distribution at the point of minimum wages. Thus many wage earners who would otherwise fall into the second cohort (2000 to 4000 NIS), fall into the 3rd cohort (4000 to 6000 NIS) as minimum wages in 2014 stood at 4300 NIS.

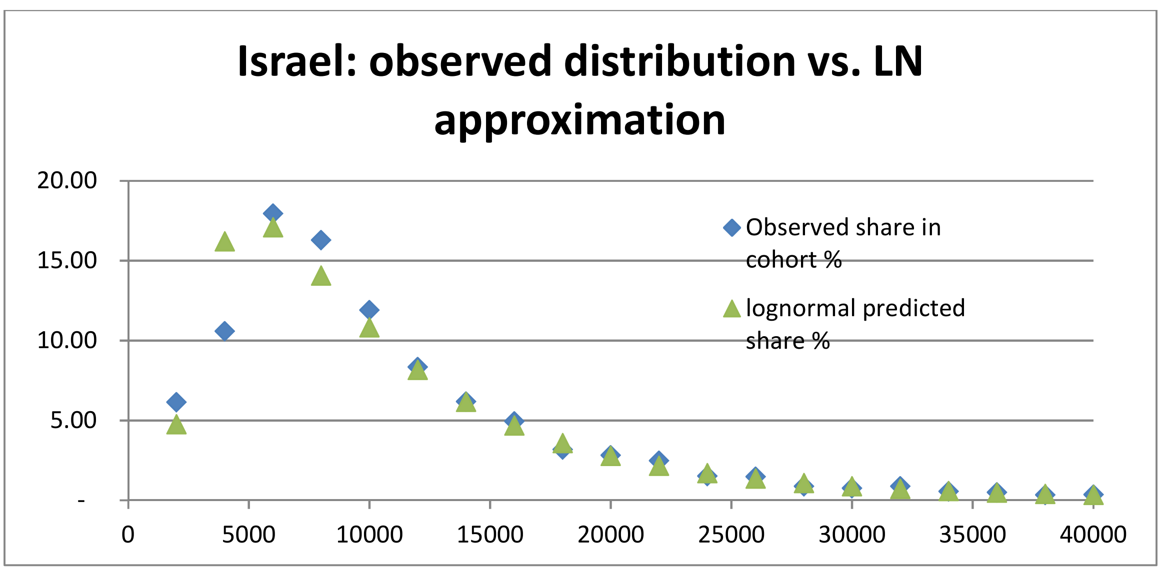
[14] The US is an exception in that respect as sample size stands at over 56,000 observations.
[15] Note that bunching at round values is also the result of the actual tendency of global salaries to be round sums.
[16] While sample data for all countries was obtained from the LIS database, the data for Israel was taken directly from the 2014 expenditures (and incomes) survey. That is in fact the same dataset which is later harmonized and used by LIS, but by using the original dataset we were able to obtain a cleaner and smoother income distribution, and for a later year (2014 instead of 2012).

Figure 8



The discrepancy (between observed and approximated frequencies) that is caused by the bunching at minimum wages, is even more pronounced when the approximation is derived from a simple log-normal distribution with no adjustments for the Pareto tails (Figure 9).

Figure 9



As evident in figures 10a and 10b, In the case of Israel, the PLN distribution was found to be the most suitable to adjust for this issue and derive a better fit. The root mean square error of the PLN approximation (from the observed distribution) was 0.99%, compare to 1.49% for the DPLN, and 1.68% for the log-normal approximation.

It is noteworthy that the superior performance of the PLN relative to the DPLN was also observed in most of the other countries that we examined. Interestingly these results differ from the results by Griffiths and Gholamreza (2012), who observed in 10 countries sample a slightly superior performance of the DPLN relative to the PLN. These differences might stem from the fact that Griffiths and Gholamreza (2012), examined the income distributions of developing countries while we examined mostly developed countries. Another possible explanation for these differences, is the fact that Griffiths and Gholamreza (2012) aimed to minimize the root mean square errors (RMSE) of income shares (s_i), while we, (due to our focus on low income groups), aimed to minimize the RMSE of the relative shares of wage earners (c_i). Indeed, our simulations also show that for the minimization of RMSE of (s_i), the DPLN's performance is close and in some cases better than that of the PLN. Thus, a possible conclusion is that the PLN performs better when the distribution of wages is of interest, and the DPLN (might) perform slightly better when the distribution of income is of interest.

Figure 10a

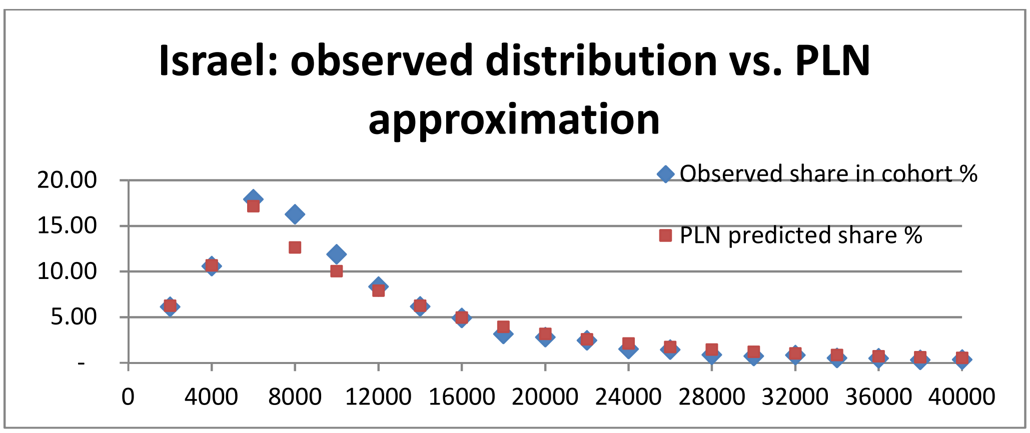
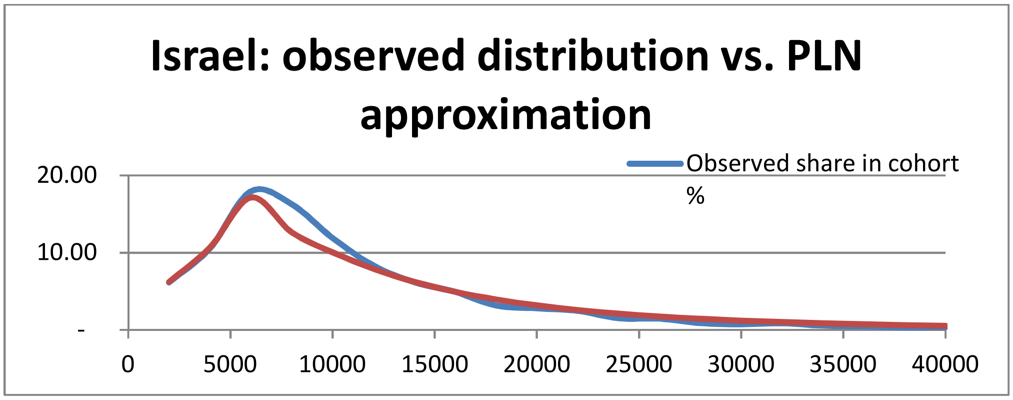


Figure 10b



A further improvement of the fit of the PLN / DPLN approximations for Israel (given the issue of bunching at minimum wages) – can be obtained by using a slightly smaller standard deviation which yields a taller, narrower distribution, which better fits the spike of the observed distribution at minimum wage level. This adjustment was conducted through an iteration process which minimized the Chi-square values of the approximated distributions relative to the observed distribution. The best fitting (log-normalized) adjusted standard deviation (in the case of Israel) was found to be eight percent smaller of the actual (log-normalized) standard deviation that was observed in the sample. This adjustment produced in all cases slightly smaller mean square errors. Thus, the (adjusted) root mean square error of the PLN approximation (from the observed distribution) was 0.97%, compare to 1.41% for the DPLN, and 1.43% for the log-normal approximation.

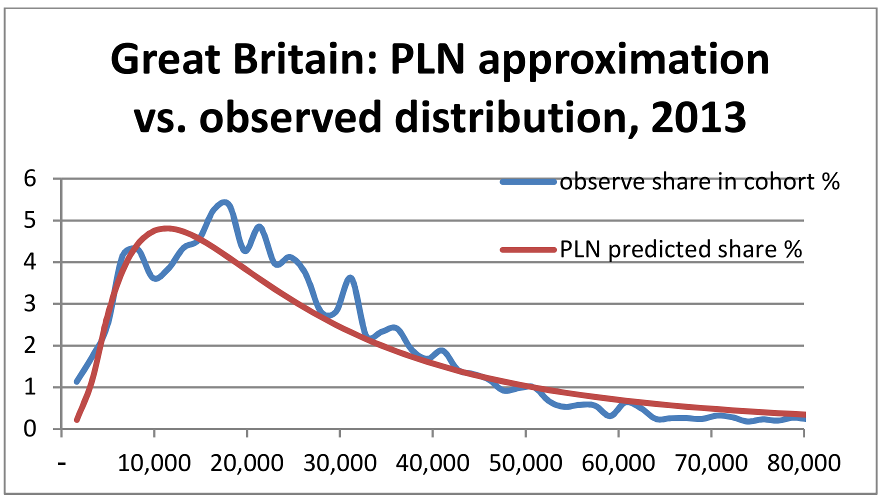
Additional challenges of the observed data

As noted, survey based income data tend to be bunched at round values and at minimum wage level. Furthermore, the quality of the sampling might not always be optimal, due to methodological issues such as inaccurate measurement of income, false reporting and censoring at high values[17]. Clearly, these issues tend to reduce the goodness of fit of the (smooth) approximated distributions to the (noisy) observed distributions. Figure 11 demonstrates this point by comparing the PLN approximated distribution of wage earners, to the sample based

[17] Typically, data for extremely rich individuals are not included in income surveys in order to prevent possible recognition and exposure of sensitive information that concerns these (potentially) high profile individuals.

(N= 17,231) observed distribution of wage earners in Britain 2013. Both distributions are represented by 50 wage intervals (bins) of equal length. As evident, the sample based observed distribution is not very smooth and perhaps a smaller number of bins could improve the fit (with the cost of some loss of information). Furthermore, it appears that for Britain, just like Israel, the issue of bunching at minimum wage level is the one that most reduces the goodness of fit. This last point is an important one because it is a phenomenon that has so far been generally ignored when conducting PLN / DPLN approximations for income distributions. Perhaps, certain adjustments (such as the standard deviation adjustments described above), could further improve the goodness of fit of these approximations in the future.

Figure 11



<u>Measures of goodness of fit</u>

Tables 2a and 2b present key statistics regarding the goodness of fit of the approximated PLN and DPLN distributions (respectively), for the 11 countries that were examined. The tables report for each country, the following statistics: sample year, number of bins sampled; optimal values of α (and β for the DPLN); root of the mean of square errors between the observed and approximated relative share of wage earners (c_i and $\hat{c}_i$)- $\mathrm{RMSE_c} = \sqrt{N^{-1}\sum_{i=1}^{N}[100(\hat{c}_i - c_i)]^2}$; root of the mean of square errors between the observed and approximated income shares (s_i and $\hat{s}_i$)- $\mathrm{RMSE_s} = \sqrt{N^{-1}\sum_{i=1}^{N}[100(\hat{s}_i - s_i)]^2}$; and the Kolmogorov-Smirnov (KS) statistic that is based on the maximum difference between actual and approximated cumulative distribution. For all statistics presented, smaller values signify a better fit. In that respect however, it is important to note that the choice of number of bins has a substantial effect on the size of the statistics. Specifically, the larger the number of bins the smaller are the $\mathrm{RMSE_c}$ and $\mathrm{RMSE_s}$. Thus, the relative goodness of fit of these approximations is only comparable when the number of bins is the same. As evident in Table 2a, for the PLN approximations the best goodness of fit is obtained for the GBR, closely followed by Israel with $\mathrm{RMSE_c}$ values of 0.52 and 0.61 (respectively). As explained above, these results stem not only from the resemblance of the observed income distributions in these countries to a Pareto Log-normal distribution – but also from the better quality of the sample data that was available for those countries. When comparing tables table 2a to 2b, it is evident that for most countries the $\mathrm{RMSE_c}$ of the PLN is smaller (i.e. a better fit) than that of the DPLN. As for the $\mathrm{RMSE_s}$ and KS statistics, one cannot say that the PLN outperforms the DPLN. In about half the countries the $\mathrm{RMSE_s}$ and KS statistics of the PLN are smaller than that of the DPLN, and in the other half the opposite occurs. Using the RMSE criteria we conclude that the PLN is a better approximation and thus we will concentrate on it.

Table 2a

Pareto log-normal approximations compared to observed distributions							
country	**year**	**# of bins**	**RMSE**	**$RMSE_s$**	**K.S. (λ=1)**	**K.S. (λ=1.2)**	**α**
Israel	2014	20	0.99	1.25	0.059*	0.025**	8.6
Israel adjusted (σ * 0.92)	2014	20	0.97	0.90	0.048**	0.020**	9.0
Israel adjusted (σ = 0.75)	2014	20	1.22	0.87	0.033**	0.014**	11.0
Israel	2014	50	0.61	0.55	0.106	0.044**	7.5
Israel adjusted (σ = 0.75)	2014	50	0.49	0.39	0.082**	0.034**	8.0
Israel	2012	50	0.75	0.66	0.117	0.049**	6.7
US	2013	50	0.87	1.06	0.137	0.057*	6.5
GBR	2013	50	0.52	0.60	0.121	0.051*	6.3
Germany	2010	50	0.96	0.88	0.227	0.095*	3.4
Brazil	2013	50	1.46	1.45	0.141	0.059*	8.0
Colombia	2013	50	1.49	1.45	0.204	0.085*	3.3
Uruguay	2013	50	0.80	0.91	0.141	0.059*	5.8
Russia	2013	50	1.66	2.18	0.130	0.054*	8.0
Spain	2013	50	0.83	0.70	0.236	0.098*	2.6
Finland	2013	50	1.31	1.28	0.347	0.145	2.5
Denmark	2010	50	1.46	1.39	0.374	0.156	2.6

* Significant at 10 percent; ** significant at 5 percent.

Table 2b

Double Pareto log-normal approximations compared to observed distributions								
Country	**year**	**# of bins**	**$RMSE_c$**	**$RMSE_s$**	**K.S. (λ=1)**	**K.S. (λ=1.2)**	**α**	**β**
Israel	2014	20	1.49	0.96	0.056*	0.023**	8.6	7.3
Israel adjusted (σ*0.92)	2014	20	1.41	0.62	0.048**	0.020**	9.0	7.6
Israel adjusted (σ=0.75)	2014	20	1.13	0.59	0.030**	0.012**	4.2	6.2
Israel	2014	50	0.73	0.43	0.092*	0.038**	7.3	7.3
Israel adjusted (σ= 0.75)	2014	50	0.52	0.39	0.065*	0.027**	4.6	8.3
Israel	2012	50	0.79	0.66	0.084*	0.035**	4.0	7.4
US	2013	50	0.92	1.08	0.177	0.074*	2.7	6.5
GBR	2013	50	0.56	0.65	0.108	0.045**	3.3	7.5
Germany	2010	50	0.97	0.89	0.238	0.099*	2.2	6.4
Brazil	2013	50	1.56	1.54	0.117	0.049**	4.6	7.1
Colombia	2013	50	1.47	1.44	0.192	0.080*	2.7	7.9
Uruguay	2013	50	0.84	0.91	0.150	0.063*	3.3	7.5
Russia	2013	50	1.67	2.18	0.118	0.049**	4.8	8.4
Spain	2013	50	0.84	0.70	0.249	0.104	1.7	5.1
Finland	2013	50	1.36	1.27	0.352	0.147	1.8	6.5
Denmark	2010	50	1.51	1.39	0.387	0.161	1.8	7.2

4.2 The impact of changes in PLN income distribution on the optimal EITC

As previously shown by (Reed and Wu 2008), Colombi (1990) and Griffiths and Gholamreza (2012), and as also evident in figures 10-11, and from tables 2a and 2b, the PLN approximation fits quite well observed distribution of wage earners. By taking advantage of this good fit, we aim to examine, in a generalized framework, how changes in income distribution affect the optimal tax schedule. For this end, we use adjust the PLN approximation for Israel's distribution of wage earners in 2014, to fit the framework of our simulations, which compute the optimal tax schedule on the guidelines of Saez (2002). This adjustment is quite straightforward. First, we set the number of bins in the PLN approximation, to match the number of income groups in the tax simulations - 8 wage groups (h_1-h_8), not including the non-employed (h_0). We then adjust the pre-tax size of the approximated shares of wage earners ($\hat{c}_i$), to account for the fact that a portion of the population is always not employed[18], by multiplying them by (a theoretical) pre-tax employment rate[19] $(1-h_0)$ so that: $\forall\, i > 0 \;\rightarrow h_i = \hat{c}_i \cdot (1-h_0)$.

Once these adjustments are made, and the approximated sizes of h_1-h_8 & w_1-w_8 are known, we proceed with our simulations and compute the optimal tax schedule on the guidelines of Saez (2002). In the first simulation, we use the benchmark case in which the PLN approximation is based on the observed (log-normalized) mean and standard deviation of the Israeli income distribution in 2014, and the optimal α that was computed by iteration. This benchmark case represents an estimate for the (desired) optimal tax schedule for Israel in 2014.

In the subsequent simulations we generate artificial PLN income distributions by making small changes in the parameters of interest, μ, σ and α (one parameter at a time). Such an exercise is quite instructive as it enables us to determine, in quite general manner, how the optimal tax schedule changes, given, for example, a change (over time in the standard deviation, of a country's income distribution. We believe that this generalized framework can help simplify tax related policy-making, by providing relatively simple and implementable insights regarding

[18] E.g. individuals with disabilities.

[19] In our simulations we use a default, pre-redistribution non-employment rate of $(\mathrm{h}_0 = 5\%)$ - which, in the simulations' post-tax equilibrium, yields plausible non-employment rates (that increase with inequality aversion).

the relationship between the evolution of a country's income distribution and the evolution of its tax schedule over time.

Before performing the simulation let us motivate the discussion. First, Pikety and Saez (2007) showed that one of the striking changes in the income distribution of developed economies is the increase of all types of income (wage, business income, capital income and capital gains), which more than doubled between 1960 and 2000 for the top 0.1 percent of the income distribution. In our analysis the increase of the right tail can be mimicked by an increase in α. Second, the UN (2012) analysis on income inequality shows[20] that in most countries there was an increase in income inequality, which affect directly the wage earners in general and the working poor in particular. This change will be mimicked in our simulations by a change in σ.

We set the benchmark case to be $\alpha_0 = 7$ (instead of 8.6)[21]; with $\sigma_0 = 0.875$ and $\mu_0 = 8.944$, corresponding to the mean and standard-deviation of observed (log-normalized) income distribution in Israel in 2014. Note that this benchmark case closely resembles the observed distribution.

After computing the optimal EITC for this benchmark case, we proceeded by deriving PLN approximations for the following four cases: $\alpha = 0.9 \cdot \alpha_0 = 6.3\,; = 0.8 \cdot \alpha_0 = 5.6\,; \sigma = \sigma_0 \cdot 1.1 = 0.963\,; \sigma = \sigma_0 \cdot 1.2 = 1.050$. In each of these cases, only one parameter of interest was changed at a time, and the other parameters kept their benchmark values. Once we derived PLN approximations for these four (simulated) income distributions, we proceeded by computing for each distribution, the optimal tax schedule – following the guidelines of Saez (2002), as described above. Table 3 presents the optimal tax schedules and additional relevant statistics for the benchmark case and the four simulated distributions. As evident from the simulations results, a reduction in the size of α is different in essence from an increase in σ. The former increases the size of the right tail of the

[20] United Nations (2012), Table 3.2.

[21] The best fitting α for the PLN approximation of Israel's income distribution was quite high (8.6). Thus, a simulated increase in σ, would make the value of $x_1 = \alpha\sigma - \frac{\ln y - m}{\sigma}$ too high, which for low values of y, would make the inverse mills ratio $R(x_1)$ equal to zero – resulting in a discontinuous PLN approximation. The insights of the analysis are not affected by this small adjustment of the benchmark case.

distribution (i.e. the relative share of the rich increases) but slightly decreases the relative share of the left tail (the working poor); while the latter, increases the size of both tails – making the pdf shorter and wider. Furthermore, a 10% change in σ seems to have a much stronger effect on the shape of the distribution than a 10% change in α. As result, a 10% change in α (relative to the benchmark distribution), barely affects the optimal tax schedule at equilibrium and the marginal tax rates are only very slightly lower than the benchmark case. In contrast, a 10% change in σ substantially, increases the marginal tax rate for the high and middle income groups, and to a lesser extent, for the low income groups. Consequently, it increases the total amount of taxes collected. This result is not surprising given the fact that an increase in the σ of the log-normalized distribution, increases the mean income of the actual distribution. It is however a bit surprising that an increase in σ slightly decreases the optimal EITC subsidy, despite the rise in inequality. This result is due to the larger share of the working poor, which raises the aggregate cost of the EITC subsidy.

Table 4 shows the average amount of taxes collected, per worker and per person. This comparison provides a good measure of scale for the effect of the change in α and in σ. While a 20% decrease in α only depresses the sum of collected taxes by 2%, a 20% increase in σ increases the sum of collected taxes by 40.4%.

Table 3: Optimal tax schedules and relevant statistics for simulated PLN distributions, v=0.25

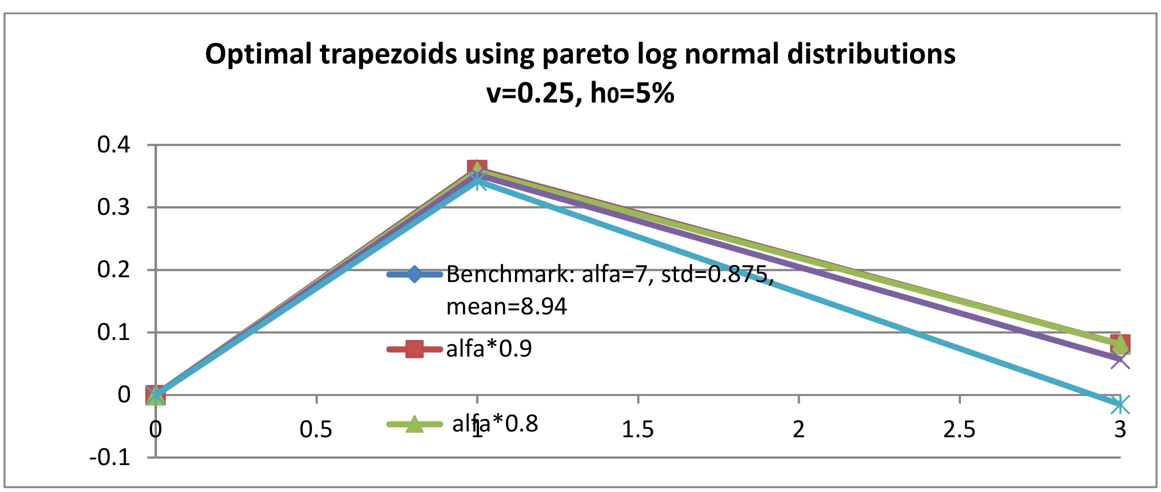
Upper bound of wage interval	0	2000	4000	6000	8000	10000	12000	14000	∞
group size before taxes (hi)	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	5.0%	4.4%	13.2%	13.9%	11.8%	9.5%	7.6%	6.0%	28.6%
$simulation\ 1: \alpha \cdot 0.9$	5.0%	4.3%	13.0%	13.7%	11.7%	9.5%	7.6%	6.0%	29.2%
$simulation\ 2: \alpha \cdot 0.8$	5.0%	4.2%	12.6%	13.5%	11.6%	9.5%	7.6%	6.1%	30.0%
$simulation\ 3: \sigma \cdot 1.1$	5.0%	5.9%	13.9%	13.1%	10.9%	8.7%	6.9%	5.6%	30.1%
$simulation\ 4: \sigma \cdot 1.2$	5.0%	9.6%	12.1%	12.4%	10.0%	8.0%	6.4%	5.2%	31.3%
group size after taxes (h'i)	h'_0	h'_1	h'_2	h'_3	h'_4	h'_5	h'_6	h'_7	h'_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	9.1%	6.0%	13.6%	11.3%	8.3%	9.5%	7.6%	6.0%	28.6%
$simulation\ 1: \alpha \cdot 0.9$	9.1%	5.9%	13.3%	11.2%	8.2%	9.5%	7.6%	6.0%	29.2%
$simulation\ 2: \alpha \cdot 0.8$	9.1%	5.7%	13.0%	11.0%	8.2%	9.5%	7.6%	6.1%	30.0%
$simulation\ 3: \sigma \cdot 1.1$	9.0%	8.0%	14.2%	10.4%	7.2%	8.7%	6.9%	5.6%	30.1%
$simulation\ 4: \sigma \cdot 1.2$	8.8%	12.8%	12.1%	9.2%	6.2%	8.0%	6.4%	5.2%	31.3%
Average wage in group	w_0	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	-	1,000	3,000	5,000	7,000	9,000	11,000	13,000	17,060
$simulation\ 1: \alpha \cdot 0.9$	-	1,000	3,000	5,000	7,000	9,000	11,000	13,000	16,470
$simulation\ 2: \alpha \cdot 0.8$	-	1,000	3,000	5,000	7,000	9,000	11,000	13,000	15,820
$simulation\ 3: \sigma \cdot 1.1$	-	1,000	3,000	5,000	7,000	9,000	11,000	13,000	19,200
$simulation\ 4: \sigma \cdot 1.2$	-	1,000	3,000	5,000	7,000	9,000	11,000	13,000	23,600
Marginal tax rate in group	Mt_0	Mt_1	Mt_2	Mt_3	Mt_4	Mt_5	Mt_6	Mt_7	Mt_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	-	-36.1%	13.9%	49.7%	59.0%	72.3%	75.0%	77.3%	39.0%
$simulation\ 1: \alpha \cdot 0.9$	-	-36.0%	13.9%	49.6%	58.8%	72.0%	74.6%	76.9%	38.0%
$simulation\ 2: \alpha \cdot 0.8$	-	-35.8%	13.9%	49.5%	58.5%	71.8%	74.3%	76.4%	36.7%
$simulation\ 3: \sigma \cdot 1.1$	-	-35.2%	14.7%	54.9%	64.4%	75.4%	78.3%	80.7%	41.8%
$simulation\ 4: \sigma \cdot 1.2$	-	-34.2%	17.8%	62.9%	71.6%	79.3%	82.2%	84.5%	46.4%
Average tax rate in group	At_0	At_1	At_2	At_3	At_4	At_5	At_6	At_7	At_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	-	-36.1%	-2.7%	18.3%	29.9%	39.3%	45.8%	50.6%	47.9%
$simulation\ 1: \alpha \cdot 0.9$	-	-36.0%	-2.7%	18.2%	29.8%	39.2%	45.6%	50.4%	47.8%
$simulation\ 2: \alpha \cdot 0.8$	-	-35.8%	-2.7%	18.2%	29.7%	39.1%	45.5%	50.2%	47.8%
$simulation\ 3: \sigma \cdot 1.1$	-	-35.2%	-1.9%	20.8%	33.3%	42.6%	49.1%	54.0%	50.1%
$simulation\ 4: \sigma \cdot 1.2$	-	-34.2%	0.5%	25.4%	38.6%	47.7%	53.9%	58.6%	53.2%
Tax payed by average group member	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	-	(364)	(82)	904	1,936	3,190	4,460	5,750	7,190
$simulation\ 1: \alpha \cdot 0.9$	-	(367)	(81)	903	1,931	3,182	4,447	5,731	6,934
$simulation\ 2: \alpha \cdot 0.8$	-	(366)	(80)	902	1,926	3,172	4,433	5,709	6,659
$simulation\ 3: \sigma \cdot 1.1$	-	(358)	(58)	1,029	2,135	3,429	4,740	6,072	8,410
$simulation\ 4: \sigma \cdot 1.2$	-	(344)	15	1,255	2,442	3,778	5,132	6,505	10,876
Average after-tax income from work	c_0	c'_1	c'_2	c'_3	c'_4	c'_5	c'_6	c'_7	c'_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	-	1,382	3,086	4,050	4,541	4,926	5,281	5,605	7,827
$simulation\ 1: \alpha \cdot 0.9$	-	1,385	3,085	4,051	4,547	4,936	5,298	5,630	7,567
$simulation\ 2: \alpha \cdot 0.8$	-	1,384	3,084	4,053	4,554	4,949	5,317	5,659	7,270
$simulation\ 3: \sigma \cdot 1.1$	-	1,376	3,061	3,919	4,283	4,612	4,909	5,174	8,387
$simulation\ 4: \sigma \cdot 1.2$	-	1,361	2,985	3,681	3,882	4,150	4,385	4,589	9,589

Table 4: Average tax per person & per worker for simulated PLN distributions (given optimal EITC)

	average T per worker	average T per person	% change in T per worker	% change in T per person
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	3,599	3,271		
$simulation\ 1: \alpha \cdot 0.9$	3,562	3,238	-1.0%	-1.0%
$simulation\ 2: \alpha \cdot 0.8$	3,525	3,205	-2.1%	-2.0%
$simulation\ 3: \sigma \cdot 1.1$	4,086	3,719	13.5%	13.7%
$simulation\ 3: \sigma \cdot 1.2$	5,038	4,594	40.0%	40.4%

Figure 12 demonstrates (for v=0.25, $h_0 = 5\%$) the change in the shape of the EITC trapezoid, relative to the benchmark, given (simulated) changes in α and in σ. As evident, the size of the EITC subsidy granted to the lowest income group (w_1), changes only very slightly given a change in α or in σ. However, the phasing-out stage becomes visibly steeper relative to the benchmark, after an increase in σ. Thus one can conclude that the EITC subsidy is quite robust to changes in the relative shares of income groups, but the phasing-out stage is more sensitive to such changes.

Figure 12



5. Simulated increase in minimum wages

In order to examine how the EITC schedule is affected by a rise in minimum wages, we take again as a benchmark, the case shown above, where $\alpha_0 = 7$, $\sigma_0 = 0.875$ and $\mu_0 = 8.944$. Again, the level of inequality aversion $v = 0.25$, and the pre-tax share of the non-employed was $h_0 = 5\%$. We then simulate two simple scenarios, in which minimum wage has risen, and compute the (new) optimal tax schedules. In the first scenario we mimic the rise in minimum wage by transferring 5 percentage points from h_2 to h_3. This change corresponds to a shift of workers from the second wage group to the third, following a rise in minimum wage, that brought their wages above the upper (wage) bound of the second group. In the second scenario we simply increase the average wage of the 2nd wage group by 1000 NIS (from 3000 to 4000). This change corresponds to a rise in minimum wage, such that the new wage does not surpass the upper (wage) bound of the second group.

Table 5 presents the optimal tax schedules and additional relevant statistics for the benchmark case and the 2 simulated scenarios. As evident, in both simulated scenarios of minimum wage rise, the optimal EITC subsidy is slightly higher than the benchmark case subsidy (36.1%). The highest subsidy (38%) is obtained in the first scenario (where there is a shift of workers from h_2 to h_3) – compared to 37% in the second scenario (where w_2 rises by 1000 NIS). However, the more substantial differences between these 3 cases are observed in the 2nd and 3rd wage groups. In both scenarios the marginal tax rate of the 2nd wage group rises relative to the benchmark case of 13.9%. Given a shift of workers from h_2 to h_3 the marginal tax rate of the 2nd group actually rises (to 16.4%), which can be a bit counter intuitive, but can be explained by the fact that the h_2 has shrunk – which makes its relative importance for the central planner smaller. More intuitive is the fact that a 1000 NIS rise in w_2 results in a rise in the marginal tax rate of the 2nd group (to 18.1%). This group now earns more and therefore more heavily taxed. But the most interesting differences between these scenarios are manifested in the 3rd wage group's marginal tax rates. While in the benchmark case the marginal tax rate for the 3rd group is 49.7%, the two simulated cases yield opposite results. A shift of workers from h_2 to h_3 results in a decline of the 3rd groups marginal tax to 40.1%; while a 1000 NIS rise in w_2 raises the 3rd group's marginal tax rate to 61%! The former result is quite intuitive: there are less working poor (h_2), and more medium-wage individuals (h_3), and therefore the (effective) tax base is broader and a smaller marginal rate can be imposed. The latter result however, requires a more complex explanation. As noted, the (significant) rise in w_2 has allowed for higher marginal taxation, but to a limited extent – as a higher marginal tax rate to this group would make the phasing-out stage too steep and creating a distortion to work incentives. At the same time, the wage gap between the 2nd and 3rd groups has been cut by half – and consequently so has the tax bracket of the 3rd group. Thus, the 3rd group's marginal tax is now higher (61%), but it is subject to this marginal rate only for a bracket of 1,000 NIS (from 4,000 to 5,000 NIS), compared to a bracket of 2,000 NIS before the rise in minimum wage. Consequently, the 3rd group's average tax rate is actually lower than in the benchmark case – which makes sense as the 2nd group now bears a bigger share of the tax burden.

Figure 12 presents the optimal EITC triangles for the benchmark case and the two scenarios described above (which simulate a rise in minimum wages). As evident, the size of the EITC subsidy is modestly affected by a rise in minimum wage but the phasing out stage becomes significantly steeper – especially for the case where w_2 rises by 1000 NIS.

Figure 12

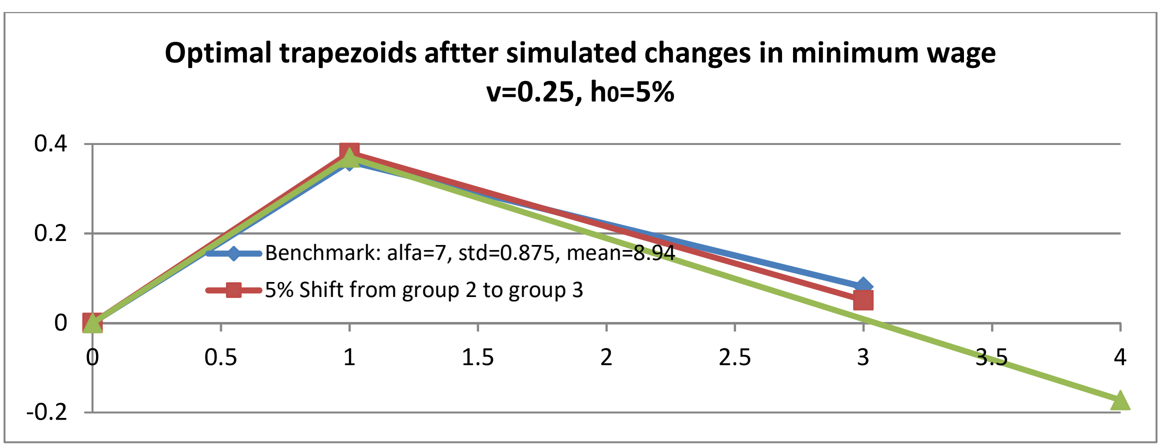


Table 5: Optimal tax schedules and relevant statistics for simulated rise in minimum wage, v=0.25

Upper bound of wage interval	0	2000	4000	6000	8000	10000	12000	14000	∞
group size before taxes (hi)	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	5.0%	4.4%	13.2%	13.9%	11.8%	9.5%	7.6%	6.0%	28.6%
$scenario\ 1: 5pp\ shift\ from\ h_2\ to\ h_3$	5.0%	4.4%	8.2%	18.9%	11.8%	9.5%	7.6%	6.0%	28.6%
$scenario\ 2: 1000\ NIS\ rise\ in\ w_2$	5.0%	4.4%	13.2%	13.9%	11.8%	9.5%	7.6%	6.0%	28.6%
group size after taxes (h'i)	h'_0	h'_1	h'_2	h'_3	h'_4	h'_5	h'_6	h'_7	h'_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	9.1%	6.0%	13.6%	11.3%	8.3%	9.5%	7.6%	6.0%	28.6%
$scenario\ 1: 5pp\ shift\ from\ h_2\ to\ h_3$	9.3%	6.1%	8.4%	16.0%	8.5%	9.5%	7.6%	6.0%	28.6%
$scenario\ 2: 1000\ NIS\ rise\ in\ w_2$	9.4%	6.1%	12.7%	11.7%	8.5%	9.5%	7.6%	6.0%	28.6%
Average wage in group	w_0	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	-	1,000	3,000	5,000	7,000	9,000	11,000	13,000	17,060
$scenario\ 1: 5pp\ shift\ from\ h_2\ to\ h_3$	-	1,000	3,000	5,000	7,000	9,000	11,000	13,000	17,060
$scenario\ 2: 1000\ NIS\ rise\ in\ w_2$	-	1,000	4,000	5,000	7,000	9,000	11,000	13,000	17,060
Marginal tax rate in group	Mt_0	Mt_1	Mt_2	Mt_3	Mt_4	Mt_5	Mt_6	Mt_7	Mt_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	-	-36.1%	13.9%	49.7%	59.0%	72.3%	75.0%	77.3%	39.0%
$scenario\ 1: 5pp\ shift\ from\ h_2\ to\ h_3$	-	-38.0%	16.4%	40.1%	59.8%	72.0%	74.7%	77.1%	38.7%
$scenario\ 2: 1000\ NIS\ rise\ in\ w_2$	-	-37.0%	18.1%	61.0%	58.5%	71.6%	74.4%	76.8%	38.4%
Average tax rate in group	At_0	At_1	At_2	At_3	At_4	At_5	At_6	At_7	At_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	-	-36.1%	-2.7%	18.3%	29.9%	39.3%	45.8%	50.6%	47.9%
$scenario\ 1: 5pp\ shift\ from\ h_2\ to\ h_3$	-	-38.0%	-1.7%	15.0%	27.8%	37.6%	44.4%	49.4%	46.9%
$scenario\ 2: 1000\ NIS\ rise\ in\ w_2$	-	-37.0%	4.3%	15.7%	27.9%	37.6%	44.3%	49.3%	46.7%
Tax payed by average group member	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	-	(364)	(82)	904	1,936	3,190	4,460	5,750	7,190
$scenario\ 1: 5pp\ shift\ from\ h_2\ to\ h_3$	-	(386)	(51)	745	1,812	3,069	4,341	5,631	7,061
$scenario\ 2: 1000\ NIS\ rise\ in\ w_2$	-	(376)	172	777	1,817	3,068	4,334	5,620	7,040
Average After tax income from work	c_0	c'_1	c'_2	c'_3	c'_4	c'_5	c'_6	c'_7	c'_8
$Benchmark: \alpha = 7, \sigma = 0.875, \mu = 8.94$	-	1,382	3,086	4,050	4,541	4,926	5,281	5,605	7,827
$scenario\ 1: 5pp\ shift\ from\ h_2\ to\ h_3$	-	1,404	3,053	4,217	4,701	5,083	5,438	5,762	7,999
$scenario\ 2: 1000\ NIS\ rise\ in\ w_2$	-	1,395	3,819	4,184	4,694	5,086	5,447	5,777	8,027

6. Summary and conclusions

While the EITC is becoming a leading program for coping with poverty, its optimal design is still a subject that merits further analysis. In this paper we build a model for calculating the optimal linear EITC and run simulations for obtaining the optimal piecewise linear schedule. We prove that in the linear model and in the presence of unemployment, an increase in inequality aversion reduces the optimal EITC. Our simulations show that the optimal piecewise linear schedule is a triangle, instead of a trapezoid, as actually implemented in countries that adopted the EITC like the U.S. or Israel. Moreover, implementing a trapezoid is subject to a social welfare loss.

An interesting extension is simulating the optimal EITC taking into account a realistic distribution of wages. By using data for several countries we show that the Pareto-lognormal distribution is a good approximation for the density of wages. By using Root Mean Squared Errors and Kolmogorov-Smirnov statistics we show that this distribution fits real data better than other distributions, like the double Pareto-lognormal distribution. By allowing changes in the parameters of the Pareto-lognormal distribution we obtained that changes in the share of the "very rich" have a lower impact on the optimal EITC than changes in the variance that affect the share of the working poor.

Finally we check the impact of an increase in the minimum wage on the optimal EITC. Our simulations show that while the size of the EITC subsidy is modestly affected by a rise in minimum wage, the phasing out stage becomes significantly steeper.

APPENDIX 1

Recall Saez's (2002) general equilibrium formula:

$$\frac{T_i - T_{i-1}}{C_i - C_{i-1}} = \frac{1}{\zeta_i h_i} \cdot \sum_{j=i}^{I} h_j \left[1 - g_i - \eta_j \frac{T_i - T_0}{C_i - C_0}\right]$$

When $\eta_i > 0 \; \forall \; i < 5$, and $\eta_i = 0 \; \forall \; i > 4$, we receive the following set of equations:

$$\frac{T_1-T_0}{C_1-C_0}=\frac{1}{h_1\zeta_1}\cdot\left[(g_0-1)h_0-h_1\eta_1\cdot\frac{T_1-T_0}{C_1-C_0}-h_2\eta_2\cdot\frac{T_2-T_0}{C_2-C_0}-h_3\eta_3\cdot\frac{T_3-T_0}{C_3-C_0}-h_4\eta_4\cdot\frac{T_4-T_0}{C_4-C_0}\right]$$

$$\frac{T_2-T_1}{C_2-C_1}=\frac{1}{h_2\zeta_2}\cdot\left[(g_0-1)h_0+(g_1-1)h_1-h_2\eta_2\cdot\frac{T_2-T_0}{C_2-C_0}-h_3\eta_3\cdot\frac{T_3-T_0}{C_3-C_0}-h_4\eta_4\cdot\frac{T_4-T_0}{C_4-C_0}\right]$$

$$\frac{T_3-T_2}{C_3-C_2}=\frac{1}{h_3\zeta_3}\cdot\left[(g_0-1)h_0+(g_1-1)h_1+(g_2-1)h_2-h_3\eta_3\cdot\frac{T_3-T_0}{C_3-C_0}-h_4\eta_4\cdot\frac{T_4-T_0}{C_4-C_0}\right]$$

$$\frac{T_4-T_3}{C_4-C_3}=\frac{1}{h_4\zeta_4}\cdot\left[(g_0-1)h_0+(g_1-1)h_1+(g_2-1)h_2+(g_3-1)h_3-h_3\eta_3\cdot\frac{T_3-T_0}{C_3-C_0}-h_4\eta_4\cdot\frac{T_4-T_0}{C_4-C_0}\right]$$

To solve this set of equations we move from cumulative tax to average tax and rewrite the above equations:

$$t_1=1-\frac{h_1(\zeta_1+\eta_1)}{(g_0-1)h_0-h_2\eta_2\cdot\frac{t_2}{1-t_2}-h_3\eta_3\cdot\frac{t_3}{1-t_3}-h_4\eta_4\cdot\frac{t_4}{1-t_4}+h_1(\zeta_1+\eta_1)}$$

$$t_1=\frac{w_2t_2}{w_1}-\frac{(w_2-w_1)}{w_1}\left[1-\frac{h_2\zeta_2}{(g_0-1)h_0+(g_1-1)h_1-h_2\eta_2\cdot\frac{t_2}{1-t_2}-h_3\eta_3\cdot\frac{t_3}{1-t_3}-h_4\eta_4\cdot\frac{t_4}{1-t_4}+h_2\zeta_2}\right]$$

$$t_2=\frac{w_3t_3}{w_2}-\frac{(w_3-w_2)}{w_2}\left[1-\frac{h_3\zeta_3}{(g_0-1)h_0+(g_1-1)h_1+(g_2-1)h_2-h_3\eta_3\cdot\frac{t_3}{1-t_3}-h_4\eta_4\cdot\frac{t_4}{1-t_4}+h_3\zeta_3}\right]$$

$$t_3=\frac{w_4}{w_3}t_4-\frac{(w_4-w_3)}{w_3}\left[1-\frac{h_4\zeta_4}{(g_0-1)h_0+(g_1-1)h_1+(g_2-1)h_2+(g_3-1)h_3-h_4\eta_4\cdot\frac{t_4}{1-t_4}+h_4\zeta_4}\right]$$

We then proceed to solve this set of equations by scanning through all possible values of t_4 (between 0 and 1), until a solution that satisfies all first order conditions is obtained. Note that, while this set of equations has multiple solutions there is always only one solution that satisfies the following conditions: 1) the marginal tax rate for each wage group must be smaller than 1; 2) the marginal tax rate must be increasing with wages.

Solving the system with endogenous social weights g_i and group size h_i using iteration

The solution for the set of equations initially takes the social weights g_i and group size h_i as exogenous. However, as Saez (2002) explains: "it is important to note that the social weights g_i are not exogenous parameters but depend on the tax schedule $(c_0,\dots,c_I)$ that is currently implemented, For example, if after-tax incomes are equalized across occupations, then there is no reason to desire further redistribution at the margin, and the marginal weights should no longer be decreasing with i". To simplify the computations Saez (2002) takes the function $g(\cdot)$ as exogenous, but also states that: "the individual weights of the classical approach can always be chosen such that the resulting g_i's match the desired marginal social welfare function $g(\cdot)$". That is to say, that a tax schedule can be found such that the post-redistribution consumption levels yield social weights that perfectly correspond to the implemented tax schedule. Similarly, each group's size h_i is also endogenous and depends on the implemented tax schedule. Thus, a tax schedule can be found such that the post-redistribution consumption levels yield a set of g_i's and group size h_i's, that (together) perfectly correspond to the implemented tax schedule. In our simulations we succeed in computing an optimal tax schedule which takes into account the endogeneity of the social weights g_i and group sizes h_i, by applying the following iterative sequence:

1. The tax schedule is initially computed using the (exogenous) social weights and group sizes that correspond to pre-tax consumption levels. I.e. – the social weights and group sizes that persisted prior to redistribution via taxes and transfers.
2. Using the tax schedule that was computed in (1), new social weights and group sizes are computed.
3. Those newly computed values of g_i and h_i are then used to compute a new tax schedule.
4. The new tax schedule is then used to compute new values of g_i and h_i, and so forth.
5. This iterative sequence is repeated until the tax schedule in iteration n perfectly matches the tax schedule in iteration n-1. The resulting values of g_i and h_i that are obtained upon convergence are the correct social weights and group sizes of the after-tax steady state equilibrium.

APPENDIX 2

The following are the details of the simulation presented in Figure 7.

Table A.2 – A Triangle vs. a Trapezoid

Average wage in group	W0	W1	W2	W3	W4	W5	W6	W7	W8
Triangle	0	1000	3000	5000	7000	9000	11000	13000	17060
Trapezoid	0	1000	3000	5000	7000	9000	11000	13000	17060
Consumption in group	**C0**	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C8**
Triangle	3743	4956	5678	5808	5836	6065	6262	6434	7769
Trapezoid	1711	2924	4924	6711	6961	7186	7381	7553	8900
Average tax rate in group	**At0**	**At1**	**At2**	**At3**	**At4**	**At5**	**At6**	**At7**	**At8**
Triangle	0.0%	-21.3%	35.5%	58.7%	70.1%	74.2%	77.1%	79.3%	76.4%
Trapezoid	0.0%	-21.3%	-7.1%	0.0%	27.2%	40.9%	49.9%	56.2%	58.8%
Marginal tax rate in group	**Mt0**	**Mt1**	**Mt2**	**Mt3**	**Mt4**	**Mt5**	**Mt6**	**Mt7**	**Mt8**
Triangle	0%	-39%	35%	75%	95%	82%	85%	86%	54%
Trapezoid	0%	-21%	0%	11%	95%	89%	90%	91%	67%
Groups' social weights	**g0**	**g1**	**g2**	**g3**	**g4**	**g5**	**g6**	**g7**	**g8**
Triangle	1.80	1.33	0.85	0.81	0.80	0.86	0.80	0.75	0.50
Trapezoid	1.80	1.33	0.85	0.81	0.80	0.86	0.80	0.75	0.50
group size after taxes (h'i)	**h'0**	**h'1**	**h'2**	**h'3**	**h'4**	**h'5**	**h'6**	**h'7**	**h'8**
Triangle	25.2%	5.4%	8.5%	5.7%	3.5%	9.5%	7.6%	6.0%	28.6%
Trapezoid	11.4%	5.4%	14.2%	13.9%	3.5%	9.5%	7.6%	6.0%	28.6%
Group's contribution to social welfare	**SW0**	**SW1**	**SW2**	**SW3**	**SW4**	**SW5**	**SW6**	**SW7**	**SW8**
Triangle	1694	353	413	269	165	495	377	290	1102
Trapezoid	368	215	604	761	195	581	441	337	1253

Note that the increase in welfare arrives mainly from re-distribution sine a triangle allows for higher resources to be delivered to the unemployed.

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